Laboratory Rayleigh-Benard convection

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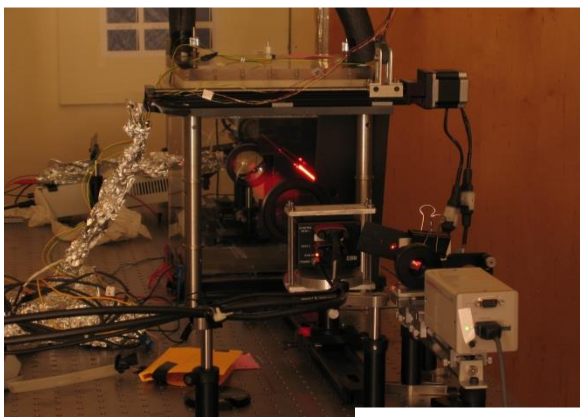
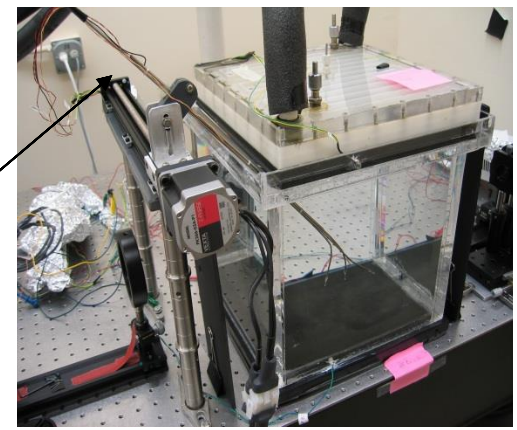
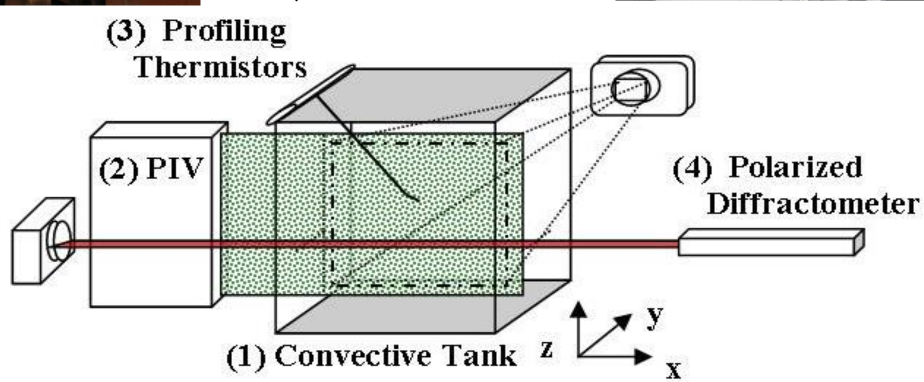
Experimental setup

- Rayleigh-Benard convection cell 0.3 m x0.3 m x0.3 m
- Fast thermistor-FP07,
- Particle Image Velocimetry (PIV),
- Linear polarization measurement system,
- *Shack-Hartmann wavefront sensor*-to optically measure temperature spectra.







Rayleigh number

- *The Rayleigh number is a non-dimensional, describing the strength of convective turbulence of a flow, given by the ratio of the destabilizing buoyancy force to the stabilizing viscous force i.e.:*

$$Ra = \frac{\alpha_T g d^3 \Delta T}{\nu D_T}$$

$\boldsymbol{\alpha_T}$ is the coefficient of thermal expansion of water at 20 ºC ($\boldsymbol{\alpha_T}$ = 2.1*10^{-4} ºC^{-1}), ***g*** the acceleration due to gravity (9.8 ms^{-2}), ***d*** the depth of the tank (**0.27 m**), ***ΔT*** the temperature difference across the tank, ν the kinematic viscosity of water at 20 ºC (**n = 1.0*10^{-6} m^{2}s^{-1}**), and D_T the thermal diffusivity of water at 20 ºC (**D_T = 1.42*10^{-7} m^{2}s^{-1}**).

- *Flow turbulent when* $Ra > 10^5$

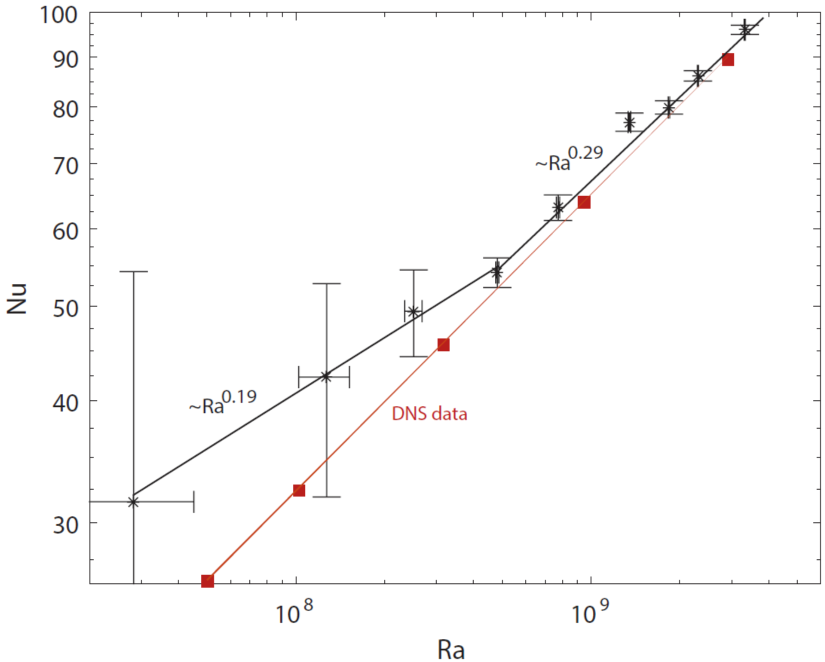
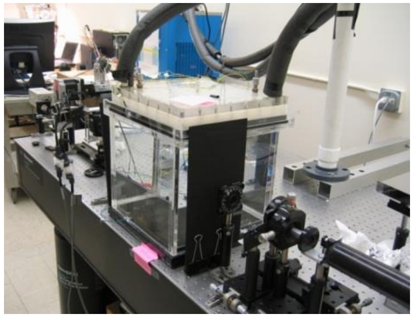


Figure 2. The Nusselt Number versus the Rayleigh Number in our experiment. At $Ra > 5 \times 10^8$ the Nusselt number $Nu \propto Ra^{0.29}$ in line with [28] results of $Nu \propto Ra^{0.297}$. For low Nu i.e.: at $Ra < 5 \times 10^8$ $Nu \propto Ra^{0.19}$ - likely an artifact of the heat loss through the sidewalls. **The error bar at each Ra signifies the standard deviation for Nu obtained from all measurements at that Ra value. The Nu values were obtained indirectly from a time signal of a heater power as an estimate of the heat flux and a time signal of the measured temperature difference between the plates.** Red line and squares - comparison with the DNS simulations of [29, 30] ($\Gamma = 1, Pr = 4.38$).



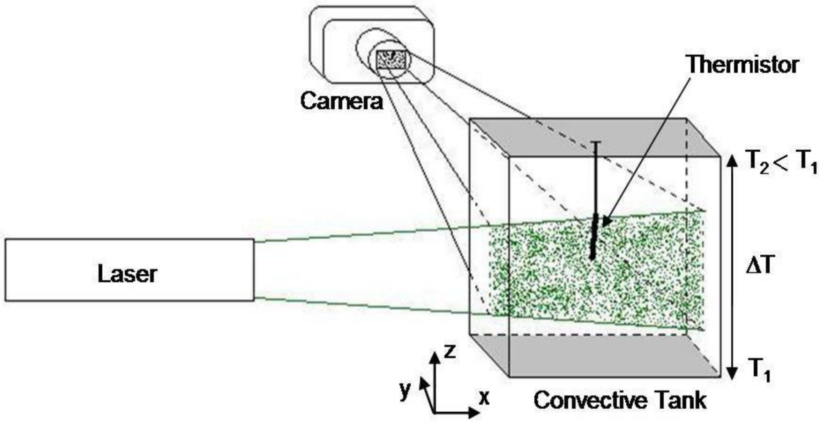
Turbulent Kinetic Energy Dissipation Rate, ε

- Dissipation rate (m^2/s^3):

$$\varepsilon = 2\nu\overline{S_{ij}S_{ij}}$$

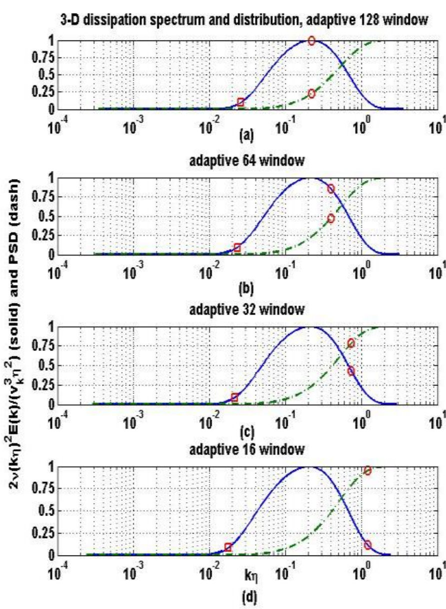
- ν - kinematic viscosity (m^2/s)
- S_{ij}-strain rate tensor:

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$



- From 2D PIV assume isotropy, PIV window size and then:

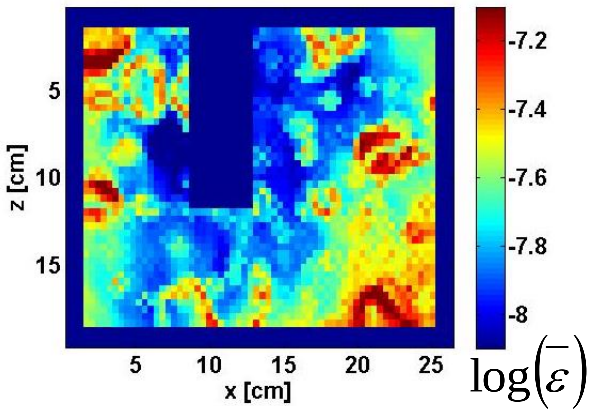
$$\varepsilon = 3\nu\left\{\frac{4}{3}\left[\overline{\left(\frac{\partial u}{\partial x}\right)^2} + \overline{\left(\frac{\partial w}{\partial z}\right)^2} + \overline{\left(\frac{\partial u}{\partial x}\frac{\partial w}{\partial z}\right)}\right] + \overline{\left(\frac{\partial u}{\partial z}\right)^2} + \overline{\left(\frac{\partial w}{\partial x}\right)^2} + 2\overline{\left(\frac{\partial u}{\partial z}\frac{\partial w}{\partial x}\right)}\right\}$$



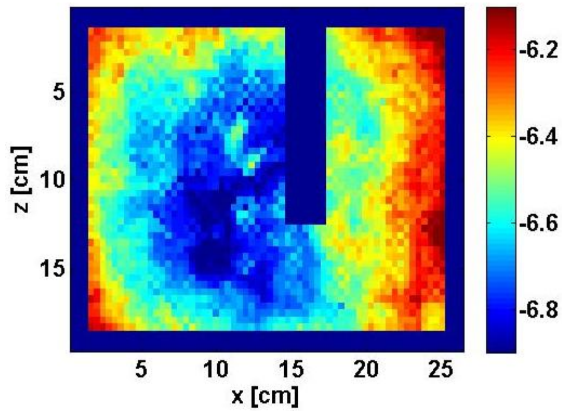
noise level

Energy dissipation rates -results

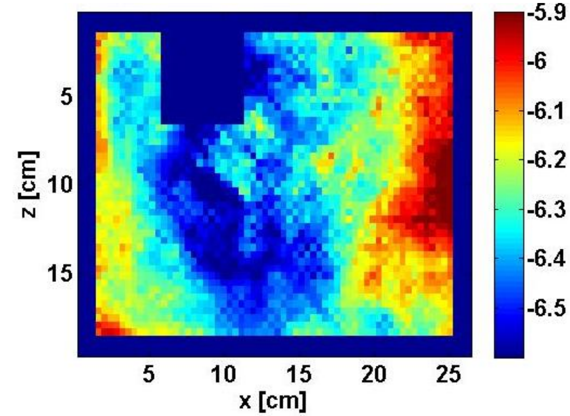
Ra = $2.6*10^8$



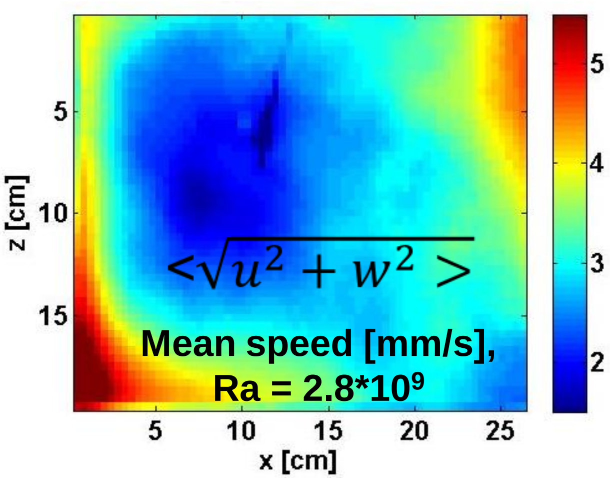
Ra = $2.2*10^9$

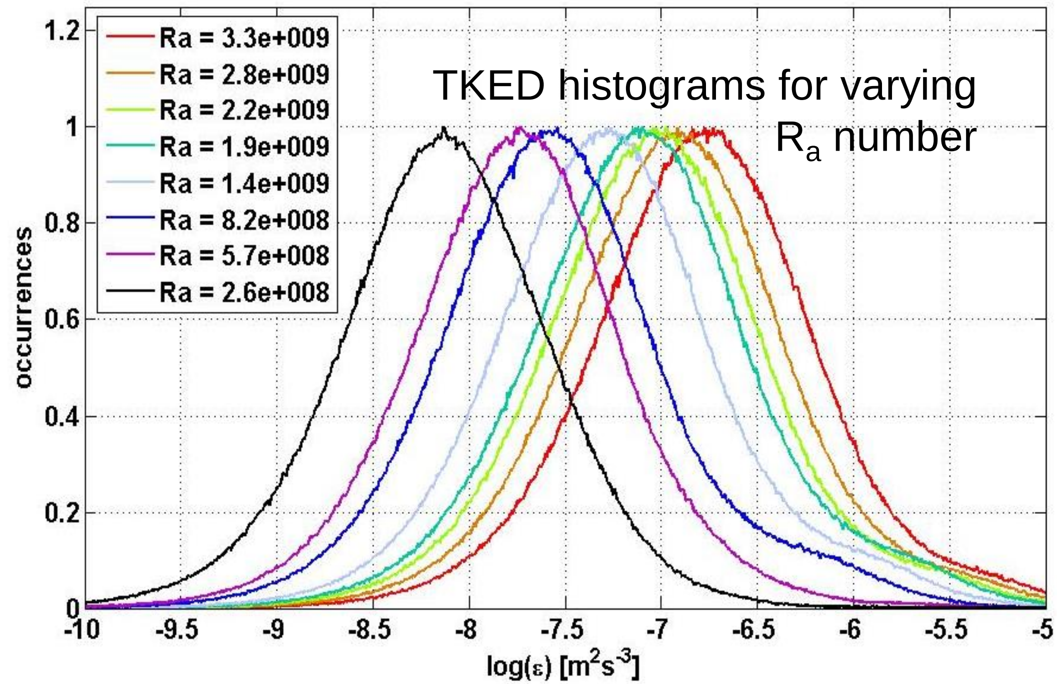


Ra = $3.3*10^9$



Note: increasing energetic layer near side walls



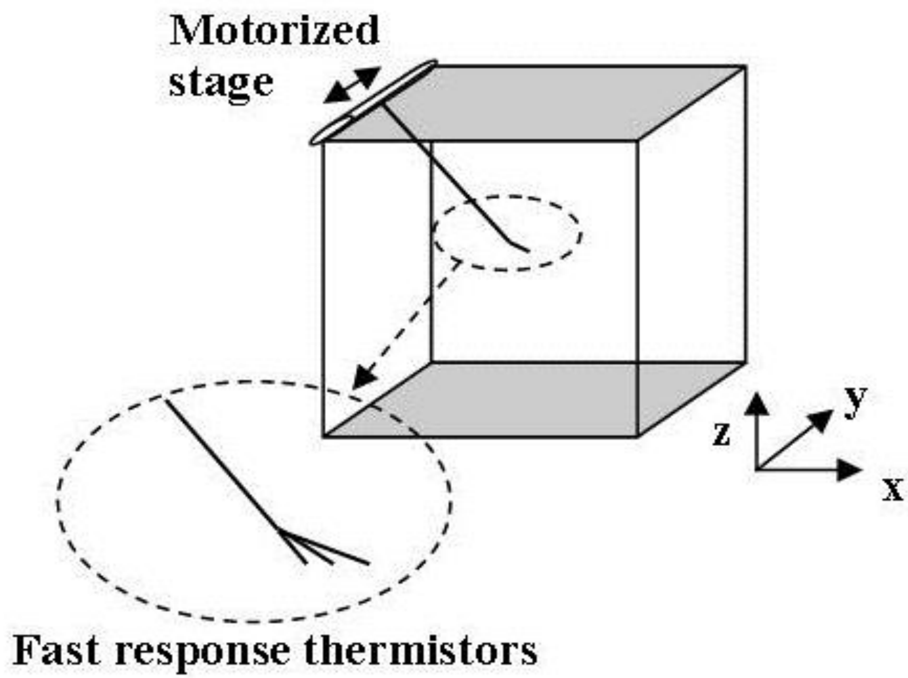


Temperature dissipation rates

- For homogeneous and isotropic turbulence, the temperature dissipation rate χ may be expressed as:

$$\chi = 6D_T \overline{\left(\frac{\partial T'}{\partial x}\right)^2} =$$

$$= 2D_T \int_0^{\infty} k^2 E_T(k)\,dk$$



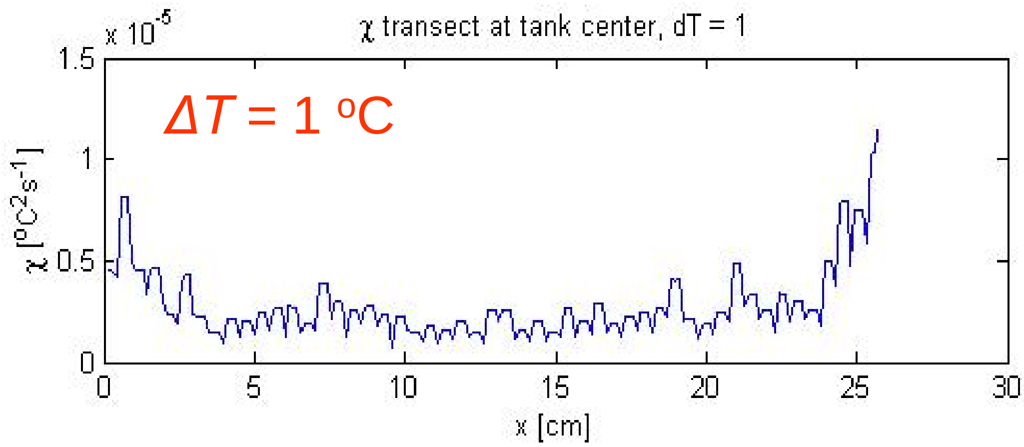
Temperature dissipation rate

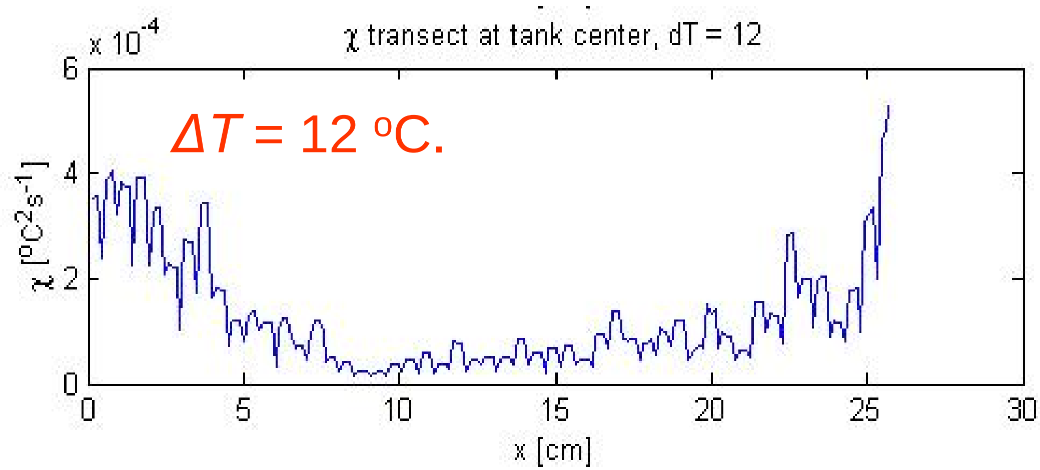
- χ horizontal variability calculated as:

$$\chi = 6D_T \overline{\left(\frac{\partial T'}{\partial x}\right)^2}$$

i.e. from horizontal transects of temperature fluctuations.

Note: the change in vertical scale between (a) and (b).

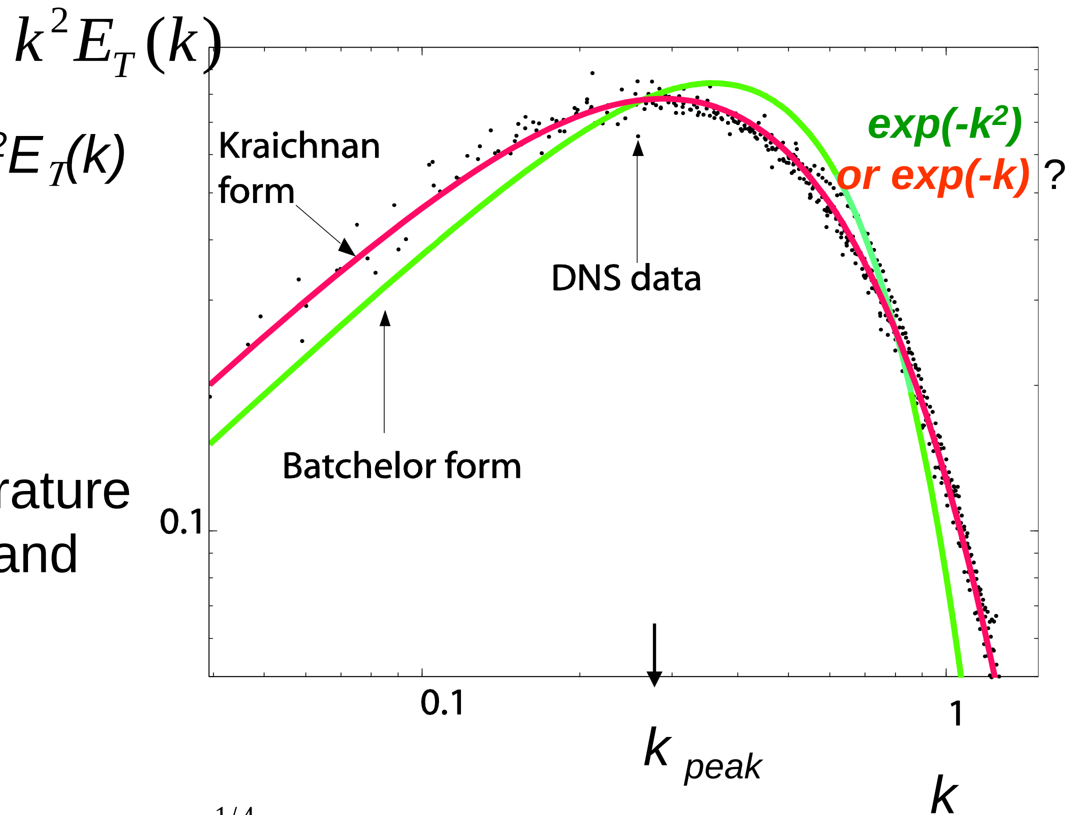




Temperature dissipation spectra, energy dissipation rate

- Dissipation spectra: $k^2 E_T(k)$

$$\chi = 2D_T \int_0^\infty k^2 E_T(k)\,dk$$



- Dissipation range temperature spectra are flow invariant and depend only on ε and χ

- Energy dissipation rate can be found from the temperature dissipation spectra as:

$$k_{peak}\left(\frac{\nu\kappa^2}{\varepsilon}\right)^{1/4} = k_{univ} \approx 0.29$$

Bogucki et. al. JFM 1996

Kraichnan vs. Batchelor spectral form

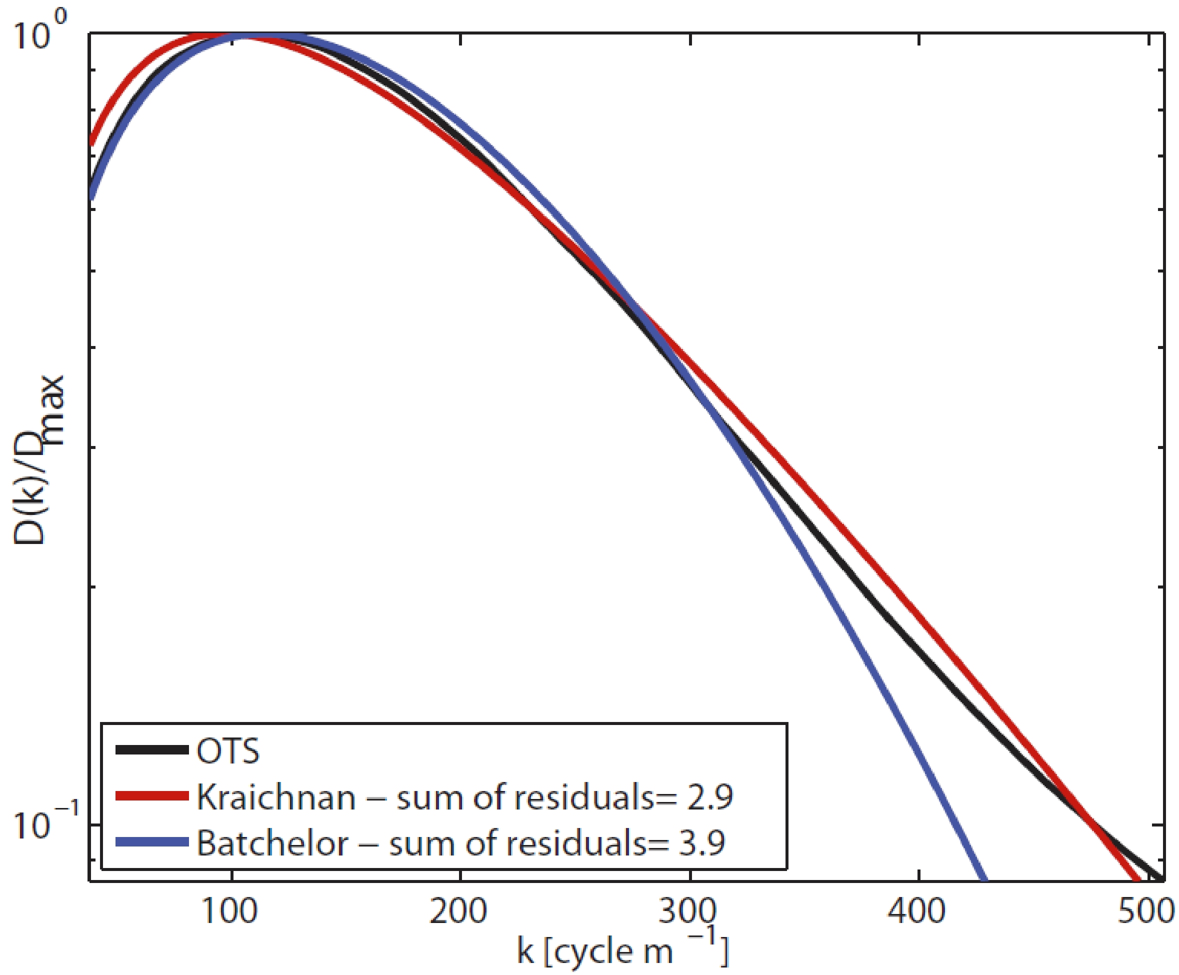


Figure 6. The least square fit of K- spectrum and B-spectrum to optically measured temperature (black) one dimensional dissipation spectra $D(k)$, normalized by its maximum value $E_{1\theta}(k)k^2/(E_{1\theta}(k)k^2)_{k=k_{max}}$. The data were collected at $Ra = 5 \times 10^9$. Note that at large wave numbers the dissipation spectrum follows $\propto exp(-k)$. The sum of residuals (difference between an observed value and the model value) describes the fit accuracy.

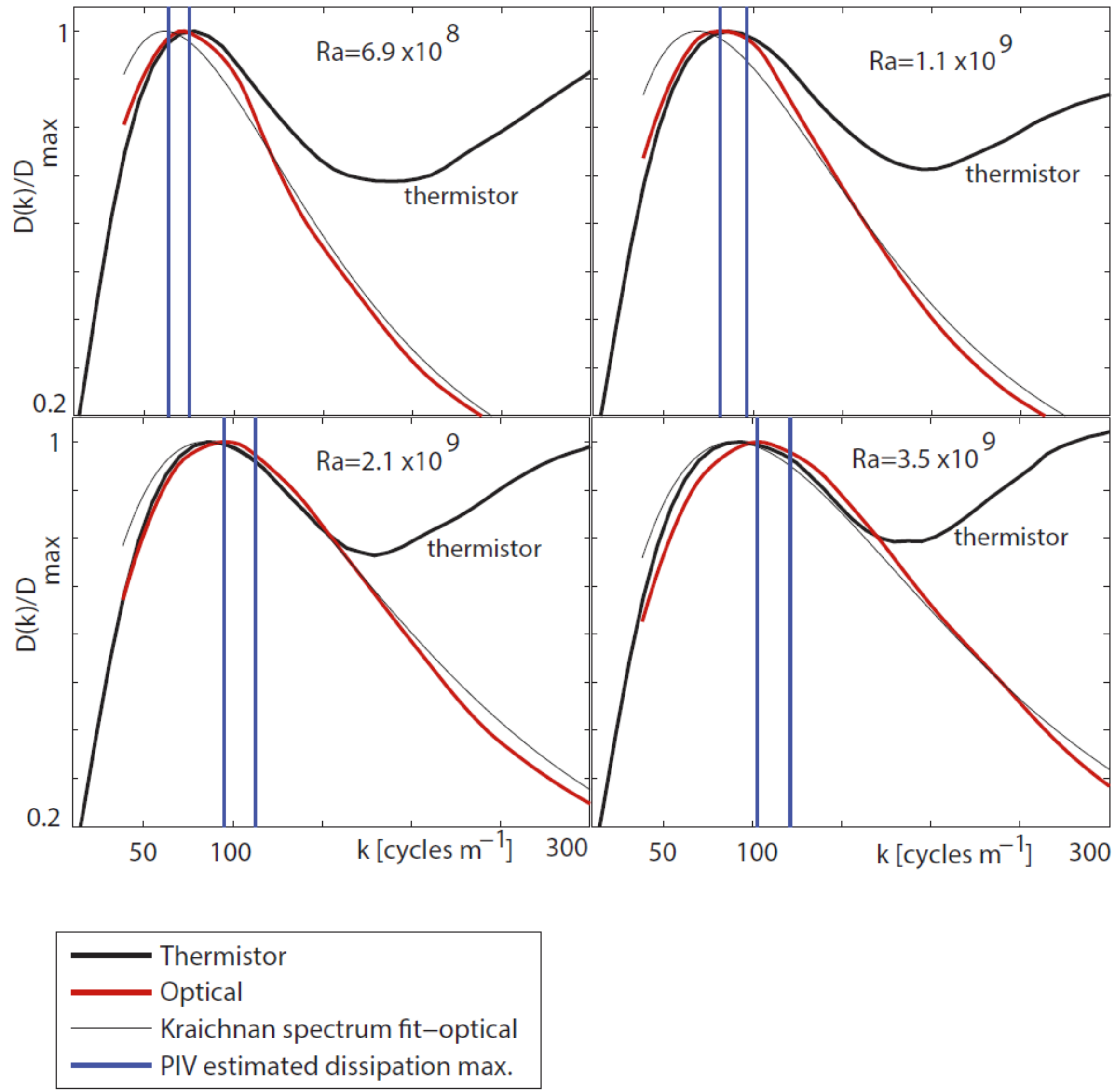


Figure 7. Comparison of optically (red) and thermistor measured temperature (black) one dimensional normalized dissipation spectra at varying Ra number. The dissipation spectra $D(k)$ were normalized by their maximum value resulting i.e.: $D(k)/D_{max} \equiv E_{1\theta}(k)k^2/[E_{1\theta}(k)k^2]_{k=k_{max}}$. Blue vertical bars are locations of the one dimensional temperature dissipation peak k_{peak} (Eq. 12) inferred from the PIV data. The thin black line is the least-square fit to K-form from the optically measured temperature dissipation spectrum.

ε,χ in experiment

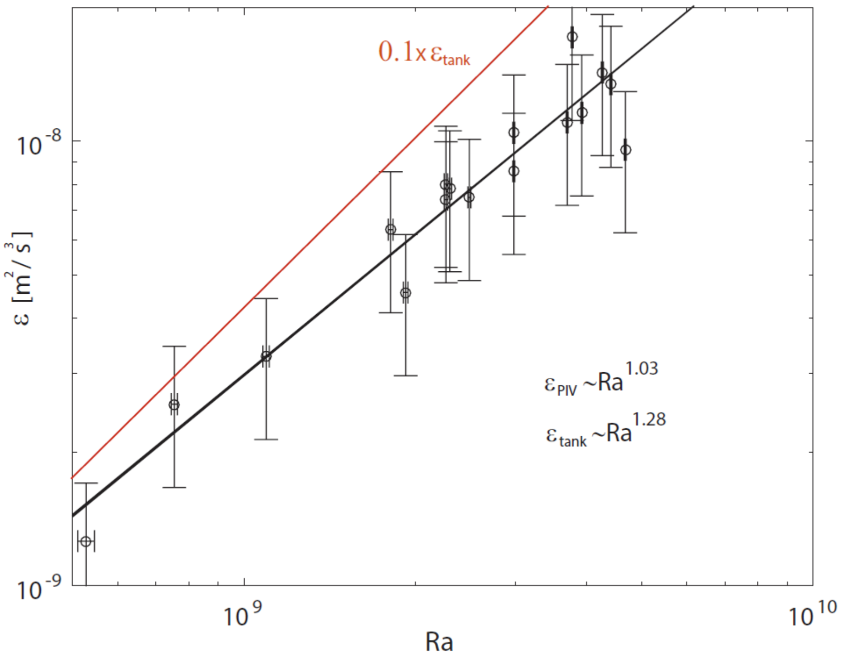


Figure 4. Summary of the domain averaged ε as a function of the Rayleigh number in the experim
The line denotes 1/10 of the total **exact mean TKED rate [19]** in the tank- ε_{tank}. **The error b**
(a standard deviation) were calculated from local ε **temporal variability.**

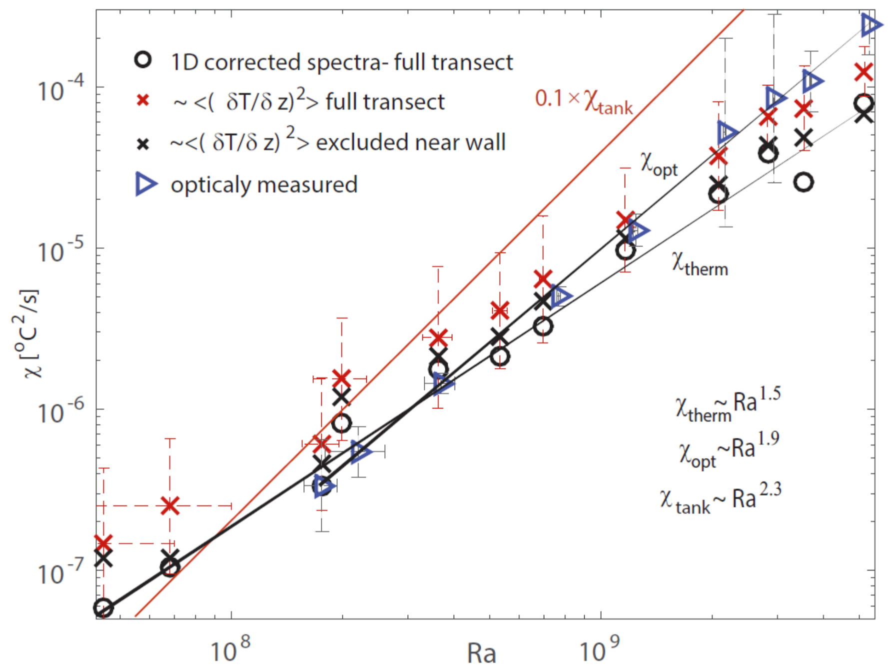


Figure 8. Comparison of optically and thermistor measured temperature dissipation rate χ. The spectrally estimated and the 'full transect' χ estimate are based on full thermistor transect (to within 0.5 cm from tank wall. The 'excluded near the wall' χ estimate comes from a subset of full thermistor transect - to within 2.5 cm from the tank wall. For comparison we plot 1/10 of the exact averaged value of the whole tank temperature dissipation rate $\chi_{tank} = \kappa \Delta T^2 Nu/d^2$ ([19]). **The error bars (a standard deviation) were calculated from local** χ **temporal variability.**