

## Confidence Intervals MATH 1442

**Note:** All procedures include the assumption that the sample is a **random sample**. The confidence intervals shown are computed using a confidence level  $1-\alpha$  and significance level  $\alpha$

### Confidence intervals for the mean

*Assumptions using the normal (z) distribution:*

- 1) The population is normally distributed or the sample size is greater than 30
- 2) The value of the population standard deviation ( $\sigma$ ) is known

$$\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

*Assumptions using the t distribution:*

- 1) The population is normally distributed or the sample size is greater than 30
- 2) The value of the population standard deviation ( $\sigma$ ) is unknown

$$\bar{x} - t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

When the population is not normally distributed and the sample size is less or equal to 30 a nonparametric method is needed

### Confidence intervals for proportions

*Assumptions*

- 1) The data comes from a binomial distribution (requirements for a binomial distribution)
- 2) There are at least 5 successes and at least five failures (equivalent to the requirement  $np \geq 5$  and  $nq \geq 5$ )

$$\hat{p} - z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

### Confidence intervals for the variance and standard deviation

*Assumptions:*

- 1) The data comes from a normal distribution (strict assumption)

For the variance

$$\frac{(n-1)s^2}{\chi_{\alpha/2, n-1}^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_{1-\alpha/2, n-1}^2}$$

For the standard deviation

$$\sqrt{\frac{(n-1)s^2}{\chi_{\alpha/2, n-1}^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_{1-\alpha/2, n-1}^2}}$$

