

Inference About Two Samples

Independent samples (Section 9.3) Alternative method

Requirements: The two samples are independent. Both samples are simple random samples. Either of these conditions is satisfied: The two samples are large (with $n_1 > 30$ and $n_2 > 30$) or both samples come from populations having normal distributions.

Test Statistic for unequal variances assumption Rule of thumb, use when $\frac{s_{\text{largest}}^2}{s_{\text{smallest}}^2} > 3$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Degrees of freedom: Conservative estimate: smaller of $n_1 - 1$ or $n_2 - 1$

Test Statistic for equal variances assumption Rule of thumb, use when $\frac{s_{\text{largest}}^2}{s_{\text{smallest}}^2} \leq 3$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

Degrees of freedom: $n_1 + n_2 - 2$

Where $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$

Dependent samples or matched pairs (Section 9.4)

Requirements: The sample data consist of matched pairs. The samples are simple random samples. Either or both of these conditions are satisfied: The number of matched pairs of sample data is large ($n > 30$) or the pairs of values have differences that are from a population having a distribution that is approximately normal.

Test Statistic Compute the difference, and proceed with a one sample test as follows:

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

Degrees of freedom = $n - 1$

Hypothesis test for two Variances (Section 9.5)

Requirements: The two populations are independent of each other. The populations are each normally distributed.

Test Statistic Compute the following ratio as follows:

$$F = \frac{s_1^2}{s_2^2} \quad (\text{where } s_1^2 \text{ is the larger of the two sample variances})$$

Numerator Degrees of freedom = $n_1 - 1$

Denominator Degrees of freedom = $n_2 - 1$