

① Phys 1402 2014-10-21

AC Circuits

$V(t)$ oscillates w/ frequency f .

$V_0, V_{\text{peak}}, V_{\text{max}}$ are all the amplitude.

$V_{\text{RMS}} = V_0/\sqrt{2}$ is the effective voltage.

Current also oscillates. Same f .

AC Ohm's Law

$$V_{\text{RMS}} = I_{\text{RMS}} Z$$

Resistor (R)

$$Z_R = R$$

Inductor (L)

$$Z_L = 2\pi f L$$

Opposes change in I

High- f has huge change in I .

This requires high- V .

$V = IZ$: Z must be high

Capacitor

$$Z_C = \frac{1}{2\pi f C}$$

Takes time to fill and empty.

High- f has rapid changes in V .

This requires high- I .

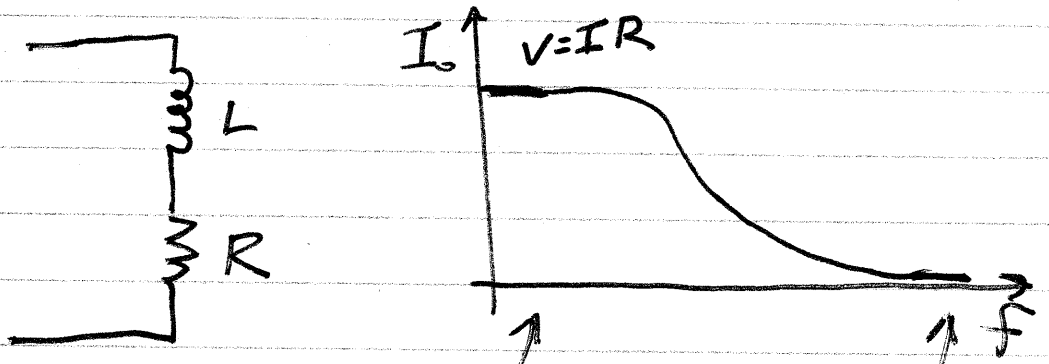
$V = IZ$: Z is Low

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Filters :

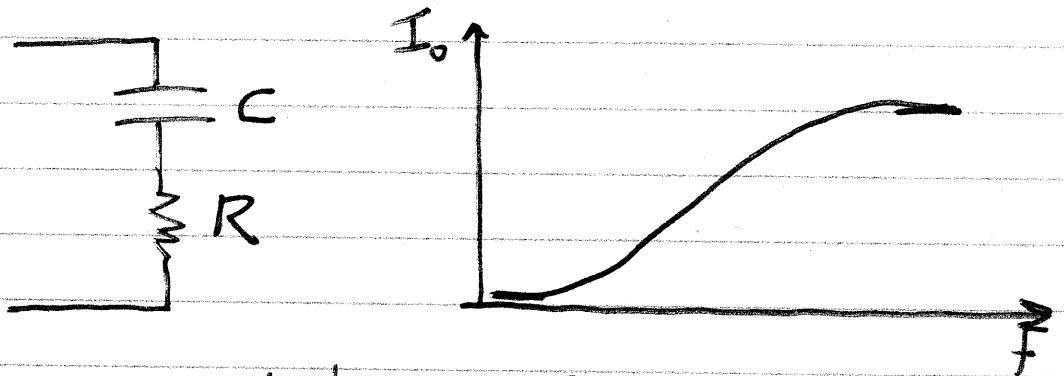
High-Z in series blocks current.

$$V_o = I_o Z \quad \text{High } Z \text{ requires high-} V$$



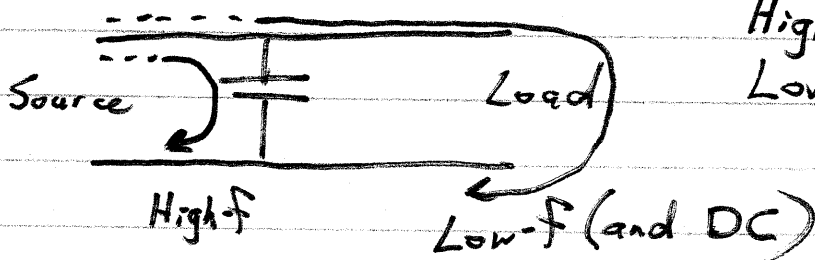
Low-f
 $Z_L = 2\pi f L \approx 0$

High-f
 $Z_L = 2\pi f L \approx \infty$



Capacitor blocks Low-f.

Capacitor in parallel



High-f go thru C.
Low-f go to Load.

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Series AC Circuit

Each component has its own Z .

Z_R , Z_L , and Z_C combine separately.

Then add as vectors.

$R \rightarrow Z_R$ in $+\hat{x}$ dir.

$L \rightarrow Z_L = 2\pi fL$ $+\hat{y}$ dir.

$C \rightarrow Z_C = \frac{1}{2\pi fC}$ $-\hat{y}$ dir.

Procedure:

$$Z_L, Z_C \rightarrow X = Z_L - Z_C$$

$$X, R \rightarrow Z = \sqrt{R^2 + X^2}$$

X : reactance = impedance from inductors & capacitors.

R always increases Z .

Z_L and Z_C can cancel!

IF $Z_L = Z_C$, $X = 0$. This gives the lowest possible Z , which is $Z = R$.

How does this happen?

$$2\pi fL = \frac{1}{2\pi fC}$$

$$(2\pi f)^2 \frac{L}{f} = \frac{1}{fC}$$

$$2\pi f = \frac{1}{\sqrt{LC}}$$

Resonant Frequency.

$$f = \frac{1}{2\pi\sqrt{LC}}$$

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$$\text{Ex: } f_R = 2.4 \text{ GHz} = 2.4 \times 10^9 \text{ Hz}$$

$$C = 1 \text{ pF} = 10^{-12} \text{ F}$$

$$f_R = \frac{1}{2\pi\sqrt{LC}}$$

Note: 10^{-12}
 $1e^{-12}$

$$2\pi f_R = \frac{1}{\sqrt{LC}}$$

$$(2\pi f)^2 = \frac{1}{LC}$$

$$L = \frac{1}{C(2\pi f)^2} = \frac{1}{(10^{-12} \text{ F})(2\pi \cdot 2.4 \times 10^9 \text{ Hz})^2}$$

$$= 4.4 \times 10^{-9} \text{ H} = 4.4 \text{ nH}$$

⑤

Power in AC

Resistor: when V is \oplus , I is \oplus , P is \oplus
 V is 0 , I is 0 , P is 0
 V is \ominus , I is \ominus , P is \oplus

Power is only used by resistor (not generated).

Power is used in pulses. $P_R = \frac{1}{2} P_{\max}$

Inductor - has energy $= \frac{1}{2} L I^2$

When I = strong, energy is high.

When I = weak, energy is low.

The inductor takes energy and gives it back!

Overall, $P_L = 0$ (on average)

Capacitor: energy $= \frac{1}{2} C V^2$

$P_C = 0$ (on average)

Transformer: shuttles energy from primary to secondary.

$$P_1 = P_2$$

Only resistors use power.

Key: Find I of resistor.

$$P = V_R I_R = \boxed{I^2 R}$$

RMS V , I go with average power.

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Ex: 120 V AC (RMS) 60 Hz
36 W Light bulb, resistive

$$P_{ave} = V_{RMS} I_{RMS} \quad (\text{for resistor})$$
$$(36W) = (120V) I_{RMS}$$

$$I_{RMS} = 0.3 A$$

$$V_{RMS} = I_{RMS} Z$$

$$Z = \frac{120V}{0.3A} = 400 \Omega$$

Since it is resistive, $R = 400 \Omega$

Put a capacitor in series.

$$C = 8.84 \mu F \quad f = 60 \text{ Hz}$$

$$Z_C = \frac{1}{2\pi f C} = \frac{1}{2\pi (60 \text{ Hz}) (8.84 \times 10^{-6} \text{ F})}$$

$$Z_C = 300 \Omega$$

$$Z_{Tot} = \sqrt{(400 \Omega)^2 + (300 \Omega)^2} = 500 \Omega$$

$$I = \frac{V}{Z} = \frac{120V}{500 \Omega} = 0.24 A$$

$$V_R = IR = (0.24 A)(400 \Omega) = 96 V$$

$$P = V_R I_R = (96V) \left(\frac{0.24A}{400 \Omega} \right) = 23 W$$