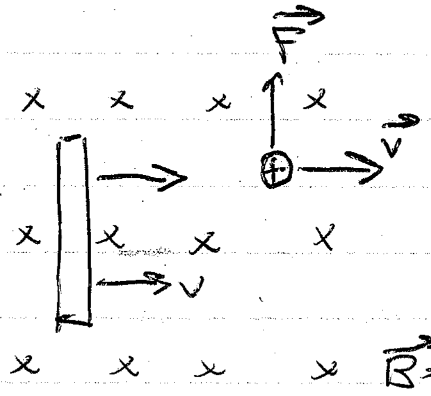
① Phys 1402   2016-10-11   Lec 14

Motional EMF

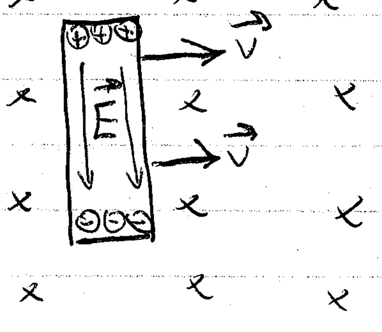
Thought 1: Move a bar into magnetic field.



A ⊕ charge feels a force $\vec{F} = |q| v B$ (up)

$\vec{F}_B$ is "up" toward the top of the page.
on ⊕ charges.

Thought 2: This is a conductor, so charges can move.



The electric force opposes the gathering of ⊕'s.

$$\vec{F}_E = qE \text{ (down)}$$

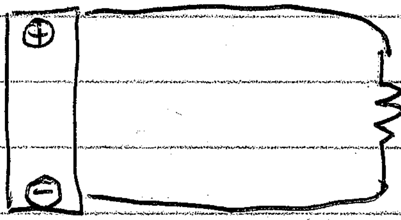
Conclusion: $E = vB$ in the moving bar.

Thought 3: Elec field is voltage per unit length.

ΔVoltage = "EMF" = $\mathcal{E} = E\ell = vB\ell$

How can we use the voltage generated in a moving bar?

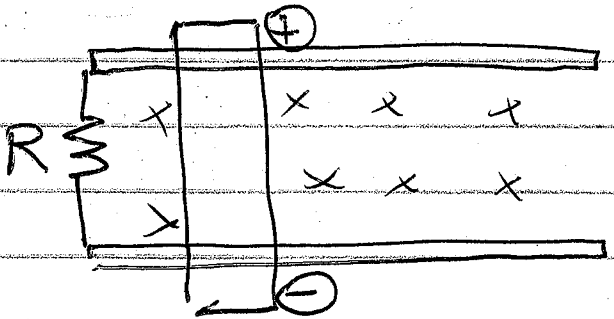
I:



Attach wires?

Doesn't work, the wires are also moving (or the resistor is moving) and that generates an opposite voltage.

II: Mount the bar on rails, connect load to rails.

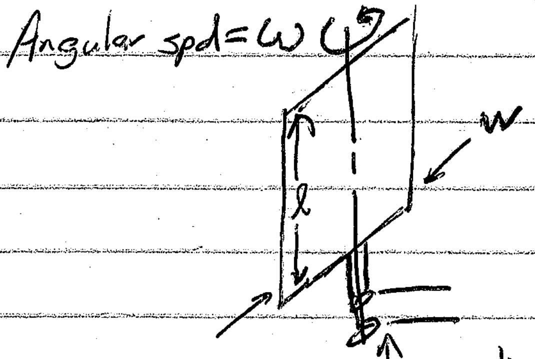


$$I = \frac{V}{R} = \frac{\mathcal{E}}{R}$$

$$\mathcal{E} = vB\ell$$

Now the rails and load are not moving. EMF pushes current thru R

III: Mount the bar on a rotating frame. (This is basically a coil.)



$$\mathcal{E} = B\ell\overset{\text{width}}{w}\overset{\text{omega}}{\omega}$$

$$= BA\,\omega \leftarrow \text{single loop}$$

$$= NBA\,\omega \leftarrow \text{coil}$$

↑point of sliding = Brushes

Converting Rotational Speed to SI Units

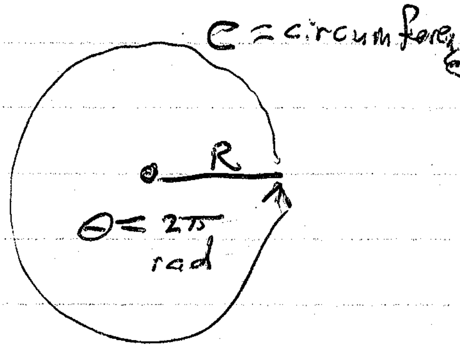
ω is measured in radians per second
(rad/s or s^{-1})

The radian is a "fake unit".
(Arc Length) = (Angle)(Radius)

$$(62.8 \text{ m}) = (2\pi \text{ radians})(10 \text{ m})$$

$$6.28 = 2\pi \text{ radians}$$

$$\underset{\uparrow \text{ no radians}}{2\pi} = 2\pi \text{ radians}$$



Radians are "dimensionless."

1 rotation = 2π radians

Ex: Car at idle $\omega = 1000$ RPM

$$1000 \frac{\text{rotations}}{\text{minute}} = 1000\left(\frac{2\pi \text{ rad}}{60 \text{ s}}\right) = 105 \text{ rad/s}$$

It doesn't matter whether we rotate the coil or the magnet!

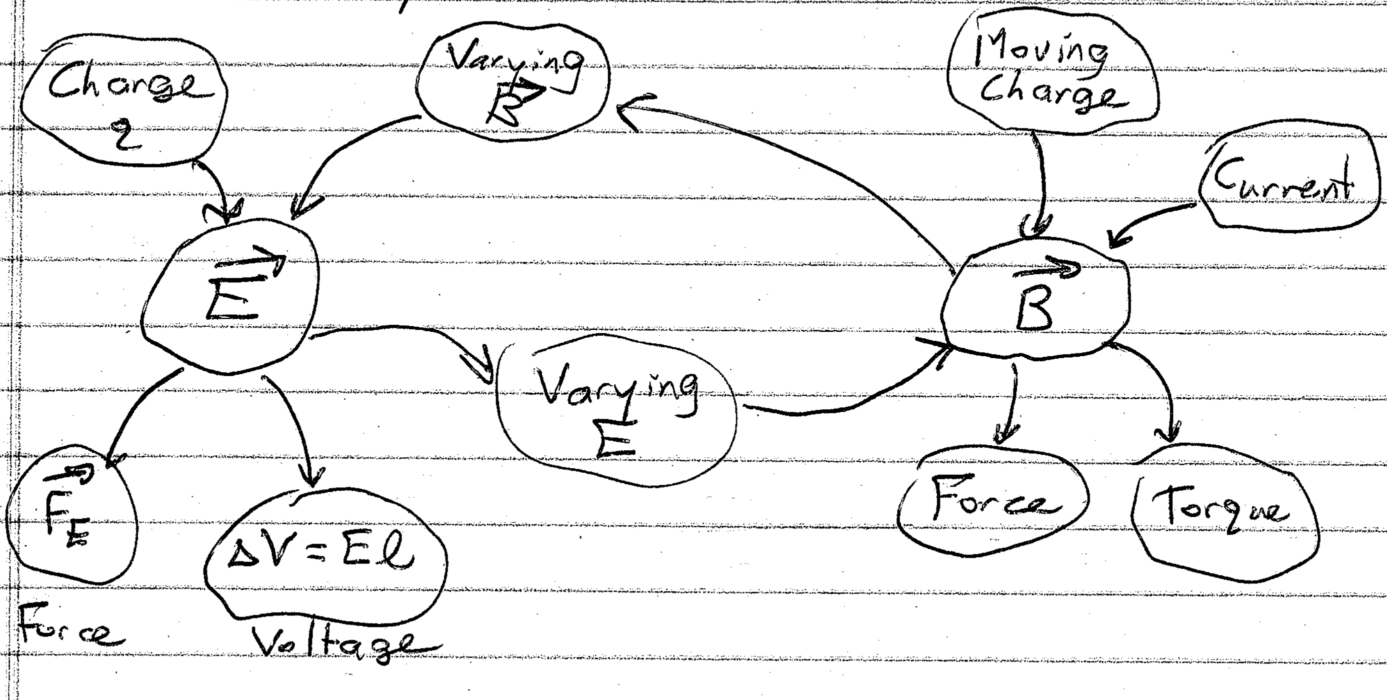
Rotating Coil → Motional EMF → Voltage

Rotating Magnet → ? → Voltage

A rotating magnet has $\vec{B} = \begin{cases} +B_0 & \text{sometimes} \\ -B_0 & \text{other times} \end{cases}$

In between, B is rapidly changing.

The changing $\vec{B}$ generates $\vec{E}$.
This $\vec{E}$ does form voltage that can push current.



⑤

Faraday's Law

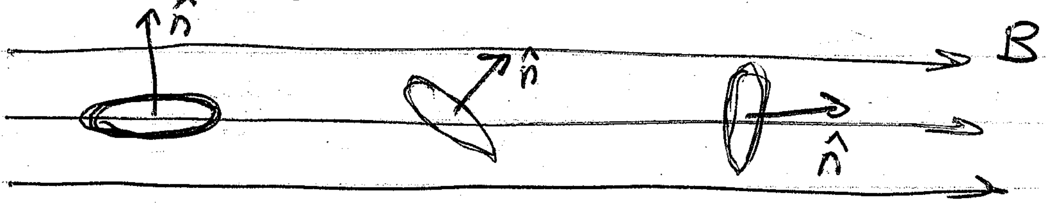
$$\mathcal{E} = -\frac{\Delta \Phi_B}{\Delta t}$$

Φ_B = magnetic flux = $NBA \cos\theta$

Essentially $\vec{B} \cdot$ Area.

N = # loops of wire → increases flux

$\cos\theta$ = angle-dependent reduction



$\theta = 90°$	θ in between	$\theta = 0°$
$\cos\theta = 0$	$\cos\theta$ in between	$\cos\theta = 1.0$
no flux		full flux
	Flux in between	