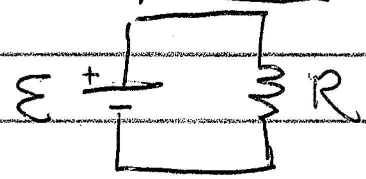
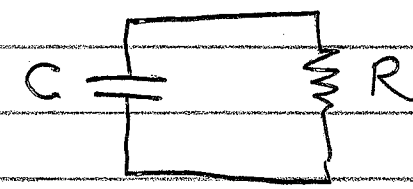
① Phys 1402 2017-09-26 Lec 7

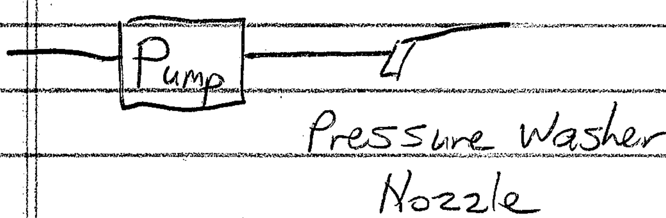
Battery Circuit

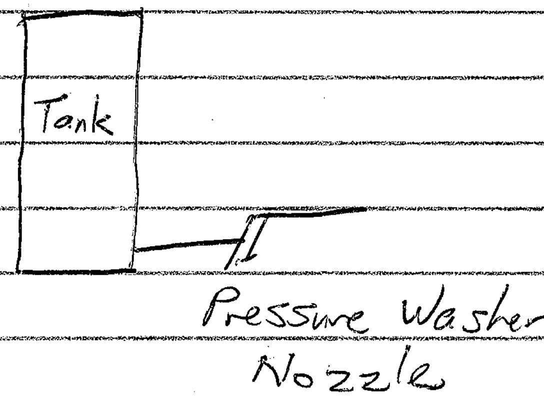


Capacitor Circuit?



Analogy:





Available Voltage:

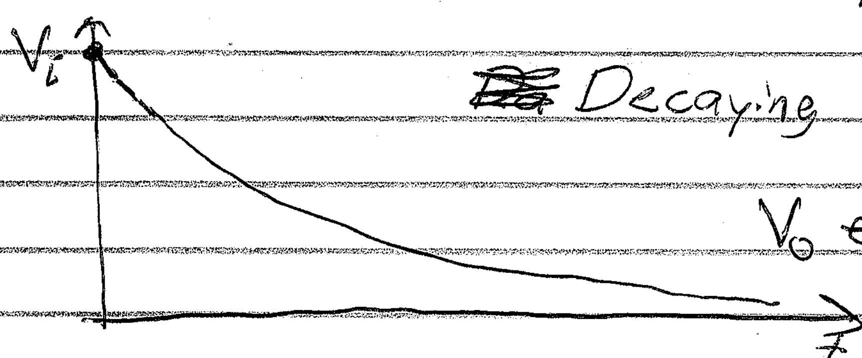
$\Delta V = \mathcal{E}$ $\qquad\qquad$ $\Delta V = Q/C$

Current:

$I = \frac{V}{R}$ $\qquad\qquad$ $I = \frac{V}{R}$

$I = -\frac{\Delta Q}{\Delta t}$

The current drains the capacitor.



~~De~~ Decaying Exponential

$V_0 e^{-t/\tau}$

$V_0 2^{-t/\lambda}$

V_0 = initial value $\quad$ τ = time constant $\quad$ λ = half life

(2)

What is $e^{-t/\tau} = 2^{-t/\lambda}$?

Exponential Function

Proportion of remaining voltage, charge, or current

t/τ	$e^{-t/\tau}$	
0	1.0	100%
1.0	0.37	37%
2.0	$0.14 = (0.37)^2$	14%
3.0	$0.05 = (0.37)^3$	5%
4.0		2%
5.0		1%

After 5 time constants, the capacitor is 99% drained.

$$e^x = \exp(x) \qquad e = 2.718$$

euler constant

At what time does the capacitor reach 50% of the original charge?

$$e^{-t/\tau} = 0.5 \qquad \ln(\,) \text{ both sides}$$

$$-t/\tau = \ln(0.5)$$

$$-t/\tau = -0.693$$

$$t = 0.693\,\tau$$

$$2^{-t/\lambda} = 0.5 = 2^{-1}$$

$$-t/\lambda = -1 \longrightarrow t = \lambda$$

The time constant comes from the circuit.

$$\tau = RC$$
$$(1.0\ s) = (1.0\ \Omega)(1.0\ F)$$

Capacitance (C) is measured in farads (F).

Bigger Capacitor → Drains "slower"
Bigger Resistor ↗

Batteries : $1\ A\cdot h = (1\ C/s)(3600\ s) = 3600\ C$

Typical smartphone
$$3\ A\cdot h \approx 10000\ C$$
$$\Delta V = 4.5\ V$$

Equivalent Capacitor:
$$Q = C\Delta V \qquad C = \frac{Q}{\Delta V} = \frac{10000\ C}{4.5\ V}$$
$$C = 2222\ F$$

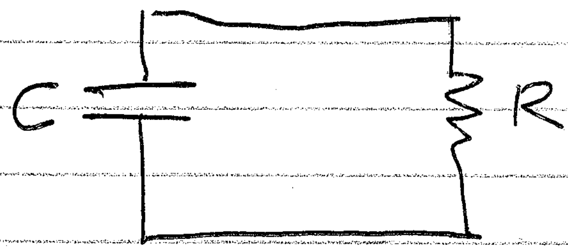
In RC Draining, many quantities decay exponentially:

$$V_c = V_0 \exp(-t/\tau)$$
$$Q = Q_0 \exp(-t/\tau)$$
$$I = I_0 \exp(-t/\tau)$$

Ohm's Law applies to resistor: $V = IR$ (R ← constant)

Capacitor voltage: $Q = CV$ (C ← constant)

RC Circuit set up for discharge



Initial voltage = 8.0 V

Resistance = 500 Ω

After 2.0 s, we measure

$I = 0.010\ A$

What is C?

Goal: Use $\tau = RC$

Try to use

$$V = V_0 \exp(-t/\tau)$$

$$(5.0V) = (8.0V)\exp(-(2.0s)/\tau)$$

$$V = IR = (0.010A)(500\Omega) = 5.0\ V$$

$$\frac{5.0V}{8.0V} = 0.625 = \exp(-t/\tau)$$

$$\ln(0.625) = -(2.0s)/\tau$$

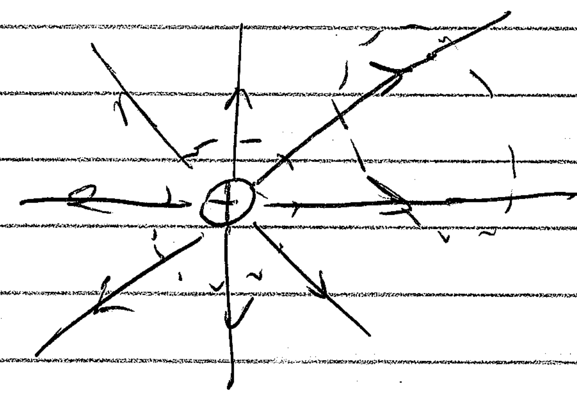
$$-0.47 = -2.0s/\tau$$

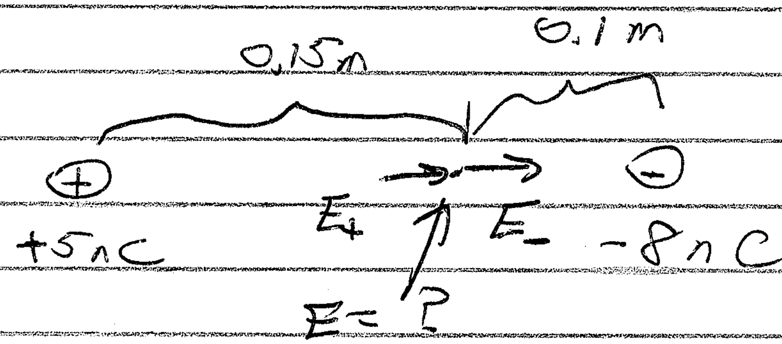
$$\tau = (2.0s)/0.47 = 4.26\ s$$

$$(4.26\ s) = (500\Omega)\,C \rightarrow C = \frac{4.26}{500} = 0.00851\ F$$

$$= 8510\ \mu F$$

5





$$E_+ = \frac{k(5nC)}{(0.15)^2} = (2000\ N/C)\ (\text{Rightward})$$

$$E_- = \frac{k(8nC)}{(0.10)^2} = (7200\ N/C)\ (\text{Rightward})$$

Total is 9200 N/C Rightward

If 8nC was (+)

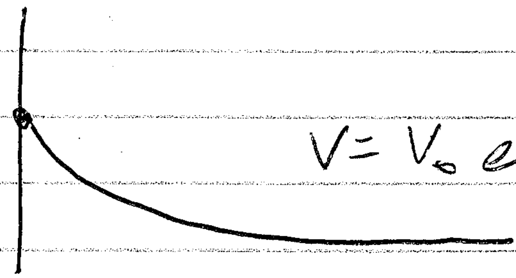
E_5 = 2000 N/C Right

E_8 = 7200 N/C Left

E_{tot} = 5200 N/C Left

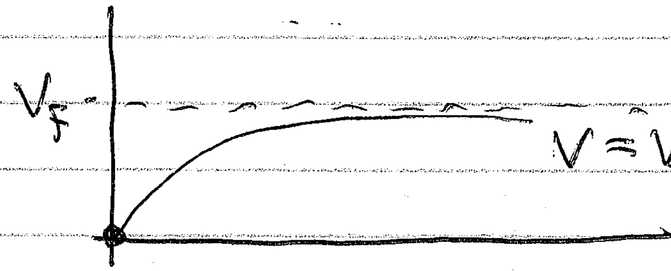
(6)

Discharge



$$V = V_0 e^{-t/\tau}$$

Charge



$$V = V_f \left(1 - e^{-t/\tau}\right)$$

Exponentially approaching a limit.