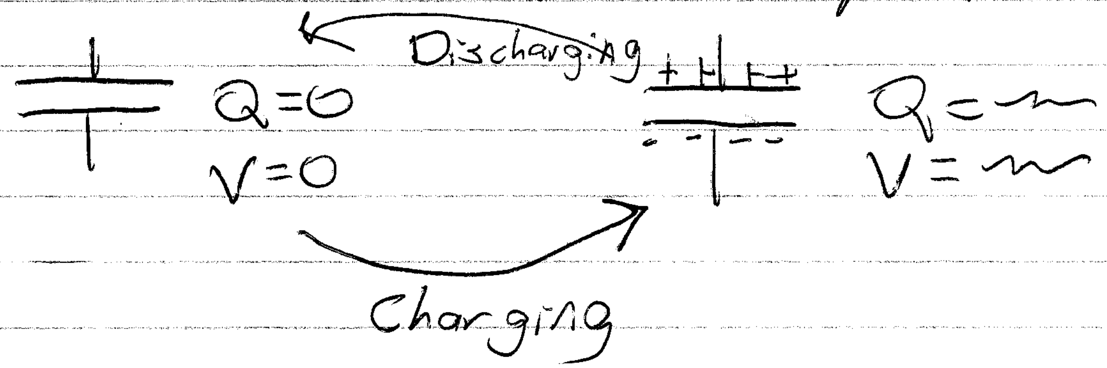
Phys 1402 2017-07-12 Lec 6

RC Circuits

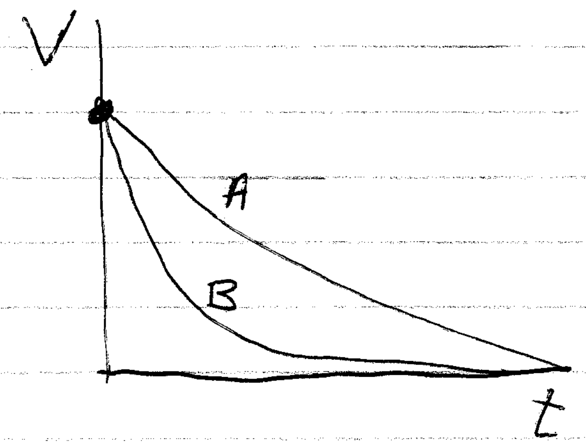
We know $Q = CV$ for a capacitor



Discharge - starting with a "full" capacitor.
V = as much as we can give it.

Phet Circuit Construction Kit (AC+DC)

Measure Voltage while Discharging



A = Slower, more time
B = Faster, less time

Shape is Exponential Decay

$$V(t) = V_0 \exp(-t/\tau)$$

Initial Value ↑ (V_0)   ↑ Time Constant (τ)

(2)

What is $\exp(-t/\tau)$?

Also written $e^{-t/\tau}$

t/τ	$e^{-t/\tau}$	
0	1	100%
1	0.37	37%
2	0.14	14%
3	0.05	5%
4	0.02	2%
5	0.01	1%
6	0.00	0%

↑ How much charge is left, fractionally?

↑ How many time constants have gone by?

Ex: $t/\tau = 2$ $\quad t = 2\tau$

This is "two time constants".

How long does it take to be 50% done?

Inverse of exponential is natural log.

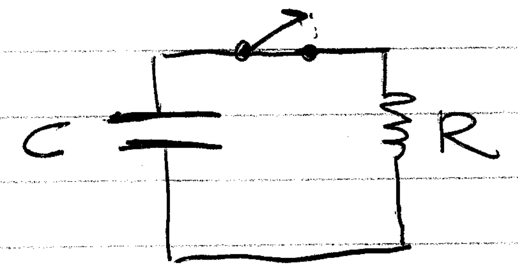
$e^{-t/\tau} = 0.5$ → $-t/\tau = \ln(0.5) = -0.693$

ln() both sides $\quad t = 0.693\ \tau$

(3)

"How fast" can mean two things:

- Relative measurements.
  It takes 0.7 time constants to drain half way.
- Absolute measurements.



Initially, cap is "charged".

When connected, current flows.

Constants:

$C = 0.2\ F$

$R = 10.0\ \Omega$

$\tau = RC$

$= (10.0\ \Omega)(0.2\ F)$

$= 2.0\ s$

Eventually, the capacitor is "empty"

In between, everything decreases exponentially.

Variables:

$V_C = V_0 e^{-t/\tau}$

$Q = Q_0 e^{-t/\tau}$

$I = I_0 e^{-t/\tau}$

$V_R = V_C$

$Q = VC$ ← capacitance

$Q_0 = V_0 C$

Ex: $V_0 = 10.0\ V$

$Q_0 = (10.0\ V)(0.2\ F)$

$Q_0 = 2.0\ C$ ← coulombs

$V = IR$

$V_0 = I_0 R$

$10.0\ V = I_0 (10.0\ \Omega)$

$1.0\ A = I_0$

(4) What is the current at $t = 7.0\ s$?

$$I = I_0 e^{-t/\tau}$$

$$\frac{t}{\tau} = \frac{7.0\,s}{2.0\,s} = 3.5$$

$$e^{-t/\tau} = e^{-3.5} = 0.03$$

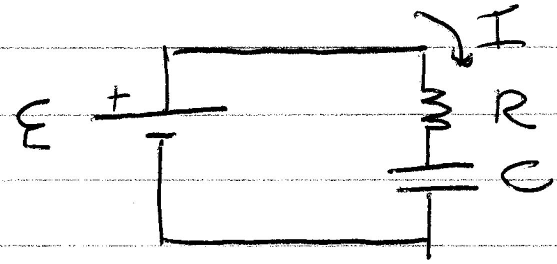
$$I = (1.0\,A)(0.03)$$
$$= 0.03\,A = 30\ mA$$
$$= 30 \times 10^{-3}\,A =$$

How long to get below 1.0 V?

$$V = V_0 e^{-t/\tau}$$

$$\frac{V}{V_0} = \frac{1.0\,V}{10\,V} = 0.1 = e^{-t/\tau}$$
$$\ln(0.1) = -t/\tau$$
$$-2.3 = -t/\tau$$
$$t = 2.3\,\tau$$
$$= 2.3\,(2.0\,s)$$
$$= 4.6\ s$$

How is "charging" different?



Kirchoff's Law for Voltage
CW Loop

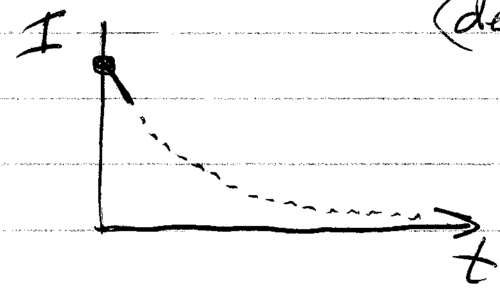
$$\mathcal{E} - IR - V_c = 0$$

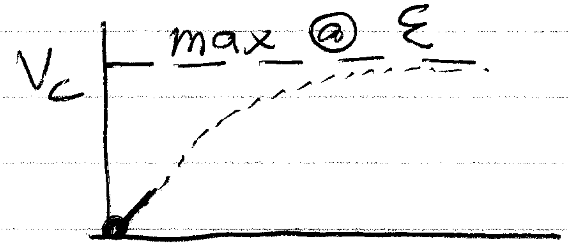
$$\mathcal{E} = IR + V_c$$

Initial: $\mathcal{E} = I_0 R + 0$

Next: $\mathcal{E} = IR + V_c$

(same) ↑ (decreases) (increase)





99% @ $t = 5\tau$

$I = I_0 e^{-t/\tau}$ $V_c = \mathcal{E}\left(1 - e^{-t/\tau}\right)$

$V_R = \mathcal{E}\, e^{-t/\tau}$ $Q = Q_{max}\left(1 - e^{-t/\tau}\right)$

Kirchoff Eventually: $\mathcal{E} = 0 + \mathcal{E}$

Ex: $C = 250\ \mu F$
$\mathcal{E} = 12\ V$

Want to get to 10.0 V on capacitor (Charging) in 5.0 s. What R?

$$V_c = \mathcal{E}\left(1 - e^{-t/\tau}\right)$$

$$(10.0\,v) = (12.0\,v)\left(1 - e^{-t/\tau}\right)$$

$$\frac{10}{12} = 0.833 = \left(1 - e^{-t/\tau}\right)$$

$$e^{-t/\tau} = 1 - 0.833$$
$$= 0.167$$

ln() both: $-t/\tau = -1.79$

$$\frac{-5.0\,s}{\tau} = -1.79$$

Mult by τ, div -1.79: $\frac{-5.0\,s}{-1.79} = \tau = 2.79\,s$

$$\tau = RC$$
$$(2.79\,s) = R\,(250 \times 10^{-6}\,F)$$

$$\frac{2.79\,s}{250 \times 10^{-6}\,F} = 11200\,\Omega = 11.2\,k\Omega = R$$

$e = 1.6 \times 10^{-19}\,C$ = 1.6e-19 C

Careful: $e = 2.718$