Phys1402 – Part 2 – Magnetism and AC

Monday, August 27, 2018 10:38 AM

- Our course notes are now broken into 3 parts on OneDrive:
  - Part 1 - Electrostatics and DC Circuits:
    https://tamucc-my.sharepoint.com/:o:/g/personal/jeffery_spirko_tamucc_edu/EhkLJvTU1_pOkB5OmwRTR4IBMn97BZRqrl-ORgvtyty7Zg
  - Part 2 - Magnetism and AC Circuits:
    https://tamucc-my.sharepoint.com/:o:/g/personal/jeffery_spirko_tamucc_edu/EuR1xpOWEK5LqEaOjXrgoQkBZ7sy3B5pL6QA4wlUziZVxQ
  - Part 3 - Oscillations, Waves, and Light:
    https://tamucc-my.sharepoint.com/:o:/g/personal/jeffery_spirko_tamucc_edu/EnXylWa4gm5FhPXWNb-bb4kB_7YX7TQsxRdh7HcSKATncg
- Our course notes are also periodically exported to PDF in:
  http://faculty.tamucc.edu/jspirko/Phys1402/

- Professor: Dr. Jeff Spirko, jeffery.spirko@tamucc.edu, NRC-1111 (inside NRC-1100 suite)
- Office Hours: See Live Caledar http://faculty.tamucc.edu/jspirko/calendar.html
- Course Web Folder: http://faculty.tamucc.edu/jspirko/Phys1402/ - Lecture Notes, Web Links
- Course YouTube Playlist: Phys1402-Fall18
- SI Info: [TBD]
- Sessions: [TBD]
- Office Hour: [TBD]
- Lab Web Folder: http://physlab.tamucc.edu/ - Lab Policies, Practice Exercises, Lab Instructions, Auxiliary Files
- Textbook: Serway/Vuille, College Physics, 11th Edition (electronic version in WebAssign).
- Homework: Will appear on WebAssign.
- Lab Reports: Due 1 week after the lab, by midnight. Submit in the Lab Reports area in the Course Menu.

Exam 1 Returned

Monday, October 1, 2018 2:00 PM

Avg: 72%

#25 $\mathcal{E} = 1.5\ V$

$R_{int} = 0.1\Omega$

$I = 0.15A$ } $V = 0.015\ V$

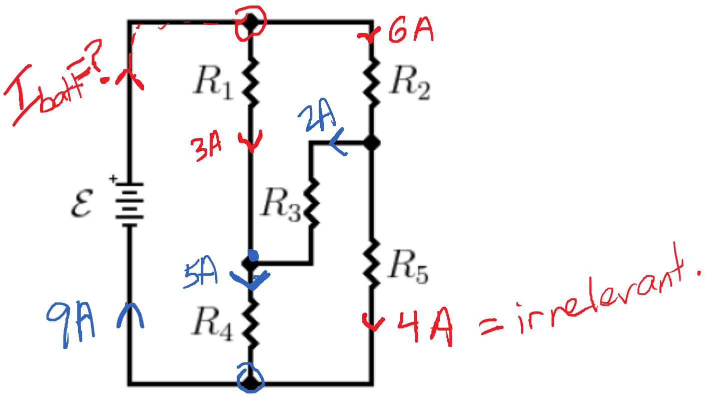
wasted inside battery

$\mathcal{E} = V_{terminal} + IR_{int}$

$1.5 = V_{term} + 0.015$

$1.485 = V_{term}$ = measured across output.

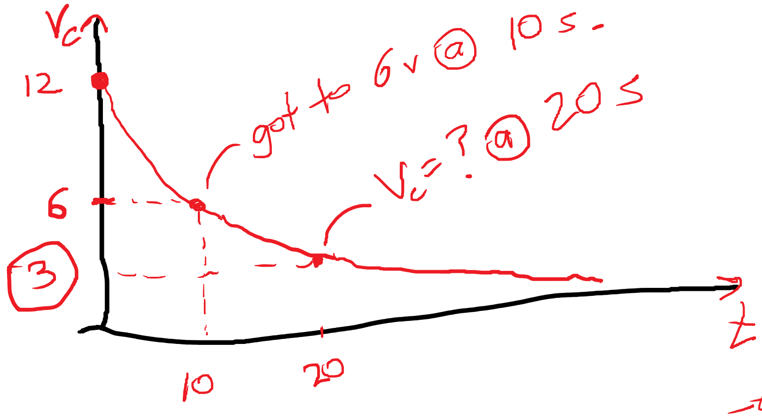
31: Keeping track of currents.



$I_{in} = I_{out}$

$I_{batt} = (6A) + (3A) = 9A$

#32: Exponential decay of RC Circuit



$V_c = V_0 e^{-t/\tau}$

Double $t \Rightarrow 2t_0$

$V_0 e^{-2t_0/\tau} = V_0 \left(e^{-t_0/\tau}\right)^2$

#33

Isolate exponential, inverse.

$V = V_0 e^{-t/\tau}$

$6 = 12 e^{-t/\tau}$ (Note: $t = 10\ s$)

$0.5 = e^{-t/\tau}$ In both sides

Isolate exp
Apply inverse.

$0.5 = e^{-t/\tau}$ ln both sides

$\ln(0.5) = -t/\tau = -10/\tau$

$\tau = -10/\ln(0.5) = 14.4$

#22: Cut dimensions of a wire in half (length and radius), what happens to resistance?

$A = \pi r^2$

$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi r^2}$$

Before:

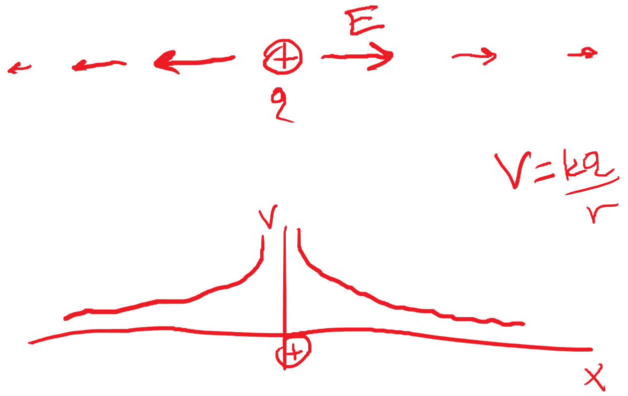
$$\frac{\rho(1)}{\pi(1)^2} = \frac{\rho}{\pi}$$

After:

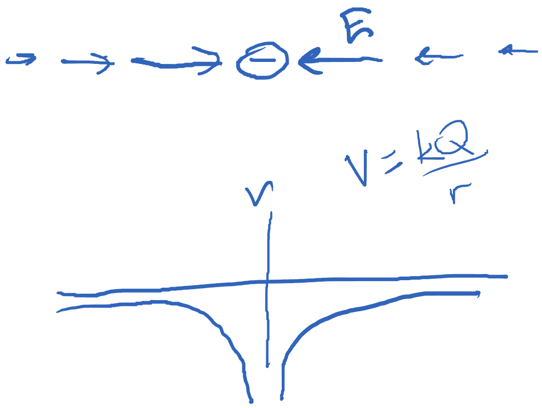
$$\frac{\rho(0.5)}{\pi(0.5)^2} = \frac{\rho}{\pi}(2)$$

#12: Electric field points from A toward B. How are the potentials at A and B related?

$A \xrightarrow{E} B$ points "downhill"

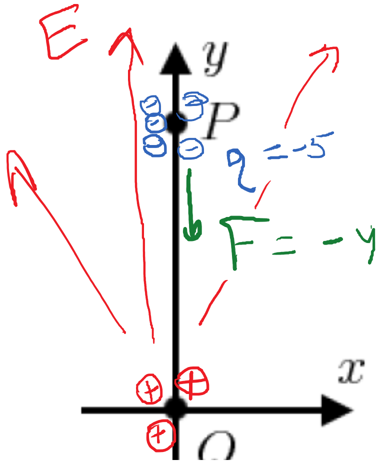


$V = \frac{kq}{r}$



$V = \frac{kQ}{r}$

E points downhill.

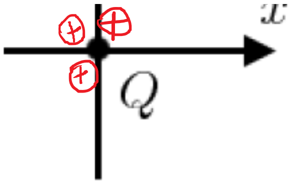


$q = -5$ placed @ point P

$F = -y$ because opposites attract.

@P: $\vec{E} = \sim +y$

$F = qE = (-\sim)(\sim +y)$



$$F = qE = (- \sim)(\sim +y)$$

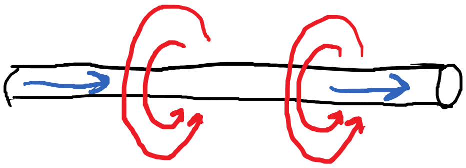
$$= (\sim)(-y)$$

Magnetism Intro

Monday, October 1, 2018 2:51 PM

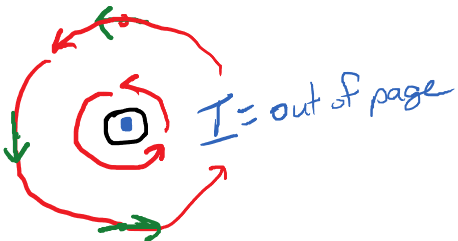
Magnetic Fields are similar to Electric Fields.

- Sources are currents and moving charges.
- Effects of magnetism are:
  - Forces on currents and moving charges.
  - Torques on current loops.
  - Induced voltages.
- Direction of magnetic fields are weird.



B loops around the source I.

↑ magnetic field



B = CCW

Describing directions in 3-D

Math	Paper	Geographic	Relative
+x, -x	Right, Left	East, West	R, L
+y, -y	Up, Down Top, Bottom (of page)	North, South	Forward, Backward
+z, -z	Out, In (of page)	Up, Down Sky, Ground	Up, Down

Magnetic Fields and Sources

Wednesday, October 3, 2018 1:56 PM

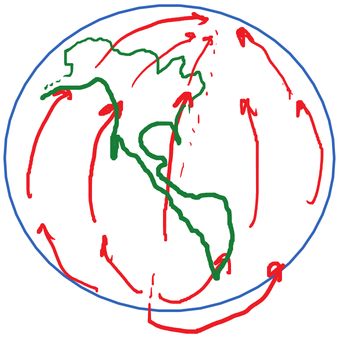
Magnetic fields ($\vec{B}$) are measured in teslas (T).

- Virtual "wind" of magnetic flux.
- Always circulate - no origin or destination.
- How do we know magnetic fields exist?
  - Charged particles get deflected. (Force)
  - Compass needles point with field. (Torque)
  - Generate electricity. (Induced voltage)
  - Form electromagnetic waves.

What creates magnetic fields?

- Moving charges - Electric current or spinning charges.
- Fluctuating electric fields

Earth's Magnetic Field

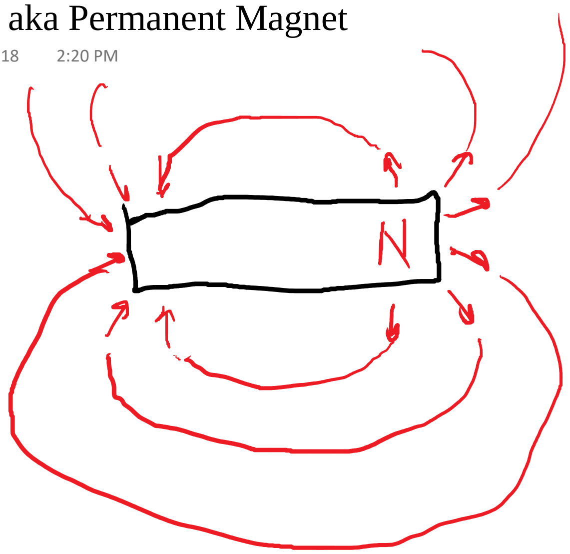


- Away from the poles, B points North.
- At the poles, B points inward or outward.

$B \approx 50\,\mu T = 0.00005\ T$

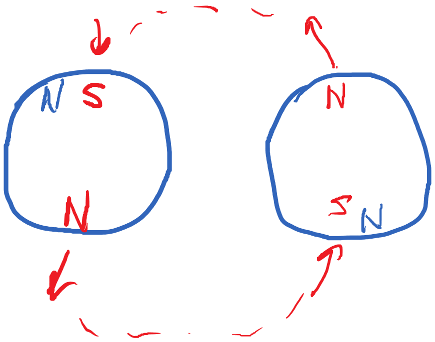
Bar Magnet - aka Permanent Magnet

Wednesday, October 3, 2018 2:20 PM



The "North" end is the end that has magnetic field pointing "outward" from the magnetic object.

Which part of Earth has the magnetic field pointing outward? (South)
So Earth is an "upside-down" magnet.

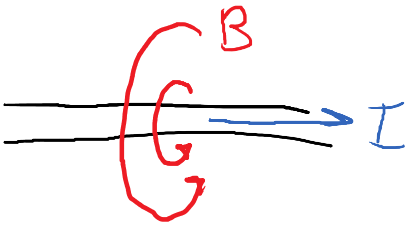


Second Earth's south geographic pole is attracted to original Earth's North.

Magnetic field of a wire

Wednesday, October 3, 2018 2:34 PM

- Long, Straight wire



- Magnitude:

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\mu_0 = 4\pi \times 10^{-7} \frac{T\cdot m}{A}$$

r = distance from center of wire, where we're finding B.

Ex: $I = 20\ A$

$r = 2.0\ cm$

$$B = \frac{(4\pi\times10^{-7})\ (20)}{2\pi\ (0.02)} = 2\times10^{-4}\ T$$

$1\ \mu T = 10^{-6}\ T$

$\frac{1\ \mu T}{10^{6}} = 1\ T$

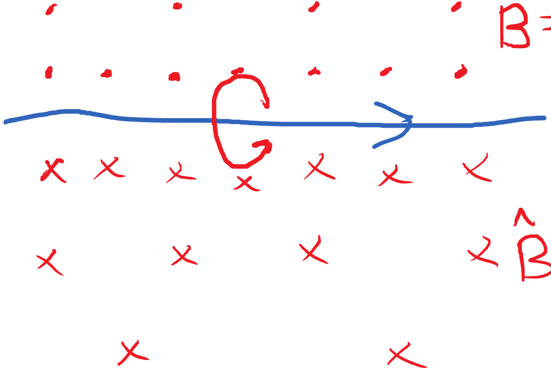
$$B = \frac{2\times10^{-4}}{10^{-6}}\ \mu T = 200\ \mu T$$

Direction of magnetic field: right-hand rule (RHR)

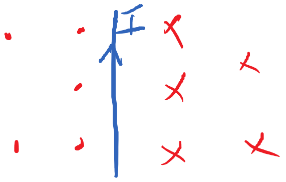
- Thumb along current
- Wrap fingers around current; they show B

"Hat" means direction.

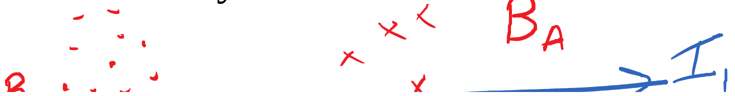
$\hat{B} = (\text{out})$

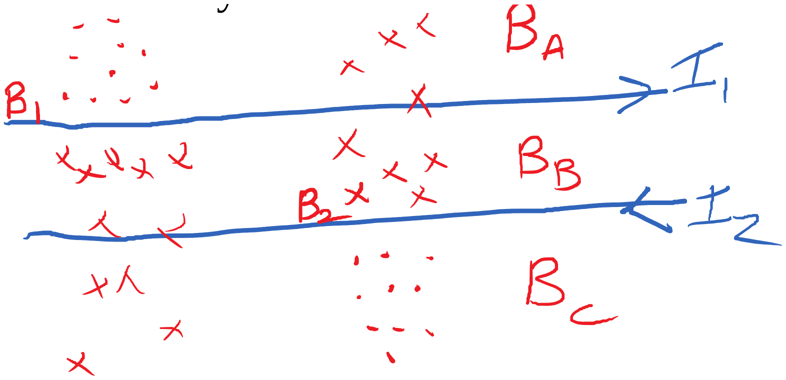


$\hat{B} = (\text{in})$



Magnetic field of two nearby wires?





$$B_A = +B_1 - B_2 = \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2}$$

$$B_B = -B_1 - B_2$$

Both contributions are into page.

$$B_C = -B_1 + B_2$$

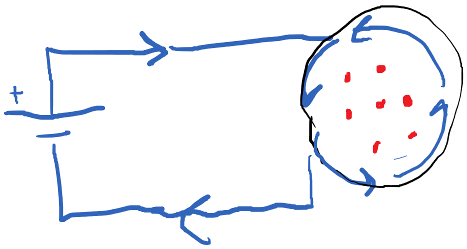
When two nearby currents flow in opposite directions:

- The magnetic field between them is amplified.
- The magnetic field outside them is reduced.

In this example, both wires are "sources" and we're looking at the magnetism that would be felt by another wire or compass in the vicinity.

Magnetic Coils

Wednesday, October 3, 2018 2:58 PM



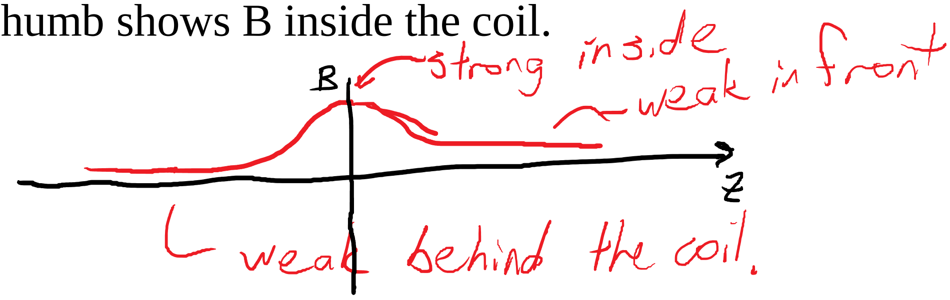
Magnetic Field inside coil:

$$B = \frac{\mu_0 N I}{2r}$$

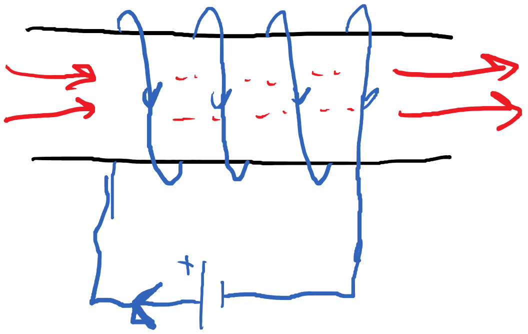
$N = \#$ loops $\quad r =$ coil radius.

Right-hand rule for a coil:

- Curl fingers with the current.
- Thumb shows B inside the coil.



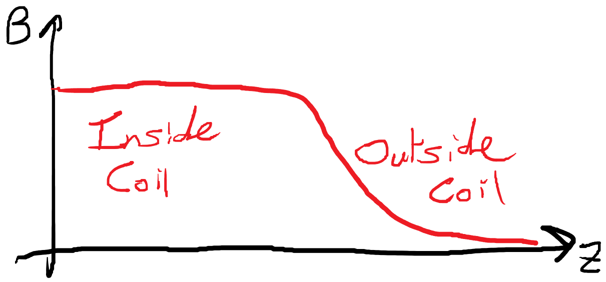
Solenoid Coil - A coil where the loops are in front of each other.



$$B = \frac{\mu_0 N I}{\ell} = \mu_0 n I$$

$\ell =$ length of coil

$n = \frac{N}{\ell} =$ turns density



Magnetic Effects

Monday, October 8, 2018 1:56 PM

- Forces
- Torques
- Induced Voltages

Magnetic Force on a Moving Charge

$$\vec{F}_B = q\vec{v} \otimes \vec{B}$$

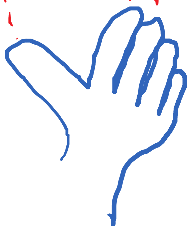
Times symbol for "cross product"

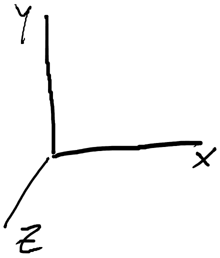
$$F_B = qvB\sin\theta_{vB}$$

$\theta = 0$, $\theta = 180^\circ$ → $\sin\theta = 0$

$\theta = 90^\circ$ → $\sin\theta = 1$ max force

Direction of F_B comes from the right-hand rule.

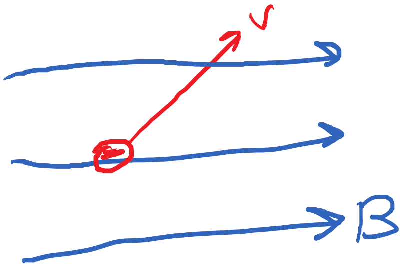




Proton moving in +y direction.
Magnetic Force in +z direction
What $\vec{B}$ can do it?

Result: B must point to the left, which is -x.

An electron has a speed of 3e6 m/s, at an angle of 45 degrees between the x- and y-axes. The magnetic field is 0.4 T in the x-direction.



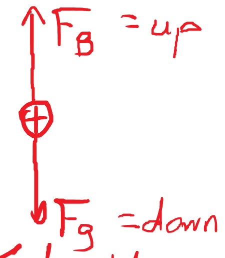
- Index up & to right
- Middle rightward, horizontal
- Thumb into screen.

⊖, so reverse direction,
F_B is out of screen.

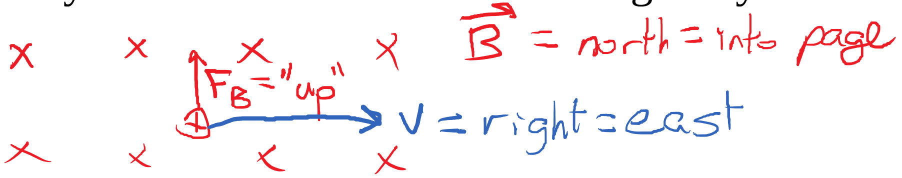
Since the charge is negative, we must reverse something.

- Option 1: Reverse the force.
- Option 2: Switch hands and use left hand for negative q.

Example: Earth's magnetic field is 0.00005 T pointing north.
In what direction could a proton fly so that it doesn't "fall" due to gravity?



Side View



Side View, Camera Faces N.

How fast should it go?

y: $F_{net} = 0 = F_B - F_g$

$$F_B = F_g$$

$$qvB = mg$$

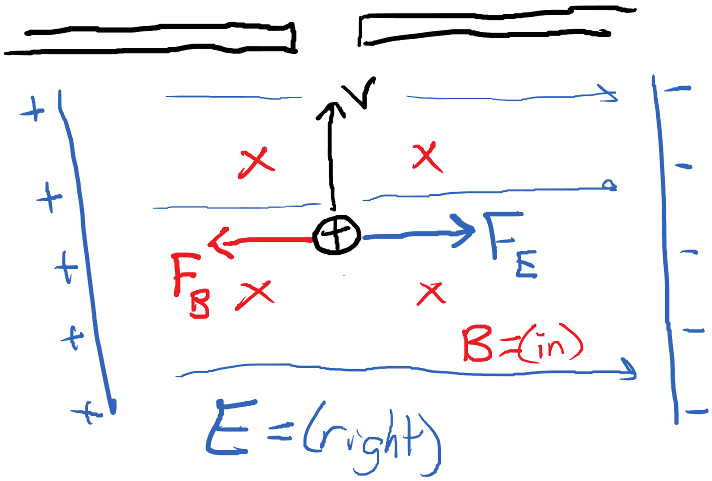
$$v = \frac{mg}{qB} = \frac{(1.67\times10^{-27})(9.8)}{(1.6\times10^{-19})(0.00005)}$$

$$= 0.0002 \text{ m/s}$$

Velocity Selector

Monday, October 8, 2018 2:37 PM

Can the electric and magnetic forces cancel?
Yes, if the 3 vectors (E, B, and v) are oriented the right ways.



$$F_E = F_B$$

$$\cancel{q}E = \cancel{q}vB$$

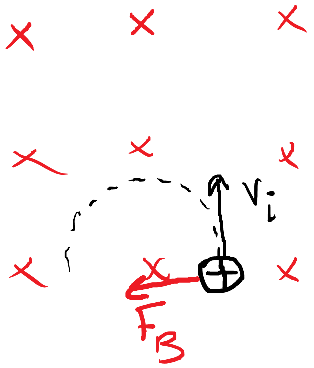
$$E = vB$$

- What if v is too fast? F_B is stronger, beam bends leftward.
- What if v is too slow? F_B is weaker, beam bends rightward.
- The particles with just the right v will go straight through. They have been selected.

Mass Spectrometer

Monday, October 8, 2018 2:45 PM

Beam of particles enters a magnetic field.



$$F_{net} = ma$$

$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv^2}{qvB} = \frac{mv}{qB}$$

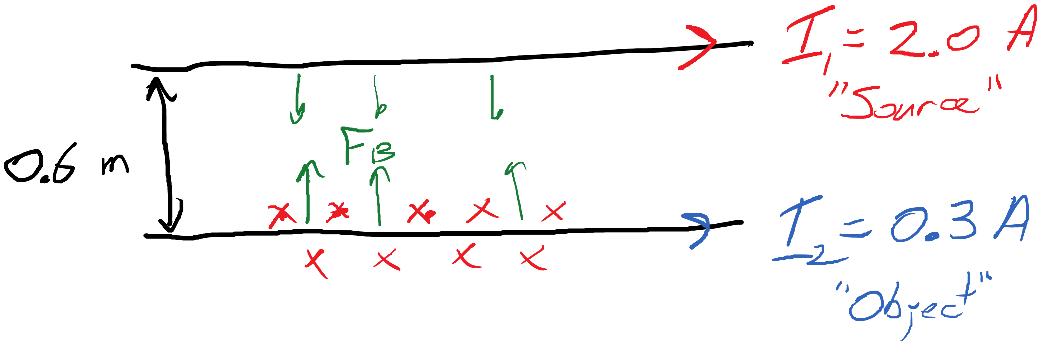
Force on a current

Monday, October 8, 2018 2:51 PM

$$F_B = I \ell B \sin\theta$$

Compare: $F_B = q v B \sin\theta$

Force between two currents.
This "between" is the force that the currents exert on each other.



$$B_1 = \frac{\mu_0 I}{2\pi r}$$

$$= \frac{(4\pi\times10^{-7})(2.0)}{2\pi(0.6)}$$

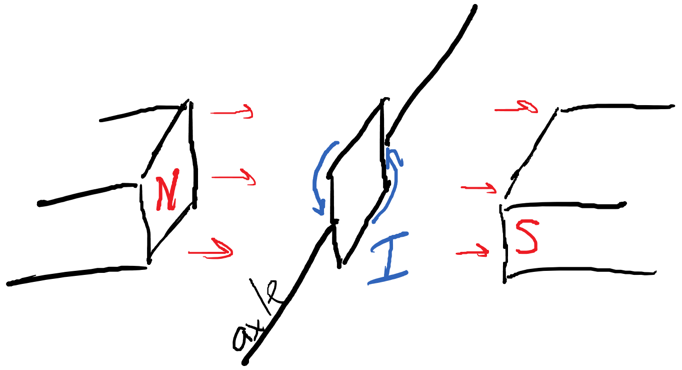
$$= 6.67\times10^{-7}\ T$$

$$F_B = I\ell B = (0.3A)\ell(6.67\times10^{-7}\ T)$$

$$\frac{F_B}{\ell} = 2\times10^{-7}\ N/m$$

Torque on a loop

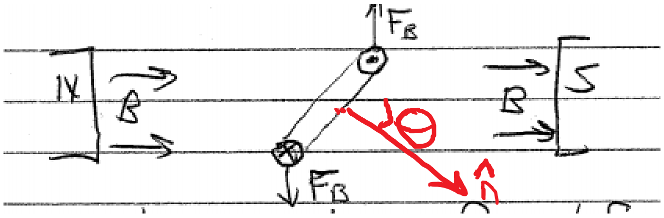
Monday, October 8, 2018 3:04 PM



Top segment:	B = right	I = out	F_B = up
Front segment:		I = down	F_B = out
Bottom		I = in	F_B = down
Back		I = up	F_B = in

This will cause zero net torque, because each force is lined up with the center of the square loop. The loop is in equilibrium and will not spin on the axle.

What if the loop is turned slightly from its given orientation?



The tilted loop feels a torque trying to restore it to its preferred orientation.

$$\tau = NBA\cos\theta$$

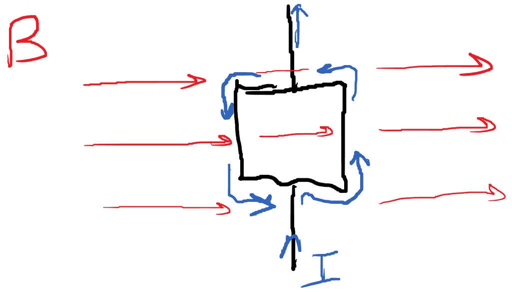
loops of wire

Area of coil

Angle between $\vec{B}$ and $\hat{n}$

Motor Coil - Easier drawing

Wednesday, October 10, 2018 1:58 PM



Segment		Force
Top/Bot		(none) not ⊥ at all.
Left	I=(down)	Out of page
Right	I=(up)	Into page

$$F_B = I\ell B$$

The combination of forces exerts a torque on the loop.

$$\ell^2 = A$$

$$\tau_{max} = I\ell B\left(\frac{\ell}{2}\right)2N = I\ell^2 BN = \underbrace{NBA}_{\text{Max Magnetic Flux thru the coil.}}\ I$$

Technically, that's the magnetic flux when the coil is oriented so that it "catches" the most flux.

The torque on the coil tries to turn the coil so that the magnetic flux gets bigger.

Electromagnetic induction

Wednesday, October 10, 2018 2:18 PM

Under the right conditions, magnetic fields can generate electric fields, and therefore electric voltages.

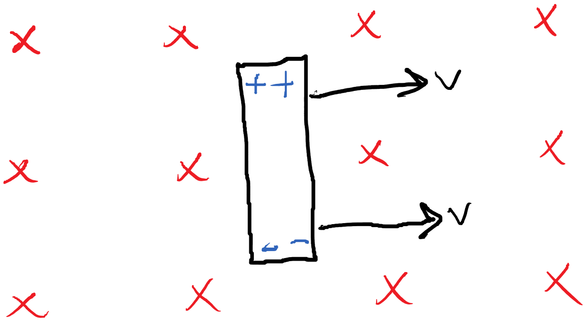
$$E = \frac{\Delta V}{d}$$

What are those conditions?

- Motional EMF
- Transformers

Motional EMF

B is into page



When the + and - charges generate the right amount of electric field, they'll stop gathering. At what point does this happen?

$$F_E = F_B$$

$$\cancel{q}E = \cancel{q}vB$$

velocity

voltage

$$\frac{\Delta V}{d} = vB$$

$$\Delta V = \mathcal{E} = vBd$$

Does Earth's magnetic field generate significant voltage?

$d = 33.0\ m$ wingspan

$B = 50\ \mu T$ Earth's B

$v = 225\ m/s$

$$\mathcal{E} = (225)(50\times10^{-6})(33)$$

$$\mathcal{E} = 0.37\ V$$

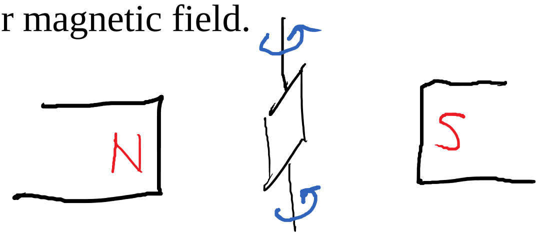
This isn't much. To generate voltage effectively, we need to do 2 things:

- Build a coil. Spin it in the magnetic field.
- Use a stronger magnetic field

speed

effectively, we need to do 2 things:

- Build a coil. Spin it in the magnetic field.
- Use a stronger magnetic field.



rotation speed in radians/s

Generated Voltage: $\mathcal{E}_{max} = NBA\omega$

Compare to motor

Torque: $\tau_{max} = NBAI$

Generator: Motion creates voltage (which pushes current)
Motor: Current creates torque (which pushes motion)

Since motors and generators are basically the same thing, they have side effects of acting like the other thing.
Generator torque: When the voltage is used to power a device, current flows. This current makes the generator produce torque that opposes your motion. The more you use a generator, the harder it is to turn the crank.

Motor EMF: We apply a voltage, which makes current flow by Ohm's Law. The current produces torque. Eventually the motor spins. The spinning generates EMF in the motor. The EMF is called "back EMF" because it opposes our current. That means that the spinning motor uses less current. A "freewheeling" motor doesn't use much current. A "loaded" motor slows down, has less back EMF, and ends up using more current.

Faraday's Law

Wednesday, October 10, 2018 2:48 PM

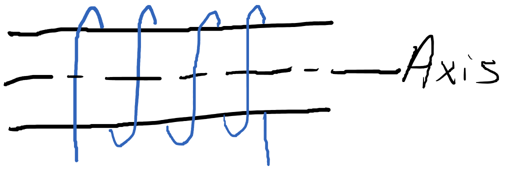
Magnetic Flux:

$$\Phi_B = NBA\cos\Theta$$

#Loops in coil

Area of each loop of coil

relative Angle between $\vec{B}$ and axis of coil.



Faraday's Law: EMF is generated whenever the flux changes.

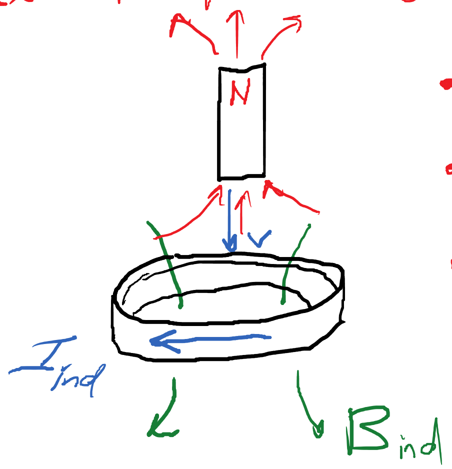
Lenz's Law: Tells us the direction of the induced voltage.

- The induced EMF tries to push current (induced current) so that the induced current creates a magnetic field which opposes the change in flux.

How do we use Lenz's Law?

- Figure out the direction of the existing magnetic field.
- Determine if the flux is increasing or decreasing in magnitude.
  - If it's increasing, our induced magnetic field should be opposite to the original magnetic field.
  - If the flux is decreasing, our induced magnetic field should be the same direction as the original magnetic field.
- The EMF and current tries to create this "induced magnetic field".

Ex: Drop a magnet thru a ring.



- B points up.
- Magnet falling closer, so Φ_B increasing.
- Induced B opposes = down
- Induced I goes left across the front, or CW viewed from the top.

Inductors and Inductance

Monday, October 15, 2018 1:55 PM

We looked at three effects of magnetic coils so far:

- Drive current (*I*) and coil creates magnetic field (*B*).
- Drive current (*I*) with external *B*, and torque is created.
- Drive rotation (ω) with external *B*, and voltage is generated.

Faraday's Law consolidates many ways of generating voltage.

$$\mathcal{E} = \frac{-\Delta\Phi_B}{\Delta t}$$

Generated Voltage

Magnetic Flux

Magnetic flux is like the overall amount of magnetism passing through a coil.

$$\Phi_B = NBA\cos\theta$$

Angle between $\vec{B}$ and coil axis.

Coil area

In this formula, *B* can be created externally, or it can be due to the coil itself.

Solenoid: $B = \frac{\mu_0 N I}{\ell}$

$$\Phi_B = N\left(\frac{\mu_0 N I}{\ell}\right) A \cancelto{1}{\cos\theta}$$

$$= \left(\frac{\mu_0 N^2 A}{\ell}\right) I$$

All geometric. Property of coil.

Electric. Result of circuit.

Inductance is the value of all of the geometric properties of the coil.

Inductor Flux: $\Phi_B = L I$

Inductance in henries (H)

Typical values range anywhere from microhenries to full henries.

How does Faraday's Law interact with this formula for inductance?

$$\Delta\Phi_B = \Phi_2 - \Phi_1$$

$$= L_2 I_2 - L_1 I_1$$

$$= L_2 I_2 - L_1 I_1$$

But L doesn't change.

$$= L I_2 - L I_1$$

$$= L(\Delta I)$$

$$\mathcal{E} = \frac{-\Delta \Phi_B}{\Delta t} = -L \frac{\Delta I}{\Delta t}$$

An inductor generates voltage out of changes in current.

- With constant current, it's "happy" and does nothing. It allows the current to flow.
- With an increasing current, it reacts by generating an opposing voltage. It tries to keep the current constant. It won't succeed. The current can actually increase.
- When current is already flowing, if you try to disconnect the inductor, it will violently oppose the sudden drop in current. It generates a huge voltage to try to keep the current flowing. This is an "inductive kick".
- While the inductor has current flowing, it stores energy.

$$\text{Energy} = \frac{1}{2} L I^2$$

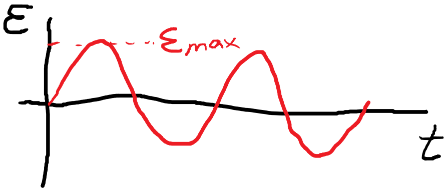
Compare to capacitor energy: $\text{Energy} = \frac{1}{2} C V^2$

Generators naturally generate AC Voltage

Monday, October 15, 2018 2:30 PM

$$\mathcal{E}_{max} = NBA\omega$$

ω = speed of rotation in radians/s



Sinewave has a frequency, f, where

$$\omega = 2\pi f$$

The period is the time of one cycle:

$$T = \frac{1}{f}$$

$$\mathcal{E} = \mathcal{E}_{max} \sin(2\pi f t)$$

In the USA, we use the same AC voltage in every household electrical outlet.

$\mathcal{E}_{max} = 170$ V $f = 60$ Hz

This is not how it's usually described.

$\mathcal{E}_{rms} = 120$ V

What's the difference between max and RMS?

- RMS is the "effective" voltage. An RMS voltage is roughly equivalent to the same DC voltage, with the same resistor. The resistor will use the same power, on average.

$$V = V_{max} \sin(2\pi f t)$$

Resistors in AC Circuits

$$V = IR$$

varies

must vary with V.

constant

$$I = \frac{V}{R} = \frac{V_{max} \sin(2\pi f t)}{R}$$

$$= I_{max} \sin(2\pi f t)$$

$$V_{max} = I_{max} R$$

What is the power used by a resistor?

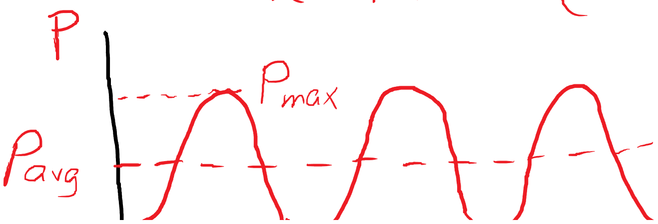
$$P = VI$$

$$P = I^2 R$$

$$P = V_{max} \sin(2\pi f t)\, I_{max} \sin(2\pi f t)$$

$$= V_{max} I_{max} \sin^2(2\pi f t)$$

$$P_{avg} = \frac{1}{2} P_{max} \quad \text{(Resistor)}$$



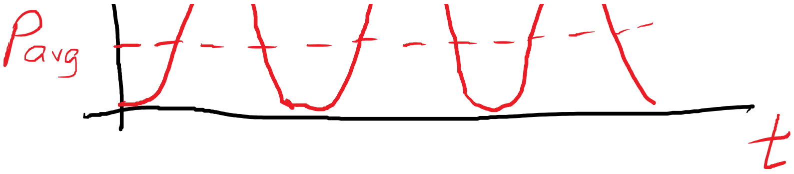
$P = I^2 R$

$$P_{avg} = I_{eff}^2 R$$

$$\frac{1}{2} P_{max} = I_{eff}^2 R$$

$$\frac{1}{2} I_{max}^2 \cancel{R} = I_{eff}^2 \cancel{R}$$

$$I_{eff} = \frac{1}{\sqrt{2}} I_{max}$$



This "effective" current is the RMS.

$$I_{RMS} = \frac{I_{max}}{\sqrt{2}}$$

The voltage that goes along with RMS current is:

$$\frac{V_{max}}{\sqrt{2}} = V_{RMS} = I_{RMS} R$$

Inductors in AC Circuits

Monday, October 15, 2018 2:51 PM

Resistors cause an opposing voltage based on current.

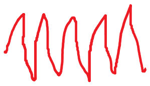
$$V_R = IR$$

Inductors cause an opposing voltage based on changes in current.

$$V_L = L\frac{\Delta I}{\Delta t}$$

To deal with this in AC, we need to know how quickly the current changes.

$$\text{Max } \frac{\Delta I}{\Delta t} = 2\pi f I_{max}$$



$$V_{L\,max} = 2\pi f L \; I_{max}$$

$$V_{L\,rms} = \underbrace{2\pi f L}_{} \; I_{rms}$$

Reactance, X_L

This looks like Ohm's Law, but I isn't multiplied by a constant. The way the inductor reacts to AC depends on the frequency.

$$X_L = 2\pi f L \qquad V_{L\,rms} = X_L I_{rms}$$

At high-frequencies, inductors have __high__ reactance. This makes the path more difficult for current to flow.

Even though reactance is like resistance, they're not quite the same.

Series Combinations: $R_T = R_1 + R_2 + \ldots$

$$X_{LT} = X_{L1} + X_{L2} + \ldots$$

R and X_L in series: $Z = \sqrt{R_T^{\,2} + X_{LT}^{\,2}}$

Impedance (Z) is the combination of resistance and reactance in an AC circuit.

$$V_{rms} = Z \; I_{rms}$$

Inductors don't use power (on average). They take and give back the same amount of energy.

$$P_{L,avg} = 0$$

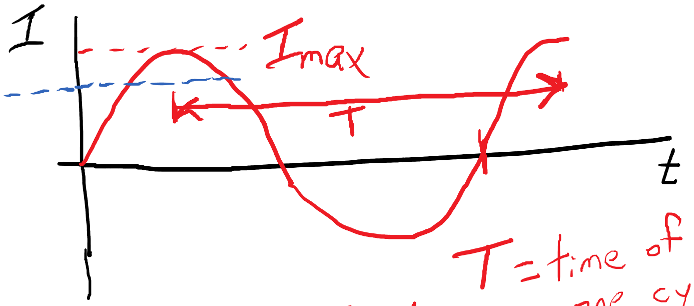
Lec 16 - AC and Capacitors

Wednesday, October 17, 2018 1:59 PM

Reminder: Exam 2 is coming up. Wed 10/31.

An AC current could be written as: $I = I_{max} \sin(2\pi f t)$

$0.707\, I_{max} = \frac{I_{max}}{\sqrt{2}} = I_{rms}$



T = time of one cycle

$T = 1/f$

Resistor in AC: $V_R = IR$ $V_{R,rms} = I_{rms} R$ $P_{R,avg} = I_{rms} V_{rms}$

Ex: 15 W Light Bulb
120 V RMS voltage }

$(15\,W) = I_{rms}(120\,V)$ $(120\,V) = (0.125\,A)\,R$

$I_{rms} = \frac{15}{120} = 0.125\,A$ $R = \frac{120}{0.125} = 960\,\Omega$

Inductors in AC: $V_L = L\frac{\Delta I}{\Delta t}$ $V_{L,rms} = I_{rms} X_L$ $P_{L,avg} = 0^{*}$

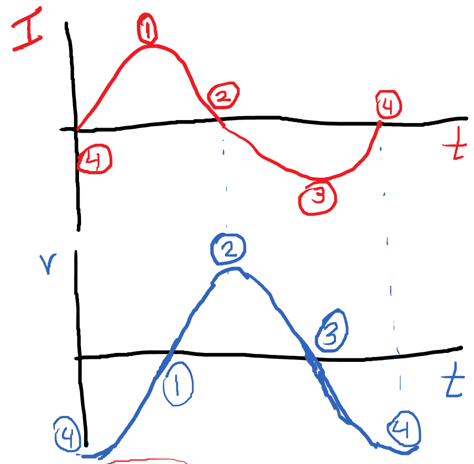
* A real inductor has resistance that does use power. The inductance doesn't.

X_L = inductive reactance = $X_L = 2\pi f L$ Like R for AC

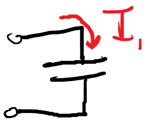
- Low-freq: X_L is tiny: Current w/o voltage
- High-freq: X_L is huge: Current is blocked.

Capacitors in AC

Wednesday, October 17, 2018 2:24 PM



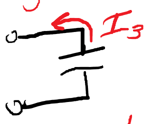
① Positive current flowing



Charge and Voltage are building.

② Current stops (briefly)
Charge and Voltage reach peak.

③ Negative Current flowing



Charge and Voltage decrease and go negative.

④ $I_4 = 0$, Q, V reach negative peak.

$$\boxed{V = \frac{Q}{C}}$$

$$Q = CV$$

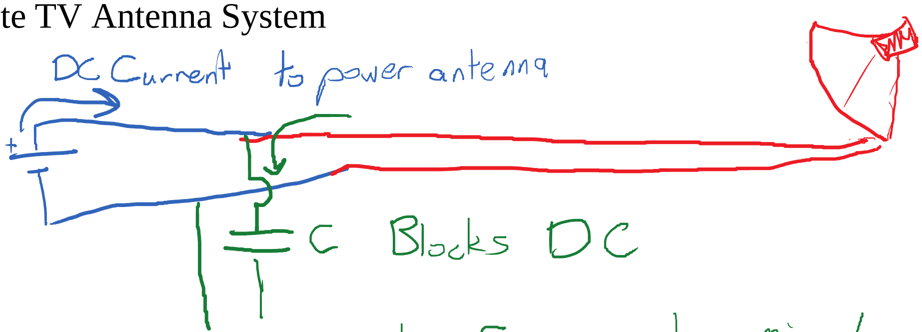
The voltage looks like a sinewave, but it's delayed by 1/4 cycle.
It's said that "voltage lags current in a capacitor".

Capacitor: $\frac{dV}{dt} = \frac{1}{C} I$ $\quad V_{C,rms} = I_{rms} X_C$ $\quad P_{C,avg} = 0$

Capacitive Reactance $= X_C = \frac{1}{2\pi f C}$

- Low freq: X_C is large Capacitor blocks low-f current.
- High freq: X_C is tiny High-f current flows easily.

Application: Satellite TV Antenna System



AC connection for signal coming back.

Ex: We used a 2.2 μF capacitor in lab this week.
What would happen if we plugged it into a wall?

$V = 120\ V \quad X_C = \frac{1}{\ } = (2\pi(60)(2.2\times10^{-6}))^{-1} = 1206\ \Omega$

Ex: We used a 2.2 μF capacitor in lab this week.
What would happen if we plugged it into a wall?

$V_{rms} = 120\text{ V}$

$f = 60\text{ Hz}$

$$X_c = \frac{1}{2\pi f C} = \left(2\pi(60)(2.2\times10^{-6})\right)^{-1} = 1206\,\Omega$$

$$V_{rms} = I_{rms} X_c$$

$$I = \frac{V}{X_c} = \frac{120}{1206} \approx 0.1\text{ A}$$

That doesn't seem like a huge amount of current.
That by itself wouldn't fry the capacitor.
Tantalum breaks down at 625 V/micro-m.
Our capacitors were rated at 25 V.

$$\frac{25\text{ V}}{625\text{ V}/\mu\text{m}} = 0.04\,\mu\text{m} = 40\text{ nm}$$. molecules ~ 1 nm

The tantalum layer is only about 40-100 molecules thick.

Series AC Circuit

Monday, October 22, 2018 1:56 PM

AC Ohm's Law $V = IZ$

Impedance (Ω)

Resistor: $Z = R$

Capacitor: $Z = \frac{1}{2\pi f C} = X_c$

Inductor: $Z = 2\pi f L = X_L$

What happens when we combine these in a circuit?

- Series circuit: Same current in everything.
- Series circuit: Voltages add.

$$V_T = V_R \sin(2\pi f t) + V_C(-\cos(2\pi f t)) + V_L \cos(2\pi f t)$$

$$= V_R \sin(2\pi f t) + (V_L - V_c)\cos(2\pi f t)$$

$$V_{T\,rms}^2 = V_{r\,rms}^2 + (V_{L\,rms} - V_{c\,rms})^2$$

The resistor voltage adds to the difference between the inductor and capacitor voltages, but using the Pythagorean theorem.

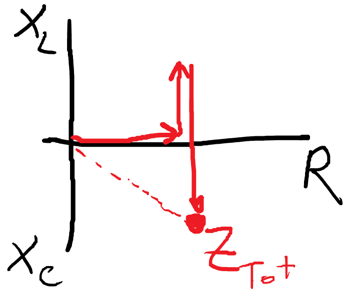
- Series circuit: Impedances add with a Pythagorean theorem.

$$Z = \sqrt{R^2 + (X_L - X_c)^2}$$

Separate for simplicity:

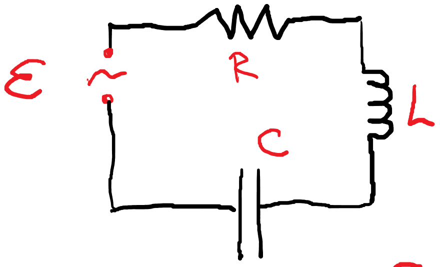
$$Z^2 = R^2 + X^2 \qquad X = X_L - X_c$$

Graphical Method: Vector Addition



Ex: Series RLC Circuit

Monday, October 22, 2018 2:21 PM



$\mathcal{E} = 140 \sin(500\, t)$

$= \mathcal{E}_{max} \sin(2\pi f\, t)$

$2\pi f = 500$

$\mathcal{E}_{max} = 140$ V

$\mathcal{E}_{rms} = \frac{140}{\sqrt{2}} = 99$ V

$f = \frac{500}{2\pi} = 79.6$ Hz

$R = 1\,k\Omega$

$R = 1000\,\Omega$

$L = 1.0$ H

$X_L = 2\pi f L = 500\,\Omega$

$C = 1.0\,\mu F$

$X_C = \frac{1}{2\pi f C} = \frac{1}{500 \times 10^{-6}}$

$= 2000\,\Omega$

$X = X_L - X_C = -1500\,\Omega$

Negative b/c Capacitor is stronger.

$Z = \sqrt{R^2 + X^2} = \sqrt{500^2 + 1500^2} = 1803\,\Omega$

$V_T = I\,Z$

$(99\,V) = I\,(1803\,\Omega) \rightarrow I_{rms} = 0.0549$ A

Now we can go back and find out more about the resistor.

$V = IR = (0.0549\text{ A})(1000\,\Omega) = 54.9$ V

$P = VI = (54.9\text{ V})(0.0349\text{ A}) = 3.2$ W

Note that this is NOT the overall voltage * current.

$VI = (99\text{ V})(0.0349\text{ A}) = 5.4$ V·A

What is the voltage across the inductor?

$V = IZ = (0.0549\text{A})(500\,\Omega) = 27.5$ V

Capacitor: $V = (0.0549\text{A})(2000\,\Omega) = 109.8$ V

Note that the capacitor voltage is more than the power supply voltage!!!

HW4-4, Magnetic Flux

Monday, October 22, 2018 2:41 PM

Loop: $\Phi_B = BA\cos\theta$

Coil: $\Phi_B = NBA\,\cancel{\cos\theta}\ 1$

Inductor: $\Phi_B = LI$

Solenoid

$$\left.\right\} B = \mu_0 \frac{NI}{\ell} \qquad L = \frac{\mu_0 N^2 A}{\ell}$$

HW4-4 $V_{rms} = 120\text{ V}$ $f = 60\text{ Hz}$ $V_{rms} = I_{rms} Z$

$$\Phi_{max} = L I_{max} \leftarrow I_{max} = I_{rms}\sqrt{2} \leftarrow I_{rms} = \frac{V_{rms}}{2\pi f L}$$

Resonance

Monday, October 22, 2018 2:48 PM

Because $X = X_L - X_C$ they can cancel.

Then: $Z = \sqrt{R^2 + X^2} = \sqrt{R^2} = R$

It's sort of like the inductor and capacitor aren't there.
So how can this happen?

$$X_L = X_C$$
$$2\pi f L = \frac{1}{2\pi f C}$$
$$(2\pi f)^2 = \frac{1}{LC}$$
$$2\pi f = \frac{1}{\sqrt{LC}}$$
$$f = \frac{1}{2\pi\sqrt{LC}}$$

For a particular L and C, this is called the resonant frequency.

What is the resonant frequency for the previous example?

$L = 1.0\ H$

$C = 1.0\ \mu F$

$$f_R = \frac{1}{2\pi\sqrt{(1.0)(1\times10^{-6})}} = 159\ Hz$$

In the previous example, we were operating at 80 Hz.
Since we were operating below resonance, the capacitor was "stronger".

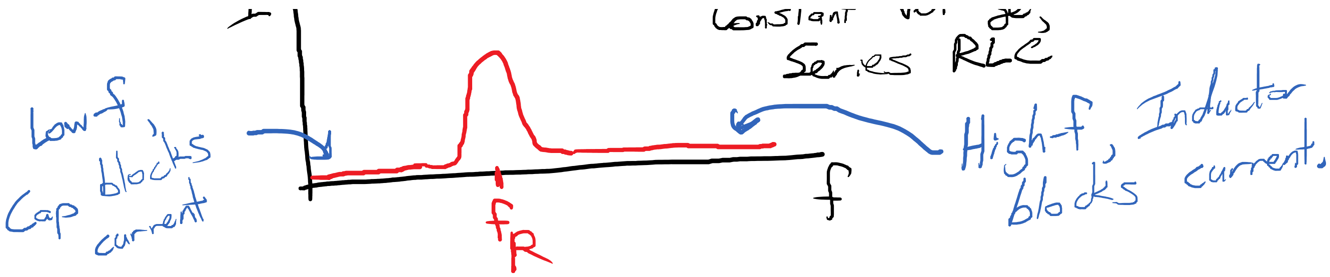
If we could operate at resonance, still with $\mathcal{E}_{rms} = 99\ V$,

$$I = \frac{V}{Z} = \frac{V}{R} = \frac{99}{1000} = 0.099\ A$$

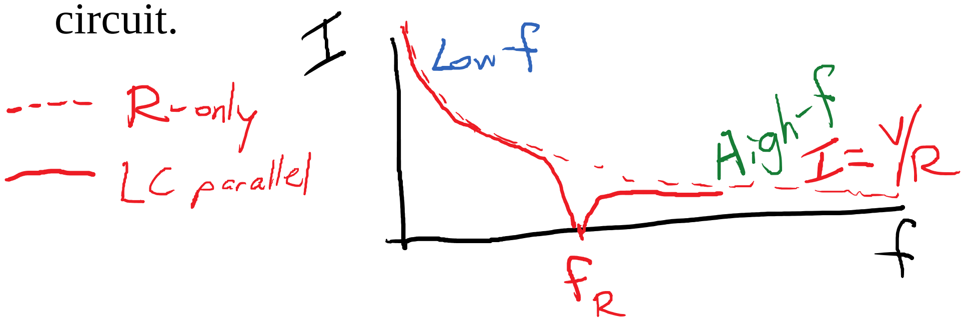
resonance

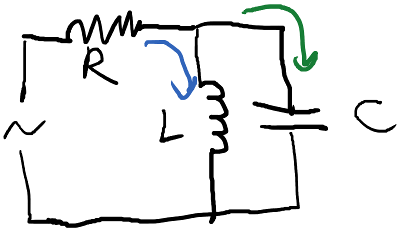
More current flows @ resonance.

Constant voltage,
Series RLC



The parallel RLC circuit is similar, but the math is harder.
The net result is that the current graph is inverted from the series RLC circuit.

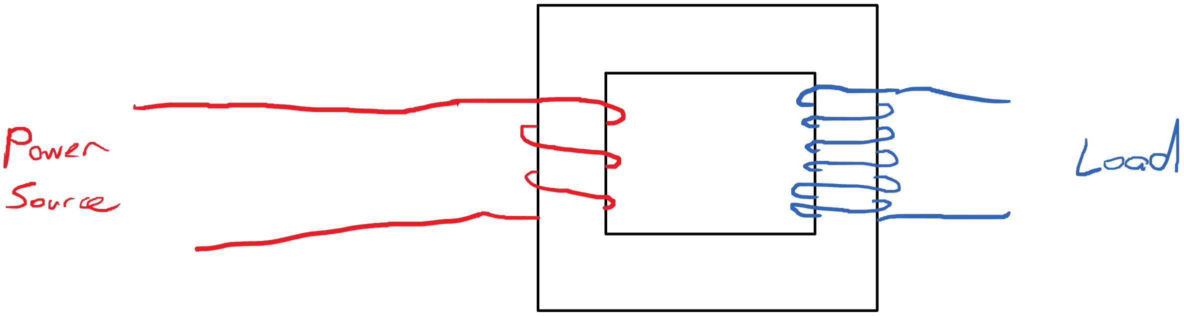




Lec 18 - Transformers

Wednesday, October 24, 2018 1:58 PM

A transformer is basically two coils that are intertwined.
They "share" the same magnetic field and magnetic flux.



The iron core serves two purposes:

- It increases the magnetism inside the coils. "Relative permeability" is a factor by which the magnetic field is increased. (~ 1000)
- It directs the magnetic field around in a loop, so that the amount of flux is the same everywhere.

Per-wire-loop Flux

$$\underset{\text{Flux}}{\Phi_B} = \underset{\text{Field}}{B}\, A \cos\theta$$

$$\Phi_{coil} = N \Phi_{loop}$$

We care about the flux because changing flux makes voltage.

- In the primary side (attached to the power source), the voltage limits the amount of current that flows. It is equal to the power source's voltage because they are in parallel.
- On the secondary side (attached to the load), the voltage acts as a power source for the load.
- It would be tough to calculate the voltages and currents, except that we know the core makes the per-loop flux the same on both sides.

$$\boxed{\frac{\mathcal{E}_s}{\mathcal{E}_p}} = \underbrace{\frac{\Delta\Phi_s/\Delta t}{\Delta\Phi_p/\Delta t}}_{\Delta\text{ Flux of each coil}} = \frac{\boxed{N_s}\, \cancel{\Delta\Phi_{loop}/\Delta t}}{\boxed{N_p}\, \cancel{\Delta\Phi_{loop}/\Delta t}}$$

So the voltage ratio is the same as the turns ratio.

What about the currents?

- An ideal transformer is very efficient.

$$\text{Effeciency} = \varepsilon = \frac{P_{out}}{P_{in}} \qquad P_s = \varepsilon P_p$$

$$\text{If } \varepsilon = 100\% \qquad P_s = P_p \qquad \text{(Avg Power)}$$

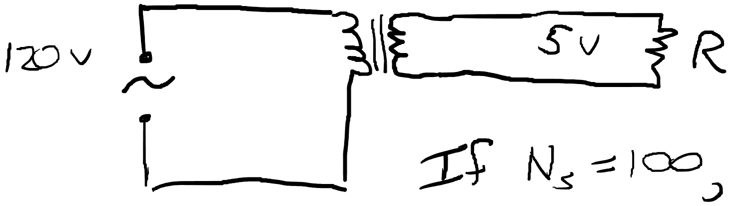
(All RMS values)

$$V_s I_s = V_p I_p$$

$$I_s = I_p \frac{V_p}{V_s}$$

$$\frac{I_s}{I_p} = \frac{V_p}{V_s} = \left(\frac{V_s}{V_p}\right)^{-1}$$

Ex: Transformer for a cell phone charger



If $N_s = 100$, how many loops in primary?

$$\frac{N_s}{N_p} = \frac{\mathcal{E}_s}{\mathcal{E}_p} \qquad \frac{5}{120} = \frac{100}{N_p}$$

$$N_p = 2400$$

If the phone draws 1.5 A, how much is drawn from the wall outlet?

$$\frac{I_s}{I_p} = \left(\frac{N_s}{N_p}\right)^{-1}$$

$$I_p = (1.5)\,\frac{100}{2400} = 0.0625\text{ A} \longleftarrow \frac{1.5\text{ A}}{I_p} = \left(\frac{100}{2400}\right)^{-1}$$

$I_p = (1.5)\ 2400 \qquad = \qquad I_p$

$$P_p = V_p I_p = (120\text{V})(0.0625\text{ A}) = 7.5\text{ W}$$

$$P_s = (5\text{V})(1.5\text{ A}) = 7.5\text{ W} \qquad \checkmark$$

Vampire Power

Wednesday, October 24, 2018 2:34 PM

Our transformers use some power when $I_s = 0$.

In this case, the transformer primary acts like an inductor with resistance.

$$L = 3.0\ \text{H} \qquad X_L = 2\pi f L = 2\pi(60\,\text{Hz})(3.0\,\text{H})$$

$$R = 2.5\ \Omega \qquad = 1130\ \Omega$$

$$Z = \sqrt{R^2 + X^2} = \sqrt{(2.5)^2 + (1130)^2} = 1130\,\Omega$$

Sqrt(2.5^2 + 1130^2) = 1130.002765483342

This draws some current from the wall.

$$V = IZ \rightarrow I = \frac{V}{Z} = \frac{120\ \text{V}}{1130\,\Omega} = 0.11\ \text{A}$$

How do we calculate the wasted (vampire) power?

$$P_R = V_R I_R = I^2 R = (0.11\,\text{A})^2(2.5\,\Omega) = 0.03\ \text{W}$$

How expensive is this, assuming we only get charged for this power?

$$P = \frac{\text{Energy}}{\text{Time}} \qquad \text{Energy} = P\Delta t = (0.03\,\text{W})(720\ \text{hours})$$

$$= 21.6\ \text{W}\cdot\text{h} = 0.0216\ \text{kWh}$$

$$\text{Cost} = \text{Rate}\cdot\text{Amount} = (\$0.12/\text{kWh})(0.0216\ \text{kWh})$$

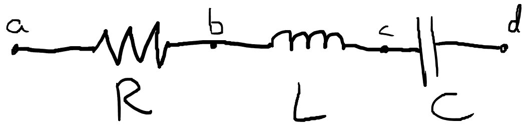
$$= \$0.002 = \left(\frac{1}{5}\right)\text{cent}$$

Building a 3.0 H transformer takes lots of material.
Could we get away with a 30 mH transformer coil instead?

- Yes, but the frequency (*f*) must be higher.
- Modern cell phone chargers start by oscillating at a very high frequency. This allows a tiny transformer that is still efficient.

Help with HW4-5

Wednesday, October 24, 2018 2:51 PM



Given: R, L, C, V_{Tot}, f

• $V_{t,rms} = \frac{V_T}{\sqrt{2}}$

$X = X_L - X_C$

$Z = \sqrt{R^2 + X^2}$

Want $V_{ab} = V_R = IR$

$V_t = I Z_T$

$V_{BC} = V_L = I X_L$

$V_{cd} = V_C = I X_C$

$(X_L - X_C)$

$V_{bd} = I Z_{LC}$

$Z_{LC} = \sqrt{X^2} = |X|$

$V_{ac} = I Z_{RL}$

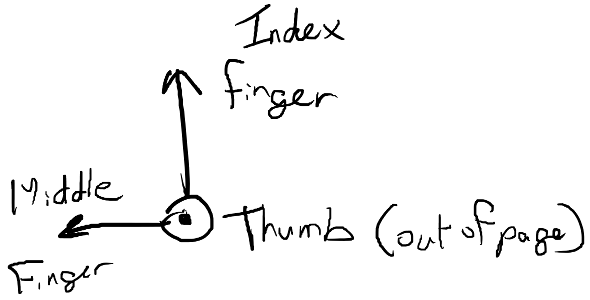
$Z_{RL} = \sqrt{R^2 + x^2}$

$X = X_L$

Right-hand Rule Review

Wednesday, October 24, 2018 3:05 PM

Electron moving Northward –
Want Force to be upward.
What B?



Flip b/c (e^-) : $B = \longrightarrow$

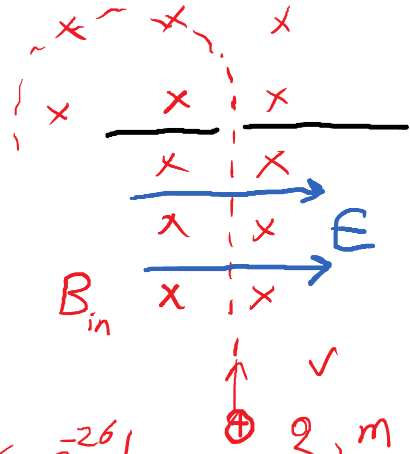
Lec 19 - Review

Monday, October 29, 2018 10:26 AM

Exam 2 Wednesday

- Magnetism
- AC Circuits

HW3-10 Mass Spectrometer



F_E = Right

F_B = Left

$qvB = mv^2/R$

$r = \frac{mv}{qB}$

$qE = qvB$

$v = E/B$

Use $q = +e = +1.6\times10^{-19}$ C

singly-charged ions

$m = 2.06\times10^{-26}$ kg

$B = 0.945$ T

$E = 935$ V/m

$\left.\right\}$ $v = E/B = 989.4$ m/s

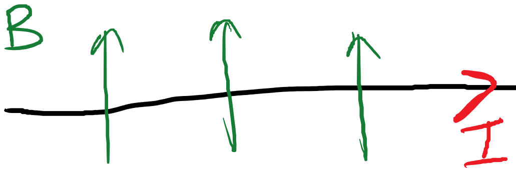
$r = \frac{mv}{qB} = 1.35\times10^{-4}$ m

Ex: Levitating a wire

Monday, October 29, 2018 2:15 PM

Current is pointing East, what B will levitate?

$$F_B = qvB$$



Want $F =$ "up"

$$\underset{\text{up}}{F_B} = \underset{\text{East}}{\underbrace{I\ell}}\,\underset{\text{Must be North}}{B}$$

AC Circuit

Monday, October 29, 2018 2:25 PM

Series RLC

$L = 2.0\ \text{H}$ $C = 50\,\mu\text{F}$ $f = 60\ \text{Hz}$

If we reduce f to 50 Hz but keep the same voltage, will the current increase or decrease?

$$f_R = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(2)(50\times10^{-6})}} = 15.9\ \text{Hz}$$

- We are tuning f to be closer to f_R.
- At resonance, $X_L = X_C$ $Z^2 = R^2 + X^2$

At resonance, X is zero, Z is minimized, and current (I) is maximized.
So the answer is: The current will increase.

$$V_{rms} = I_{rms} Z$$

Cost of energy

Monday, October 29, 2018 2:34 PM

$$P = \frac{\text{Energy}}{\text{Time}}$$

$$\text{Rate} = \frac{\text{Cost}}{\text{Energy}} \approx \$0.12/\text{kWh}$$

$$\text{Energy} = \underset{\text{In kW}}{P}\;\underset{\text{In hours}}{\Delta t}$$

A cell phone uses 100 mA at 5 V DC. How much does it cost to operate a phone continuously for a month?

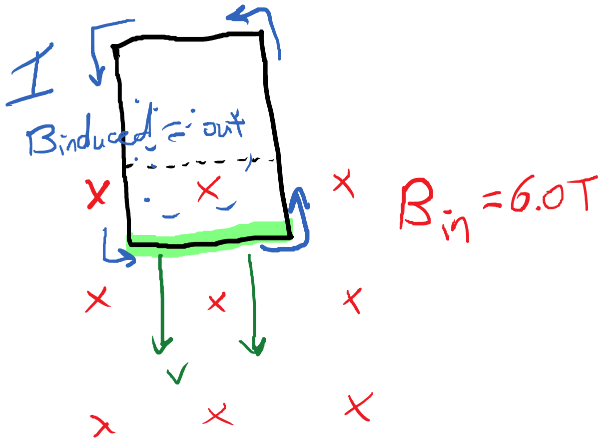
$$P = VI = (5\text{ V})(100 \times 10^{-3}\text{ A}) = 0.5\text{ W}$$

$$\text{Energy} = P\Delta t = (0.5\,\cancel{\text{W}})(1\,\cancel{\text{month}})\left(\frac{1\text{ kW}}{1000\,\cancel{\text{W}}}\right)\left(\frac{30\,\cancel{\text{day}}}{1\,\cancel{\text{month}}}\right)\left(\frac{24\text{ hr}}{1\,\cancel{\text{day}}}\right)$$

$$= 0.36\text{ kWh}$$

$$\text{Cost} = \text{Rate} \cdot \text{Amount} = (\$0.12/\text{kWh})(0.36\text{ kWh}) = \$0.04$$

$\Phi_B = \cancel{N}BA\cancel{\cos\theta}$



Φ_B is increasing

Lenz's law: B_{loop} = opposite to B_{in}

B_{loop} = out of page

I_{loop} = CCW

width = 2.0 m
height = 3.0 m
mass = 0.6 kg

If loop falls w/ constant velocity,

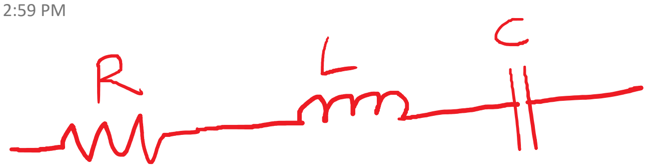
$F_B = F_g$

$I\ell B = mg$

$I = \frac{(0.6)(9.8)}{(2.0)(6.0)} = 0.49\,A$

Series RLC

Monday, October 29, 2018 2:59 PM



$Z^2 = R^2 + X^2$

f = 60 Hz

$\mathcal{E} = 120\,V$ RMS

$I = 0.08\,A$ RMS

$R = 900\,\Omega$

$L = 5.3\,H$

$C = ?$

$Z = \frac{120\,V}{0.08\,A} = 1500\,\Omega$

$X_C = \frac{1}{2\pi f C}$

$X_L = 2\pi f L$

$= 2\pi(60)(5.3)$

$= 2000\,\Omega$

$1500^2 = 900^2 + X^2$

could be ⊖

$X = \sqrt{1500^2 - 900^2} = 1200\,\Omega$

$X_L - X_C = X$

$2000 - X_C = 1200$

$X_C = 800\,\Omega$

$800 = \frac{1}{2\pi(60)C}$

$C = \frac{1}{2\pi(60)(800)} = 3.32\times10^{-6}\,F = 3.3\,\mu F$

$P = V_R I = I^2 R = (0.08)^2(900) = 5.76\,W$