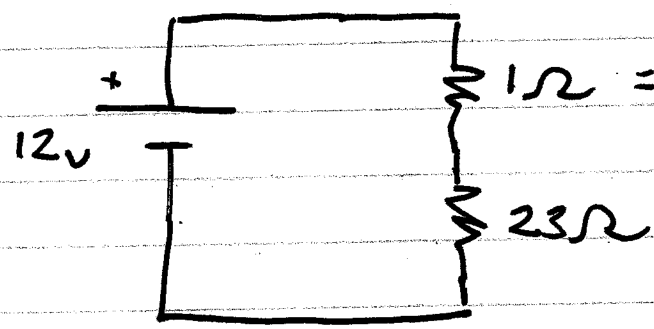
Phys 2426 2014-09-24 Lec 8

Related to HW2 8(b):



1Ω = internal resistance

$(12\ v)I$ = Power produced

For 1Ω resistor $V = 0.5\ V$

$(0.5\ v)I$ = Power wasted internally

For 23Ω resistor $V = 11.5\ V$

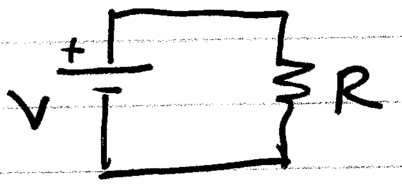
$(11.5\ V)I$ = Power delivered to load

$$\text{Efficiency} = \frac{P_{delivered}}{P_{produced}} = \frac{11.5}{12} = \frac{23}{24}$$

$$\text{Waste} = \frac{0.5}{12} = \frac{1}{24}$$

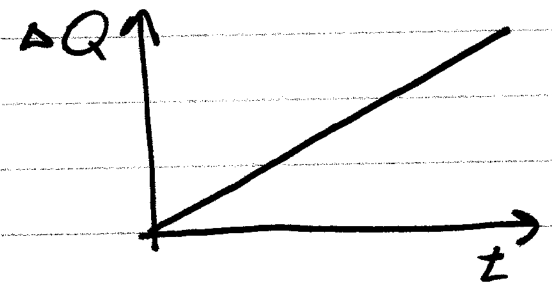
Charging & Discharging capacitors

With a battery:

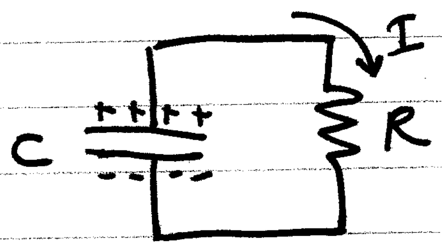


$$I = \frac{V}{R} = \text{const}$$

$$I = \frac{\Delta Q}{\Delta t} \rightarrow \Delta Q = \int I\,dt$$



With a capacitor



$$V_c = Q/C \qquad V_R = IR$$

$$I = \frac{V_R}{R} = = \frac{Q}{RC}$$

As Q decreases, I decreases.

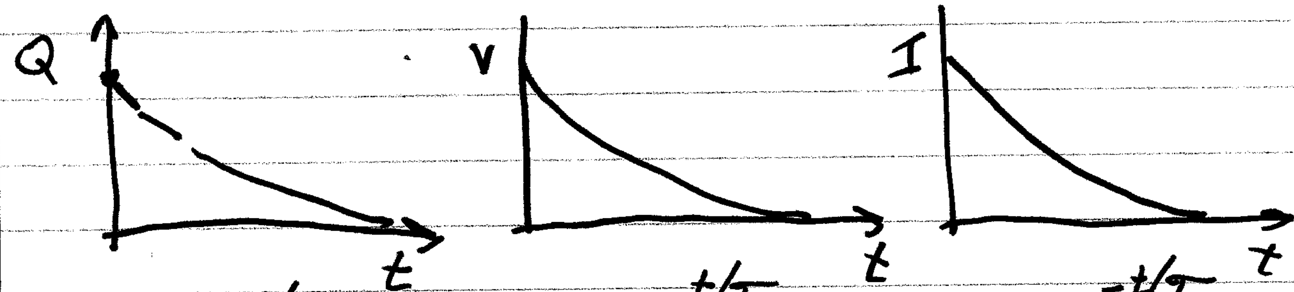
$$I = \frac{1}{RC} Q = \frac{-\Delta Q}{\Delta t} \qquad \frac{dQ}{dt} = \frac{-1}{RC} Q$$

Solution: $Q = Q_0 e^{-t/RC}$

$$\frac{dQ}{dt} = \underbrace{Q_0 e^{-t/RC}}_{Q} \left(\frac{-1}{RC}\right)$$

Discharging a Capacitor

	Initially	...and then
Q	Q_0	decreases
V	$V_0 = Q_0/C$	decreases
I	$I_0 = V_0/R$	decreases



$Q_0 e^{-t/\tau}$ $\quad$ $V_0 e^{-t/\tau}$ $\quad$ $I_0 e^{-t/\tau}$

τ = "time constant" = RC

Bigger R = slower drain

Bigger C = bigger bucket

Ex: $V_0 = 100\text{ V}$ $\qquad$ $\tau = 5\text{ s}$

t	V
$t=0$	$V = V_0 = 100\text{ V}$
1	$V = (100\text{V})\,e^{-1/5} = (100\text{V})(0.82) = 82\text{ V}$
2	$V = \qquad = 67\text{ V}$
⋮	
$\tau = t = 5\text{ s}$	$= 37\text{ V}$

After one time constant, V is 37% of V_0.

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$$V = V_0 e^{-t/\tau}$$

V = voltage at a given time
t = current time on stopwatch
V_0 = initial voltage
τ = time constant

$\frac{V}{V_0}$ = fraction of remaining charge.

$\frac{t}{\tau}$ = current time in "time constants"

Half-life is time to reach 50%.

$$\frac{V}{V_0} = 0.5 = e^{-t/\tau}$$

$$\ln(0.5) = -t/\tau$$

$$t = -\tau \ln(0.5) = 0.693\tau$$

Useful Eqns:

$V_C = Q/C \qquad V_R = IR \qquad \tau = RC$

Ex: At $t=0$, $V = 6\,V$ $\qquad V_0 = 6$
$t = 2.5\,s$ $\quad V = 4\,V$
$R = 1\,k\Omega$
$C = \tau/R$

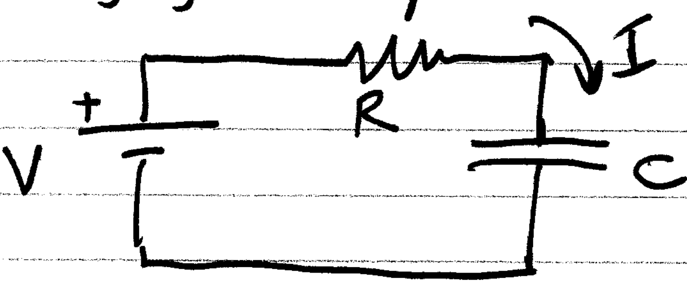
$$e^{-2.5/\tau} = \frac{4}{6}$$

$$-2.5/\tau = \ln(4/6)$$

$$\tau = -2.5/\ln(4/6)$$

$$= 6.17\ s$$

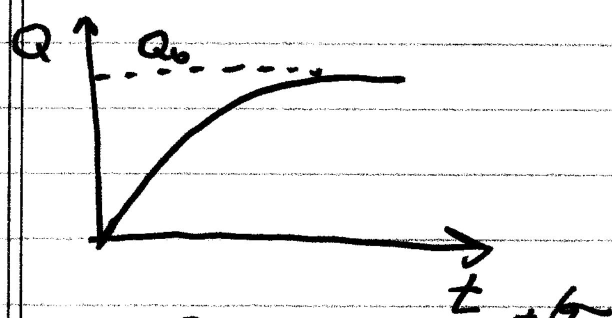
Charging a capacitor



$$V_{batt} = V_R + V_C$$

	Initially	...and then	Eventually
Cap	$Q = 0$	Q increases	$Q_0 = CV_0$
	$V_c = 0$	V_c increases	$V_c = V_0$
Batt	V_0	V_0	V_0
R	$V_R = V_0$	V_R decreases	$V_R = 0$
	$I = V_0/R$	I decreases	$I = 0$

Initially, cap acts like a wire.



$$I = I_0 e^{-t/\tau}$$

$$Q_0 - Q = Q_0 e^{-t/\tau}$$

$$Q = Q_0\left(1 - e^{-t/\tau}\right) \qquad V = V_0\left(1 - e^{-t/\tau}\right)$$

$\left(1 - e^{-t/\tau}\right)$ is fraction of final charge

$e^{-t/\tau}$ is fraction of process remaining