Phys 2426 2014-10-08

Magnetic Sources

Long, Straight Wire	Coil	Solenoid
$B = \frac{\mu_0 I}{2\pi r}$	$B = \frac{\mu_0 I}{2R}$ (@ Center)	$B = \mu_0 I N/l$ (Inside)

Calculating B in general:

1) Biot-Savart Law - break source into bits and add up contributions of the bits.
2) Ampere's Law - The integral of the mag field depends simply on the source.

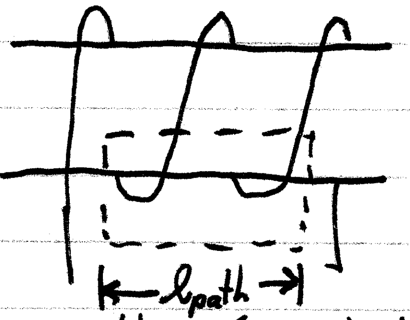
$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{enc}$$

$\oint$ Integration takes place along an enclosed ~~pat~~ looping path

$\vec{dl}$ is length of part of path w/ direction.

• is a dot product
If $\vec{B}$ and $\vec{dl}$ are in the same dir, just Bdl.
If they are ⊥, the dot prod is zero.

Ampere's Law for a Solenoid



① Choose a path (Dashed Line)

② Determine if $B=0$ anywhere. (Bottom segment of path)

③ Determine if $B \perp d\ell$ anywhere. (Left + right segments)

④ Evaluate $\oint \vec{B} \cdot d\vec{\ell}$ for rest of path.

For top segment $\oint \vec{B} \cdot d\vec{\ell} = B \, l_{path}$

⑤ How much current is enclosed?

$$I_{enc} = I \, N_{enc}$$

Easy derivation: assume $l_{path} = l_{solenoid}$ then $N_{enc} = N$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

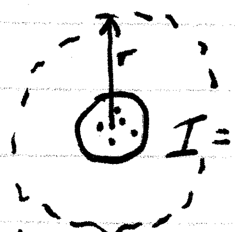
$$B\ell = \mu_0 I N$$

$$B = \mu_0 I N / \ell$$

③

Ampere's Law in Cylindrical Symmetry

Current flowing in a long, straight, thick wire.



I = spread throughout cross-section

Outside the wire:

$$\oint \vec{B} \cdot d\vec{l} = B\, l_{path} = 2\pi r B$$

$$I_{enc} = I = \text{entire current}$$

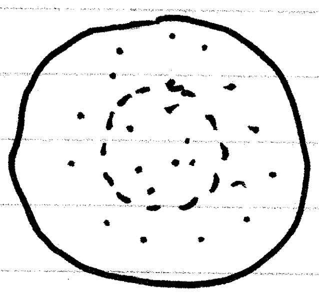
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$2\pi r B = \mu_0 I \longrightarrow B = \frac{\mu_0 I}{2\pi r}$$

If we are outside the wire, the thickness doesn't matter.

<u>Inside the Wire:</u>

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$



$$2\pi r B = \mu_0 I \frac{A_{path}}{A_{wire}}$$

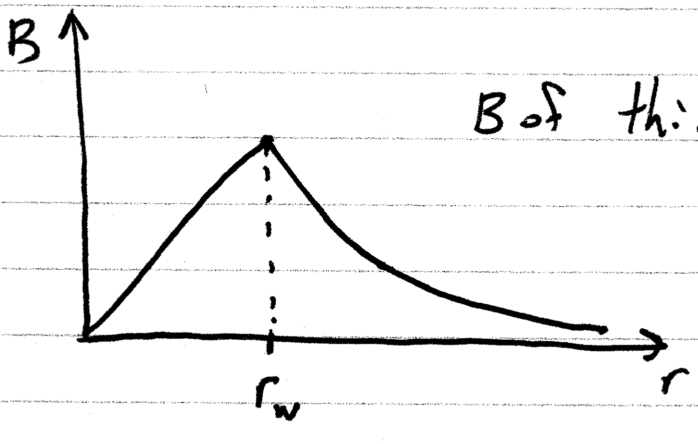
$$= \mu_0 I \frac{\pi r^2}{\pi r_w^2}$$

→ r = path radius

⟶ r_w = wire radius

$$B = \frac{\mu_0 I r}{2\pi r_w^2}$$

④



B of thick wire

$$\frac{\mu_0 I r}{2\pi r_w^2} \qquad \frac{\mu_0 I}{2\pi r}$$

Summary of result: $B = \dfrac{\mu_0 I_{enc}}{2\pi r}$

Ex: 5A flowing in a 1 cm radius wire.

(a) 5 cm from middle $\quad B = \dfrac{\mu_0 (5A)}{2\pi (5 cm)} = 20\,\mu T$

(b) surface of wire $\quad B = \dfrac{\mu_0 (5A)}{2\pi (0.01\,m)} = 100\,\mu T$

(c) 0.5 cm from middle $\quad B = \dfrac{\mu_0 (1.25A)}{2\pi (0.005\,m)} = 50\,\mu T$

$I_{enc} = 1.25A = \frac{1}{4}(5A)$

$$B = \frac{\mu_0 (5A)(0.005\,m)}{2\pi (0.01\,m)^2}$$

Magnetic Effects

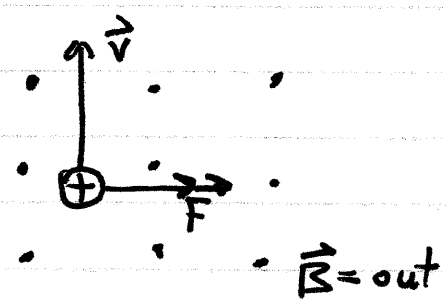
Force on moving charge — cross product

$$\vec{F} = q\vec{v} \otimes \vec{B}$$

RHR for cross product:

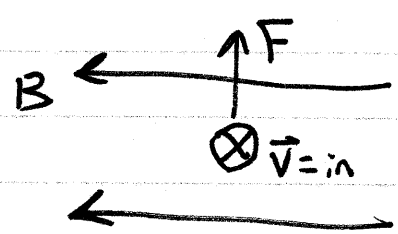
① Thumb is the result (Force on ⊕)
② Index finger is 1st vector ($\vec{v}$)
③ Middle finger is 2nd vector ($\vec{B}$)

Ex: Proton moving toward top of page.
Mag field pointing out of page



Force is to right.

Ex: Proton moving down into page. Want F toward top of page. } What $\vec{B}$? To Left.



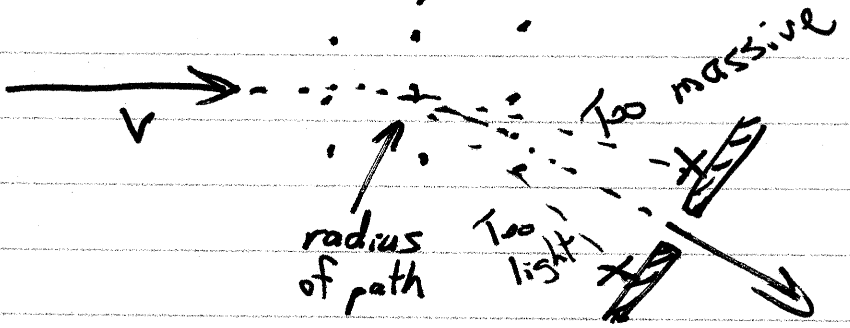
⑥

Ex: $\vec{B}$ points North.
Drop an electron.
What dir is the force?
RHR has thumb point East.
⊖ electron means force is west.

What path does the particle take?
Hint: F is always ⊥ $\vec{v}$.
The path is circular.

$$F = ma$$
$$qvB = mv^2/r$$
$$r = \frac{mv}{qB}$$

Used for mass spectrometer



If particle is too massive: The path is straighter.
" " " too light: The path bends more.

(7)

Velocity Selector

Make F_E and F_B equal but opposite.

$$\left.\begin{aligned} F_E &= qE \\ F_B &= qvB \end{aligned}\right\} \quad \begin{aligned} E &= vB \\ v &= E/B \end{aligned}$$