① Phys 2426 2014-10-20

## AC Waveform

$$V(t) = V_0 \sin(2\pi f t + \phi_0)$$

Amplitude ($V_0$) Frequency ($f$) Phase ($\phi_0$)

$$= V_0 \sin(2\pi f (t - t_0))$$

Resistor: @ every instant $V = IR$

$$I(t) = \frac{V_0}{R} \sin(2\pi f t + \phi_0)$$

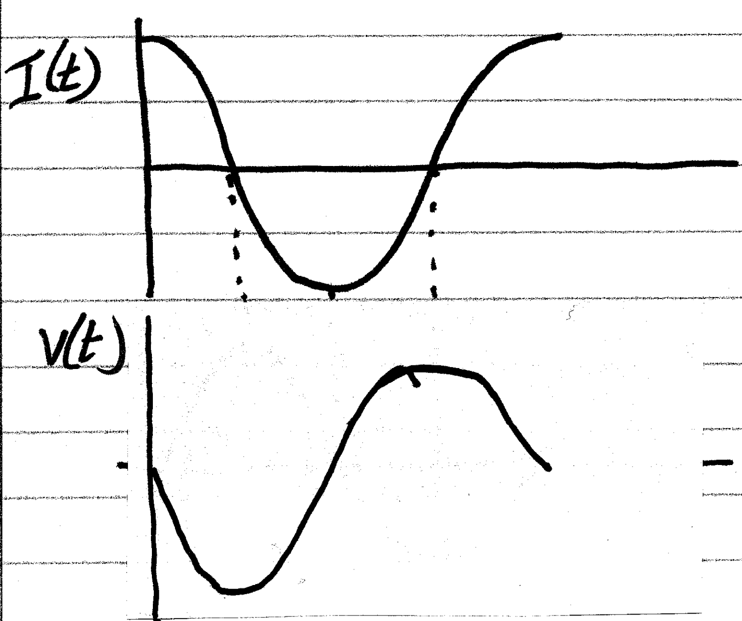
$$V_0 = I_0 R$$

Current Amplitude ($\frac{V_0}{R}$)

No change in $f$ or $\phi_0$.

Inductor:

$$V = +L \frac{dI}{dt}$$




$$I(t) = I_0 \cos(2\pi f t + \phi_0)$$

$$V = +\bar{L} I_0 \sin(\sim)(2\pi f)$$

$$= \underbrace{\bar{I}_0\, 2\pi f L}_{V_0} \sin(\sim)$$

$$V_0 = I_0 X_L$$

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$X_L$ = inductive reactance in ohms ($\Omega$)

Also called $Z_L$ = inductive impedance

$$Z_L = 2\pi f L \qquad V_0 = I_0 Z_L$$

Ohm's Law for AC

Note: f didn't change.

$$\sin(\theta) = \cos(\theta - \pi/2)$$

The current and voltage in the inductor are not in phase.

Also, $Z_L$ depends on f!

Low-f: $Z_L = 2\pi f L$ = small.

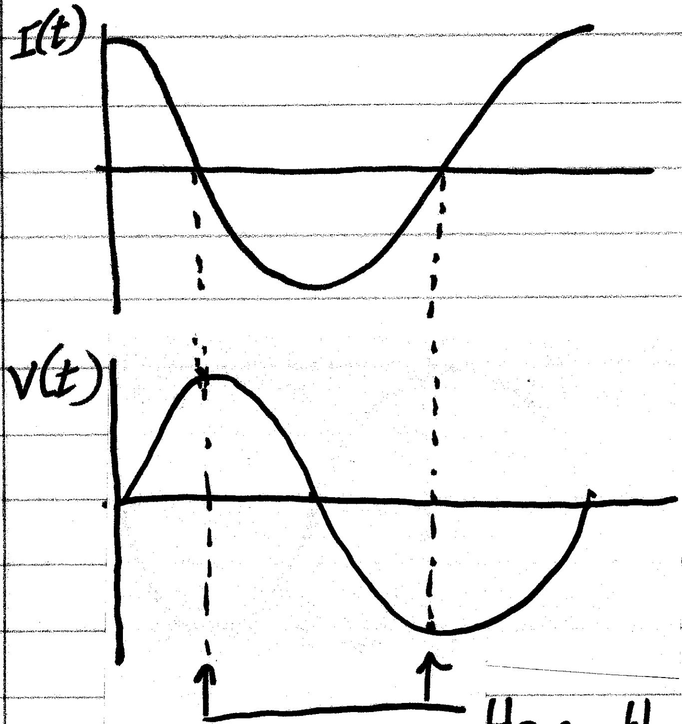
Current can be high w/o voltage.

High-f: $Z_L = 2\pi f L$ = huge

High-f current is blocked.

Remember: Inductor hates change (in current)

## 3 Capacitor in AC




Here, the cap is "charged"

$$V_c = Q/C$$

The change in V is a change in Q.
As Q decreases, $I$ goes out.
$I(t)$ is current flowing ~~out~~ in
is ~~⊕~~ ⊖

$$V = Q/C$$

$$\frac{dV}{dt} = \frac{-I}{C}$$

$$V = V_0 \sin(2\pi f t + \Phi_0)$$

$$I = -C\frac{dV}{dt} = -C V_0 \cos(2\pi f t + \phi_0)(2\pi f)$$

$$= -\underbrace{2\pi f C V_0}_{I_0 \,=\, \text{current amplitude}}$$

$$Z_c = \frac{1}{2\pi f C} \qquad V_0 = I_0 Z_C$$

Low-f $\rightarrow$ high $Z_C$
$f=0 \rightarrow Z_C = \infty$
High-f $\rightarrow$ Low $Z_C$

# Combining Components - Series AC

DC: Resistances add

AC: Impedances add like vectors

$Z_R = R$ in x-direction

$Z_L = 2\pi f L$ in y-direction

$Z_C = \frac{1}{2\pi f C}$ in (-y)-direction

$|\vec{Z}_{Tot}|$ is net impedance in series

$$\left.\begin{array}{l} Z_R = \text{total } R \\ \left.\begin{array}{l} Z_L = \text{total } Z_L\text{'s} \\ Z_C = \text{total } Z_C\text{'s} \end{array}\right\} X = Z_L - Z_C \end{array}\right\} Z = \sqrt{R^2 + X^2}$$

More resistors in series always increases Z.

More L's or C's could increase or decrease Z.

Z depends on f!

Ex: 120 V

30 $\Omega$ resistor

$$I = \frac{V}{R} = \frac{120\,V}{30\,\Omega} = 4A$$

If $f = 60$ Hz, what C would have same Z?

$$Z_c = \frac{1}{2\pi f C} \rightarrow C = \frac{1}{2\pi f Z_c} = 8.8 \times 10^{-5}\,F$$
$$= 88\,\mu F$$

What L would have the same Z?

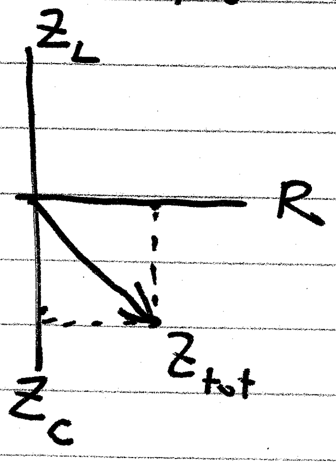
$$Z_L = 2\pi f L \qquad L = \frac{Z_L}{2\pi f} = 0.080\,H = 80\,mH$$

(For ref: "Outer coil" had $L = 78\,\mu H$)

What if $R = 30\,\Omega$ and $C = 88\,\mu F$ are in series?

$$\left.\begin{array}{l} Z_R = 30\Omega \\ X = Z_L - Z_c = -30\Omega \end{array}\right\} Z = \sqrt{30^2 + 30^2} = 42.4\,\Omega$$

What about R + L?




What about L + C?

$$\left.\begin{array}{l} X = 0 \\ R = 0 \end{array}\right\} Z = 0$$

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If $Z_L = Z_C$ it is called resonance.

The inductor & capacitor cancel each other.

$$Z_L = 2\pi f L \qquad Z_C = \frac{1}{2\pi f C}$$

$$2\pi f L = \frac{1}{2\pi f C}$$

$$(2\pi f)^2 = \frac{1}{LC}$$

$$2\pi f = \frac{1}{\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

@ resonant $f$, $X = 0$ for series $LC$

@ high-$f$ $Z_L$ is larger and dominates.

@ low-$f$ $Z_C$ is larger and dominates.

@ $f_R$ $R$ is largest