1 Phys 2426 2014-11-03

Oscillations - Chap 15

Any back-and-forth pattern in time.

- Frequency f in hertz (Hz)
  #oscillations per second
- Period T
- Amplitude - deviation from equilibrium.
- Equilibrium - middle value; steady state
- Restoring Force
- Momentum
- Energy

Example: Spring & mass

Spring Force $F = -k x_s$

x_s = "stretch" of spring

k = spring constant

Mass Force $F_{net} = m a$

Combined $-kx = ma$

$$a = \frac{-k}{m} x$$

$$-\omega^2 A \sin(\omega t) = \frac{-k}{m} A \sin(\omega t)$$

$$\omega^2 = k/m$$

$$\omega = \sqrt{k/m}$$

Guess $x = A \sin(\omega t)$

$\dot{x} = \omega A \cos(\omega t)$

$\ddot{x} = -\omega^2 A \sin(\omega t)$

(2)

Current Solution $x = A\sin(\omega t)$

AC Voltage $V = V_{max}\sin(2\pi f t)$

Amplitude	A	V_{max}
Frequency	$f = \frac{\omega}{2\pi}$	f

$$f = \frac{1}{2\pi}\sqrt{k/m}$$

Energy of mass-spring system

Spring: $PE = \frac{1}{2}k x_s^2$

min when $x = 0$

max when $x = \pm A$

Mass: $KE = \frac{1}{2}mv^2$

min when $x = \pm A$

max when $x = 0$

Max KE = Max PE

$$\frac{1}{2}mv^2 = \frac{1}{2}kA^2$$

$$V_{max} = \sqrt{\frac{k}{m}}\,A$$

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Ex: Car suspension

Static: $m = 1000\ kg$ causes $x = 10\ cm$ deflection

$$F_g = mg = 9800\ N$$

$$|F_s| = kx$$

$$9800\ N = k\,(0.1\ m)$$

$$k = 98\,000\ N/m$$

Dynamic: $f = \frac{1}{2\pi}\sqrt{k/m}$

$$= \frac{1}{2\pi}\sqrt{\frac{98000\ N/m}{1000\ kg}}$$

$$= \frac{1}{2\pi}\sqrt{98} = 1.6\ Hz$$

$$T = \frac{1}{f} = \frac{1}{1.6\ Hz} = 0.63\ s$$

Things that don't affect the freq:

Amplitude: A can be any amount

Phase; Time can be shifted

$$\sin(2\pi f t) \rightarrow \sin(2\pi f t + \phi) \rightarrow \cos(2\pi f t)$$

$$\cos(\pi/2 - \theta) = \sin\theta = \cos(\theta - \pi/2)$$

Back to the car: $m = 1000\ kg$
$k = 98000\ \cancel{kg}\ N/m$

Energy to push down 5 cm?

$$PE_s = \frac{1}{2}kx^2 = \frac{1}{2}(98000\ N/m)(0.05\,m)^2$$
$$= 123\ J$$

Max speed of bumper when you let go:

$$KE = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2 \cdot (123J)}{1000\ kg}} = 0.5\ m/s$$

Example Oscillators:

- Mass + spring
- Pendulum for small swings
- Pressure in a sound wave
- Water height in a water wave
- Displacement of a string
- Electric + magnetic Fields

On Wed: Chap 16.