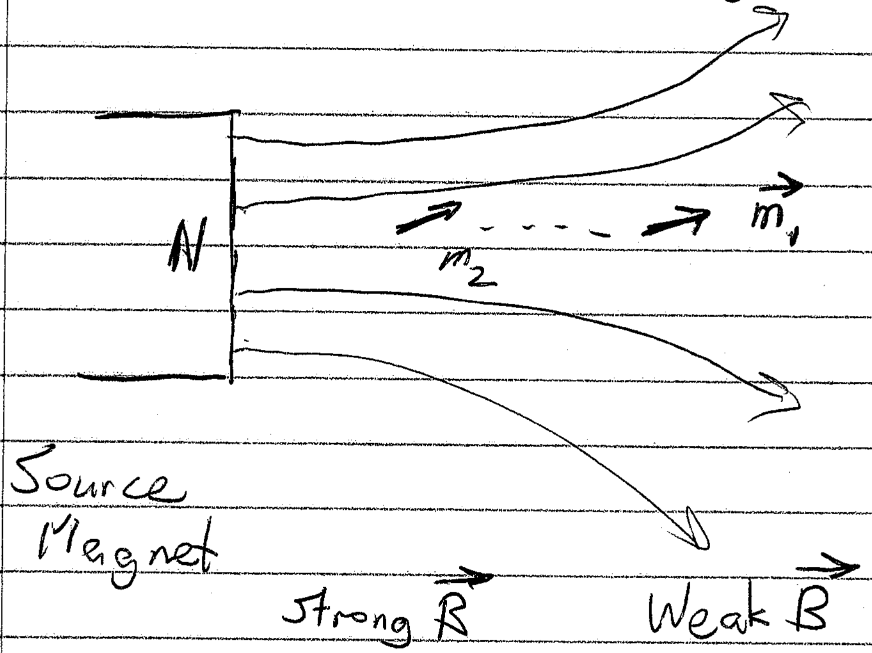
Phys 2426 2015-10-06 Lec 12

Effects of Magnetism - magnetic Moment



$$\overset{\text{pot energy}}{U} = -\underset{\text{moment}}{\vec{m}} \cdot \underset{\text{mag field}}{\vec{B}}$$

U is low (negative) if $\vec{m} \parallel \vec{B}$
This aligns compass needle w/ $\vec{B}$

Derivative of $-U$ is a force

$$U = -mB\cos\theta \qquad \frac{-dU}{d\theta} = (-)(-mB)(-\sin\theta)$$
$$= -mB\sin\theta$$

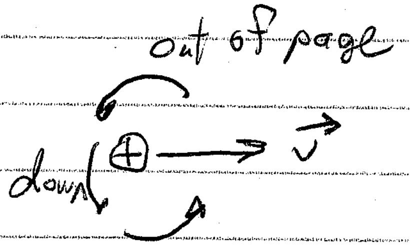
$$|\tau| = mB\sin\theta$$

$\vec{m} \propto \vec{B}$	Low U	Zero τ
$\vec{m} \perp B$	Zero U	high τ
$\vec{m} \propto -\vec{B}$	High U	zero τ (unstable)

$m = NAI$ (coil)

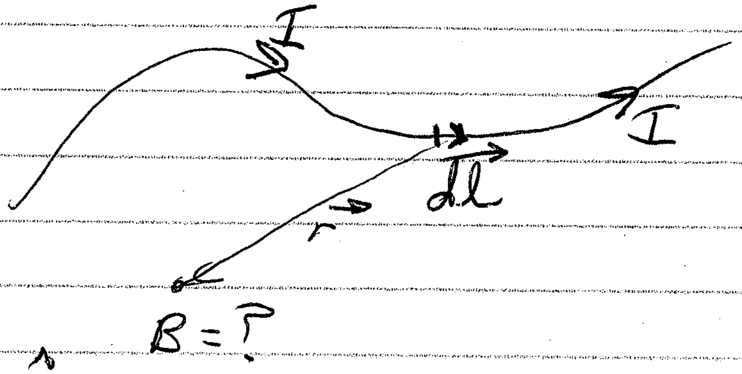
② Sources of $\vec{B}$

Moving charge



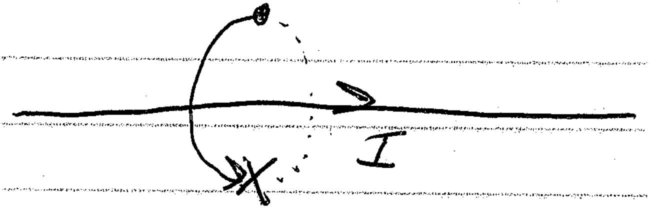
B is strong near the charge, weak further away.

Bit of Current



$$\vec{B} = \int \frac{\mu_0 I \vec{d\ell} \otimes \hat{r}}{4\pi r^2}$$

Long Straight Wire

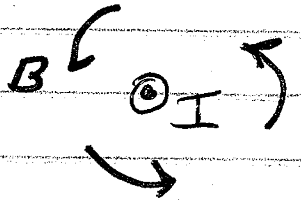


• = out of page
× = into page

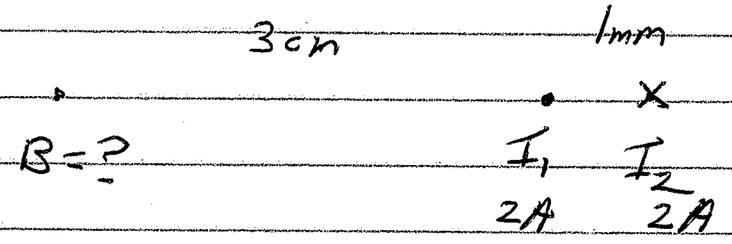
$$B = \frac{\mu_0 I}{2\pi r}$$

RHR for straight wire

- Thumb in dir of current →
- Fingers curl in dir of B



Parallel Wires



$$B_1 = \frac{\mu_0 (2A)}{2\pi (0.03\,m)} = 13.3\,\mu T \qquad \text{(down)}$$

$$B_2 = \frac{\mu_0 (2A)}{2\pi (0.031\,m)} = 12.9\,\mu T \qquad \text{(up)}$$

Total : $B = 13.3\mu T - 12.9\mu T = 0.4\mu T$ (down)

Analytical Approx

$$B_1 = \frac{\mu_0 I}{2\pi R} \qquad\qquad B_2 = \frac{\mu_0 I}{2\pi (R+\delta)}$$

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{R} - \frac{1}{R+\delta}\right)$$

$$(1+x)^n \approx 1+nx$$

$$= \frac{\mu_0 I}{2\pi}\frac{1}{R}\left(1 - \frac{1}{1+\delta/R}\right)$$

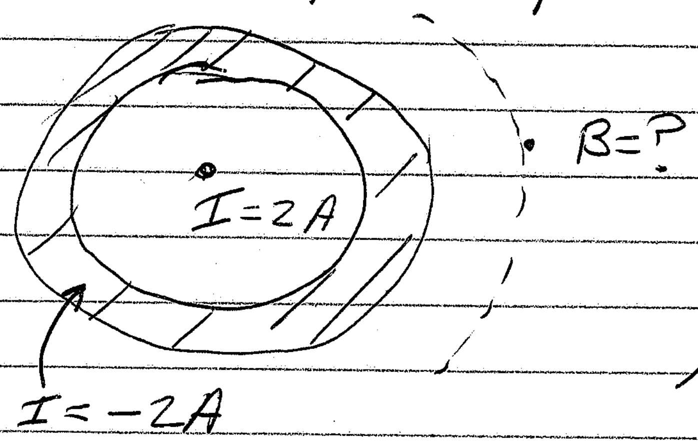
$$= \sim\!\sim \left(1-\left(1-\delta/R\right)\right) = \frac{\mu_0 I \delta}{2\pi R^2}$$

⑤

Ampere's Law - relates source and mag field without integrating source.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{enc}$$

Coaxial Symmetry



$$\oint \vec{B} \cdot \vec{dl} = 2\pi r B$$

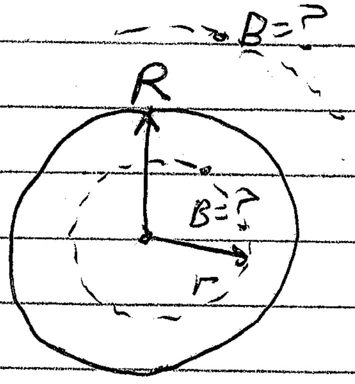
$$\mu_0 I_{enc} = \big((2A) - (2A)\big)\mu_0$$
$$= 0$$

B=0 outside coax.

B inside a wire

Outside: $2\pi r B = \mu_0 I_{Tot}$

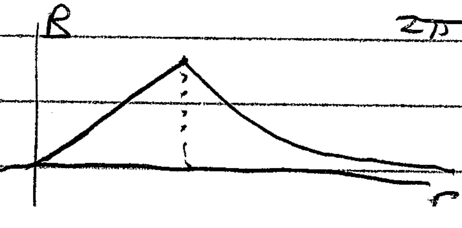
$$B = \frac{\mu_0 I}{2\pi r}$$



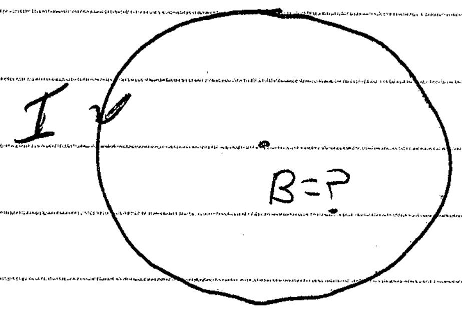
Inside: $2\pi r B = \mu_0 I_{Tot} \left(\frac{r}{R}\right)^2$

$$B = \frac{\mu_0 I r}{2\pi R^2}$$

$I_{Tot} = 2A$
$R = 5cm$



Coil or Loop



$$\vec{B} = \int \frac{\mu_0 I \, \vec{dl} \otimes \hat{r}}{4\pi r^2}$$

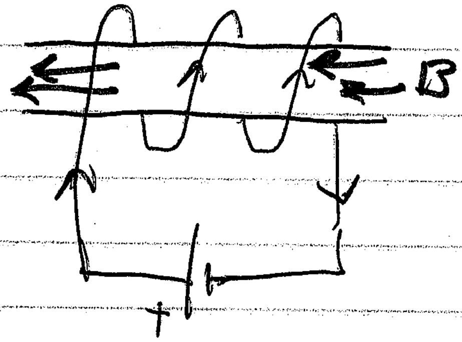
$$= \frac{\mu_0 I}{4\pi r^2} \cancelto{2\pi r}{\int dl}$$

$$B = \frac{\mu_0 I}{2R}$$

RHR for Coil

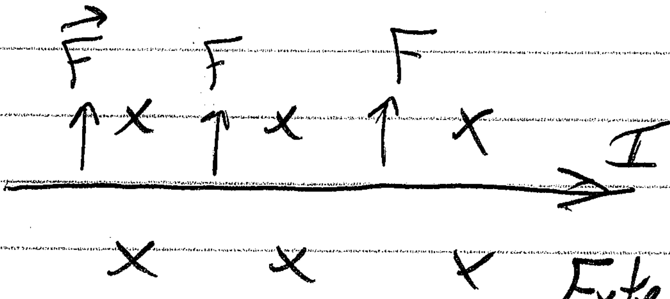
- Curl fingers in dir of current
- Thumb points in dir of $\vec{B}$ inside loop,

Solenoid Coil



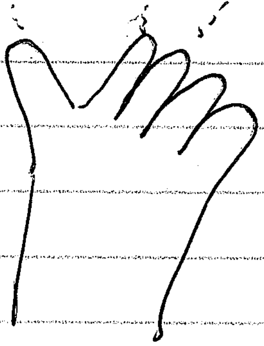
$$B = \frac{\mu_0 I N}{l}$$

Force on a wire



Externally-caused
B

$F = I l B \sin\theta$



Index = I
Middle = magnetic
Thumb = F

Proton beam pointing up.
Mag Field points North.
Force = ? → West