

① Phys 2426

2015-10-20

Lec 16

Exam 2 Thu 10/29

Covers	Magnetism	Chap	29-30
	EM Induction		31-32
	AC Circuits		33

AC Voltages Oscillate

Ex: $V_{\max} \sin(2\pi ft)$

$$V_{\max} \sin(2\pi ft + \phi)$$

↑
Phase angle

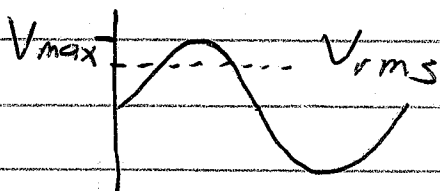
$$V_{\max} \sin(2\pi f(t - t_0))$$

$$V_{\max} \cos(2\pi ft)$$

$$V_1 \sin(2\pi ft) + V_2 \cos(2\pi ft)$$

→ $V_{\max} = \sqrt{V_1^2 + V_2^2}$

$$\sqrt{2} V_{\text{rms}} \sin(2\pi ft)$$



$$V_{\text{rms}} = \frac{1}{\sqrt{2}} V_{\max}$$

②

Resistors Ohm's Law $V = IR$

In AC: $V(t) = I(t) R$

$$\sqrt{2} V_{rms} \sin(2\pi ft) = I(t) R$$

$$I = \sqrt{2} \frac{V_{rms}}{R} \sin(2\pi ft)$$

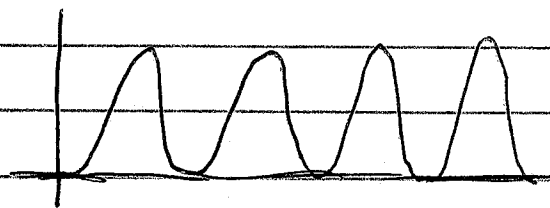
I_{rms}

Ohm's Law for R in AC: $V_{rms} = I_{rms} R$

Power: $P = VI$

$$P(t) = \sqrt{2} V_{rms} \sin(2\pi ft) \sqrt{2} I_{rms} \sin(2\pi ft)$$
$$= 2 V_{rms} I_{rms} \sin^2(2\pi ft)$$

$$\sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta)$$



$$P(t) = V_{rms} I_{rms} (1 - \cos(2 \times (2f)t))$$

$$P_{avg} = V_{rms} I_{rms}$$

Pulses are at
double the freq.

③

Inductors

$$\mathcal{E} = -L dI/dt$$

Inductor's EMF opposes change in current.

Resistors oppose current but $V = IR$

with no minus sign. This is the "voltage drop" of the resistor.

So, voltage drop of inductor is:

$$V_L = L \frac{dI}{dt}$$

Suppose $I = \sqrt{2} I_{rms} \sin(2\pi f t)$

$$V_L = L \left(\sqrt{2} I_{rms} \cos(2\pi f t) 2\pi f \right)$$

$$= \sqrt{2} \underbrace{(2\pi f L)}_{\text{Like R}} I_{rms} \cos(2\pi f t)$$

$$V_{rms} = \underbrace{2\pi f L}_{\text{Like R}} I_{rms}$$

$X_L = 2\pi f L$ inductive reactance

Just inductor: $Z = X_L$ impedance

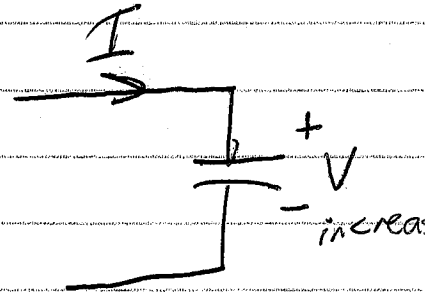
$$V_{rms} = I_{rms} Z \quad \text{AC Ohm's Law}$$

(4)

Capacitors

$$Q = CV$$

$$I = \frac{dQ}{dt} = C \frac{dV}{dt}$$



$$\sqrt{2} I_{rms} \sin(2\pi ft) = C \frac{dV}{dt}$$

$$\frac{dV}{dt} = \frac{\sqrt{2} I_{rms} \sin(2\pi ft)}{C}$$

$$V(t) = \frac{\sqrt{2} I_{rms} (-\cos(2\pi ft) \frac{1}{2\pi f})}{C}$$

$$= -\sqrt{2} \underbrace{\frac{1}{2\pi f C}}_{V_{rms}} I_{rms} \cos(2\pi ft)$$

$$V_{rms} = \frac{1}{2\pi f C} I_{rms}$$

↳ Like R

$$X_C = \frac{1}{2\pi f C}$$

Capacitive Reactance

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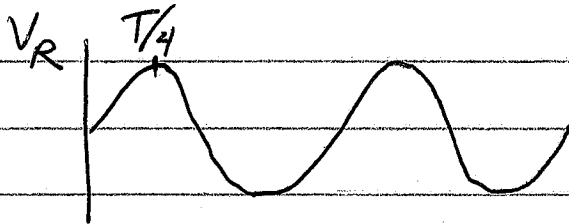
$$V_{rms} = I_{rms} Z$$

$$I \propto \sin(\omega t)$$

Resistor: $Z = R$

doesn't care about Freq.

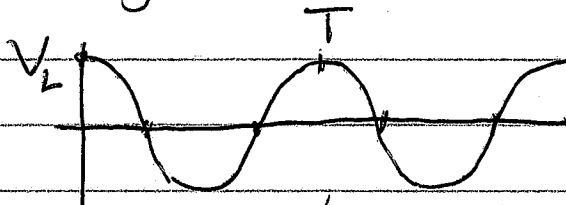
$$V_R \propto \sin(\omega t)$$



Inductor: $Z = 2\pi fL$

hates change
high-f is blocked

$$V_L \propto \cos(\omega t)$$



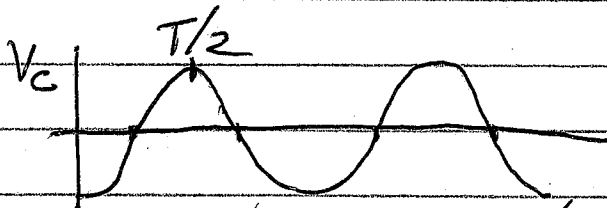
Voltage peaks @ $t=0$
Current peaks @ $t=T/4$

"Current lags" in an inductor.

Capacitor: $Z = 1/(2\pi fC)$

blocks DC
almost blocks low-f

$$V_C \propto -\cos(\omega t)$$



Current peaks @ $t=T/4$
Voltage peaks @ $t=T/2$

"Voltage lags" in a capacitor.

⑥

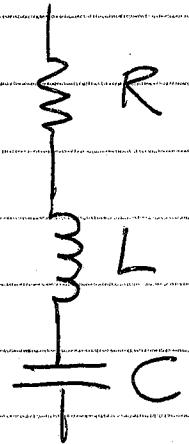
Series RLC Circuit

$$I = \sqrt{2} I_{rms} \sin(2\pi f t)$$

$$V_R = \sqrt{2} I_{rms} R \sin(2\pi f t)$$

$$V_L = \sqrt{2} I_{rms} (2\pi f L) \cos(2\pi f t)$$

$$V_C = \sqrt{2} I_{rms} \left(\frac{1}{2\pi f C}\right) (-\cos(2\pi f t))$$

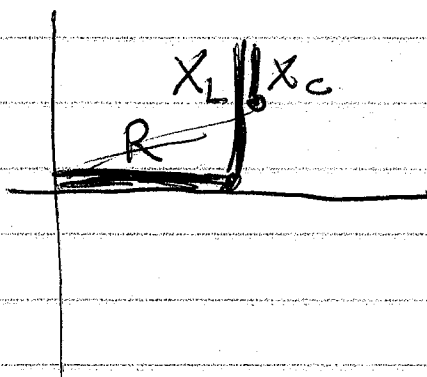


$$V_{tot} = V_R + V_L + V_C \quad (\text{As functions of time})$$

$$= V_1 \sin(\omega t) + V_2 \cos(\omega t)$$

$$= \sqrt{2} V_{rms} \sin(\omega t + \phi)$$

Adding impedances as vectors



R
L
C

X
Y
 $X_L = 2\pi f L$
 $-X_C = -1/(2\pi f C)$

Tot R $(X_L - X_C)$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$V_{rms} = I_{rms} Z$$

$$\phi = \text{atan} \left(\frac{X_L - X_C}{R} \right)$$

$$P_{avg} = V_{rms} I_{rms} \cos \phi$$