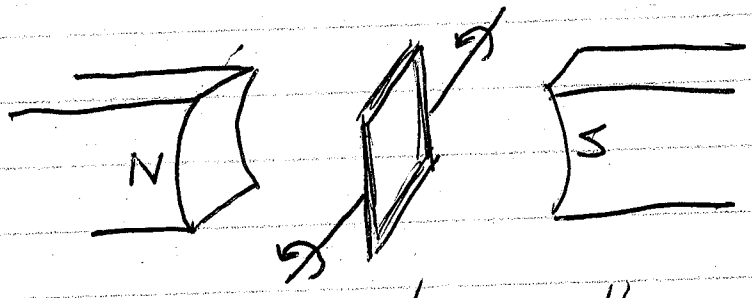
① Phys 2426 2016-10-18 Lec 16

AC Electricity

Alternator



Spinning Coil generates voltage.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad \Phi = NBA\cos\theta$$

$$= -\frac{d}{dt} NBA\cos\theta = -NBA\left(-\sin\theta \underbrace{\frac{d\theta}{dt}}_{\omega}\right)$$

$$= \underbrace{NBA\omega}_{\text{Amplitude}} \sin\theta \qquad \theta = \omega t = 2\pi f t$$

$\sin\theta$ repeats when $\theta = 2\pi$

$$2\pi f t = 2\pi$$

$$ft = 1$$

This is a special time, $t = T$ = Period

$f = 1/T$ = Frequency

Frequency f in $\boxed{\text{Hz}}$ $\quad 1\ \text{Hz} = 1\ \text{cycle/s}$

Angular Freq ω in s^{-1} $\quad 1\ s^{-1} = 1\ \boxed{\text{rad/s}}$

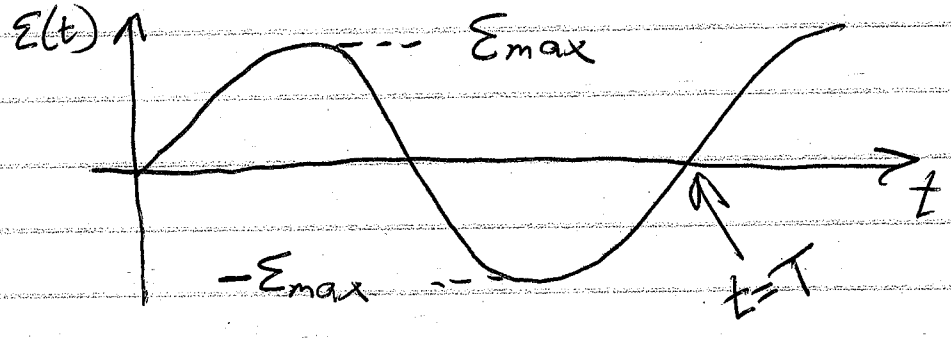
$$\omega = 2\pi f$$

$$\sin(\omega t) = \sin(2\pi f t)$$

US AC: $\quad \mathcal{E}_{max} = 170\ V \quad \mathcal{E}_{rms} = 120\ V$

$\quad f = 60\ Hz \quad \omega = 377\ s^{-1}$

Graph of AC Signal



The voltage is usually between 0 and $\mathcal{E}_{max}$.
The average-ish voltage is:

$$\mathcal{E}_{RMS} = \frac{1}{\sqrt{2}}\mathcal{E}_{max}$$

└ Root Mean Square

$$\frac{170\ V}{\sqrt{2}} = 120\ V$$

Resistors in AC

Ohm's Law $V = IR$

osc — must oscillate — const

$$I = \frac{V}{R} = \frac{\mathcal{E}_{max} \sin(2\pi f t)}{R}$$

$$I_{max} = \frac{\mathcal{E}_{max}}{R}$$

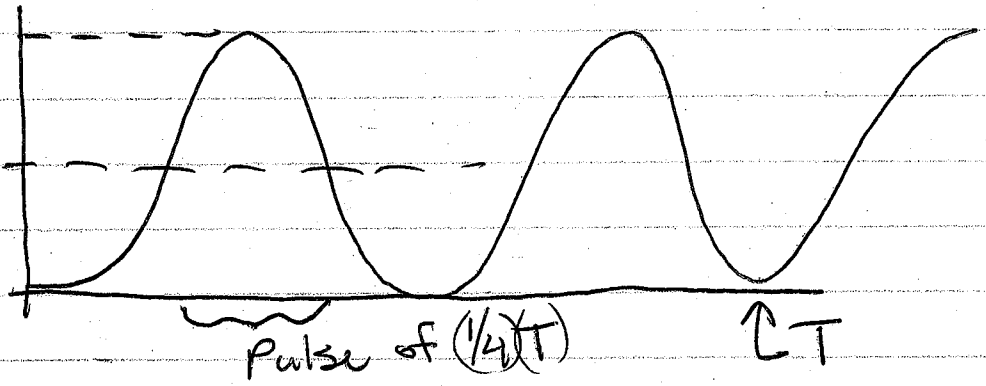
Power: $P = VI$

$$= \mathcal{E}_{max} \sin(2\pi f t) \frac{\mathcal{E}_{max}}{R} \sin(2\pi f t)$$

$$= \frac{\mathcal{E}_{max}^2}{R} \sin^2(2\pi f t)$$

$\frac{\mathcal{E}_{max}^2}{R} = P_{max}$

$\frac{1}{2} P_{max} = P_{avg}$



Average Power

For Resistor: $P_{avg} = \frac{1}{2} P_{max}$

$$P_{avg} = \frac{\frac{1}{2}\mathcal{E}_{max}^2}{R} = \frac{\mathcal{E}_{rms}^2}{R}$$

In DC: $P = \frac{V^2}{R}$

$$\frac{\frac{1}{2}\mathcal{E}_{max}^2}{R} = \frac{\mathcal{E}_{RMS}^2}{R}$$

$$\mathcal{E}_{rms} = \frac{1}{\sqrt{2}}\mathcal{E}_{max}$$

For a resistor

$P_{avg} = I_{rms} V_{rms}$ $\quad$ $V_{rms} = I_{rms} R$

Energy & Power

$$P = \frac{d\,Energy}{dt} \qquad P_{avg} = \frac{\Delta Energy}{\Delta t}$$

$$Energy = P_{avg}\,\Delta t$$

(5)

Ex: 60 W Light Bulb

Current: $P_{avg} = V_{rms} I_{rms}$

$(60\,W) = (120\,V) I_{rms}$

$0.5\,A = I_{rms}$

Resistance: $V_{rms} = I_{rms} R$

$(120\,V) = (0.5\,A) R$

$240\,\Omega = R$ ~ Hot, cold /

Measured w/ ohmmeter: $R = 60\,\Omega$

Energy use in a month:

$$\text{Energy} = P_{avg}\,\Delta t = (60\,W)(720\,hr)\left(\frac{3600\,s}{h}\right)$$

$$= 1.55 \times 10^{8}\,J$$

$$= (60\,W)\left(\frac{1\,kW}{1000\,W}\right)(720\,hr) = 43.2\,kWh$$

$$\text{Cost} = \text{Rate} \cdot \text{Amount} = \left(\frac{\$0.12}{kWh}\right)(43.2\,kWh)$$

$$= \$5.18$$

LED Bulb Cost $= (\$5.18)\,(9/60)$

$= \$0.78$

Inductors in AC

Assume $I = I_{max} \sin(2\pi f t)$

$$V_L = L \frac{dI}{dt} = L I_{max} \cos(2\pi f t)(2\pi f)$$

$$V_L = \underbrace{(2\pi f L) I_{max}}_{V_{max}} \cos(2\pi f t)$$

$$V_{max} = I_{max} (2\pi f L)$$

Differences from Ohm's Law

- Proportionality depends on frequency.

$X_L = 2\pi f L$ = Reactance

Z_L = Impedance

- sin() changed to cos().

$$\cos(\theta) = \sin(\theta + \pi/2)$$