1 Phys 2426 2016-10-25 Lec 18

Reddit Educational GIFs (Don't get Lost)
3-Phase AC Motor

Resistor - Opposes Current $V = IR$

$V_{Rms} = I_{rms} R$

Inductor - Opposes Change in Current $V_L = L \, dI/dt$

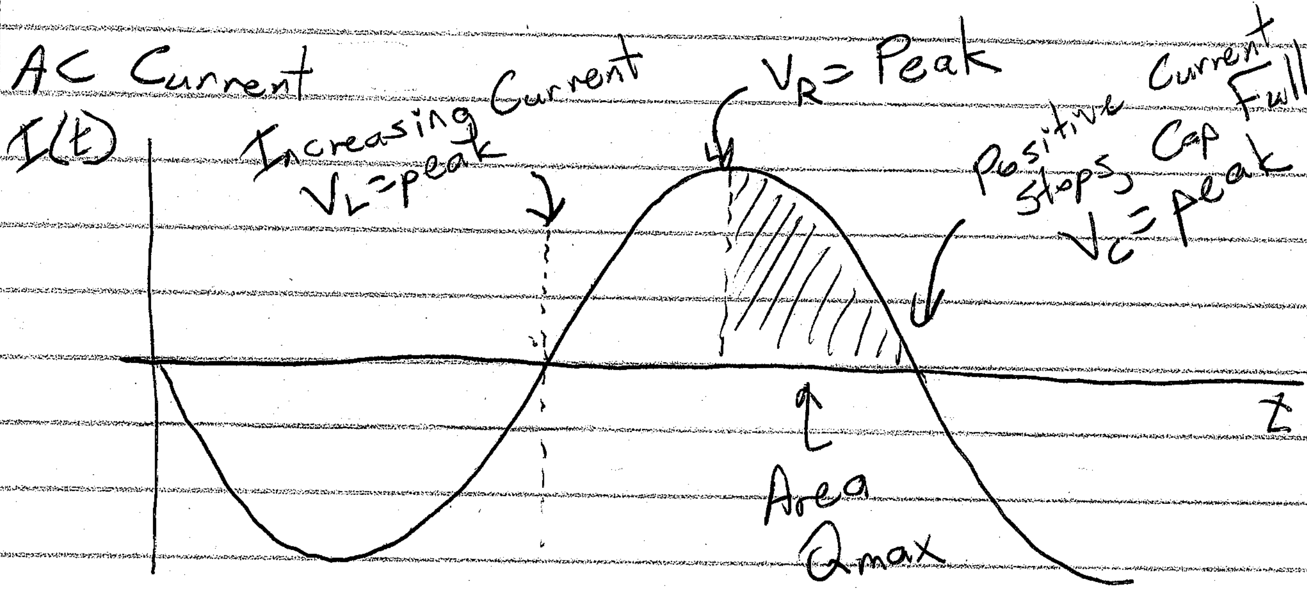
$X_L = 2\pi f L$ $V_{rms} = I_{rms} X_L$

Capacitor - Opposes high Charge. $V_C = Q/C$

Current changes charge $I = dQ/dt$

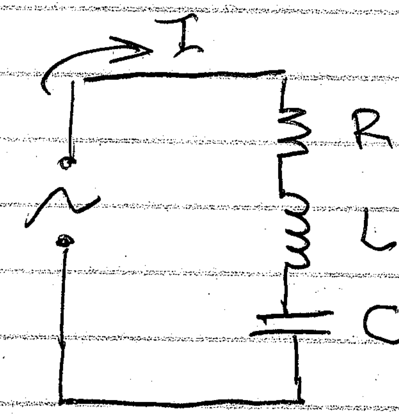
$X_C = \frac{1}{2\pi f C}$ $V_{rms} = I_{rms} X_C$

AC Current



When $V_C = \max$, $V_L = -(\max)$

② Phys 2426 2016-10-25



Series AC Circuit

$I = I_0 \sin(2\pi f t)$

$I_R = I_L = I_C = I$

$V_R = I_0 R \sin(2\pi f t)$

$V_L = L \frac{dI}{dt} = L I_0 2\pi f \cos(2\pi f t)$

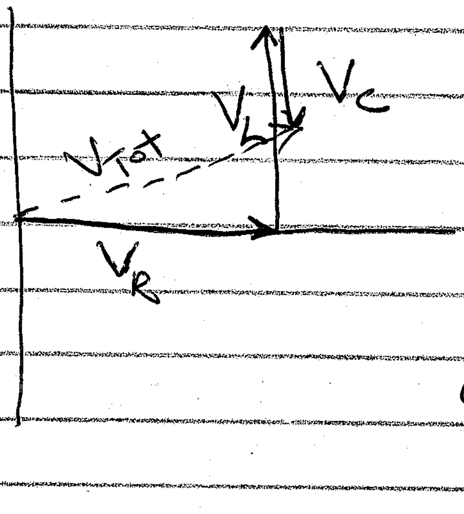
$= I_0 (2\pi f L) \cos(\sim) = I_0 X_L \cos(\sim)$

$V_C = \frac{Q}{C} = \frac{-1}{2\pi f C} I_0 \cos(\sim)$

$V_{Tot} = V_R + V_L + V_C$ as functions

Two Methods:

- Use trig identities
- Vector Addition



x: V_R

y: $V_L - V_C$

mag: $\sqrt{V_R^2 + (V_L - V_C)^2}$

$\phi = \text{atan}\left(\frac{V_L - V_C}{V_R}\right)$

 Phys 2426 2016-10-25

$$V_{tot} = V_R + V_L + V_C \quad \text{(vectors)}$$

$$IZ = IR + IX_L + IX_C \quad \text{(vectors)}$$

↑ Impedance

① Reactance $X = X_L - X_C$

② Impedance

$$Z = \sqrt{R^2 + X^2}$$

③ $V_{net} = IZ$

Ex: 1 kΩ, 1.0 H, 1 μF in series

$V = 140 \sin(500\,t) = V_{max}(\sin(2\pi f t))$

$V_{rms} = \frac{140}{\sqrt{2}} = 99\ V$ $\qquad 2\pi f = 500$

$X_L = 2\pi f L = (500)(1.0) = 500\ \Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{(500)(1\times10^{-6})} = 2000\ \Omega$

$X = -1500\ \Omega$

$Z = \sqrt{1000^2 + 1500^2} = 1803\ \Omega$

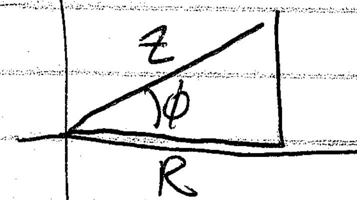
$I = \frac{V}{Z} = \frac{99}{1803} = 0.0549\ A$

Power: $P = I^2 R = 3.02\ W$

$VA = IV = (0.0549)(99) = 5.44\ VA$

④ Phys 2426 2016-10-25

$$\frac{R}{Z} = \frac{1000}{1803} = 0.555 = \cos\phi$$



$$\frac{P}{I V_{net}} = \frac{3.02}{5.44} = 0.555$$

$$3.02 = (0.555)(5.44)$$

$$P_{avg} = \cos\phi \; V_{rms} I_{rms}$$

$\cos\phi$ — Power Factor

Pure R: $\cos\phi = 1$

Mix LR: $\phi > 0$ $\cos\phi < 1$

$$\tan\phi = \frac{X}{R} \qquad p.f. = \cos\phi = \frac{R}{Z}$$

$$P_{avg} = (V_{rms})(I_{rms})\cos\phi = I_{rms}^2 R$$

Resonance - If $X_L = X_C$, they cancel.

$$X_L = X_C$$

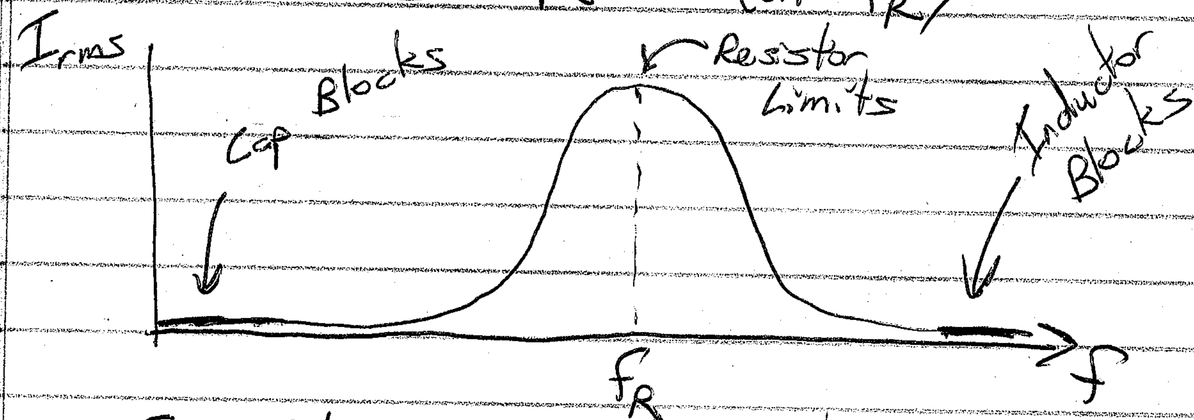
$$2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{(2\pi)^2 LC}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

This is a special frequency: resonant frequency.
The inductor & capacitor cooperate to pass current at f_R. Only the resistor impedes the current.

$$Z = R \quad (\text{at } f_R)$$



Same V_{Rms} @ every point