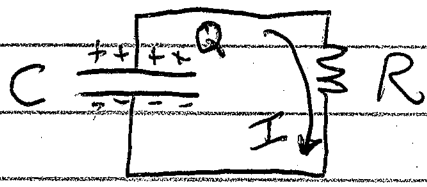
(1) Phys 2426 2017-09-26 lec 7

Trying to power a resistor with a capacitor:



Capacitor: $Q = CV$

Resistor: $V = IR$

This current drains the cap: $I = -\frac{dQ}{dt}$

$$\frac{V}{R} = -C\frac{dV}{dt} \longleftarrow \frac{V}{R} = -\frac{d}{dt}(CV)$$

$$\frac{dV}{dt} = \frac{-1}{RC} V$$

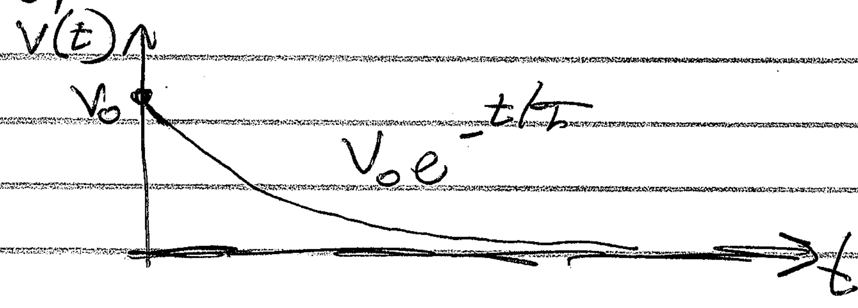
Note: $\frac{d}{dt} e^{t} = e^{t}$ $\qquad \frac{d}{dt} e^{-t/\tau} = \frac{-1}{\tau} e^{-t/\tau}$

$$\frac{d}{dt}\left(V_0 e^{-t/\tau}\right) = V_0 \frac{-1}{\tau} e^{-t/\tau}$$

$V = V_0 e^{-t/\tau}$ is the solution

as long as $\tau = RC$

Analogy: Drain a tank of water thru a straw.



(2)

What is $e^{-t/\tau}$?

Fraction of charge remaining.

What is t/τ?

Amount of time since start, measured as a "number of time constants"

	t/τ	$\exp(-t/\tau)$	
Start	0.0	1.0	100%
	0.5	0.61	61%
Half-Life →	0.693	0.5	50%
	1.0	0.37	37%
	2.0	$(0.37)^2 = 0.14$	14%
	⋮		
	5.0	0.01	1%

After 5.0 time constants, basically done.

Note: $e = 2.718$ vs. $e = 1.6 \times 10^{-19}$ C

$\tau = RC$

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RC Circuit Set up for Discharge

Initial Voltage = 8.0 V

Resistor = 500 Ω

After 2.0 s, current is measured:

$I = 0.010$ A

What is C?

How? Want to use $\tau = RC$

Need τ:

$$V = V_0 \exp(-t/\tau)$$

$$(5.0\,V) = (8.0\,V)\exp(-(2.0\,s)/\tau)$$

$$V = IR = (0.010\,A)(500\,\Omega) = 5.0\,V$$

$$\frac{5.0\,V}{8.0\,V} = 0.625 = \exp(-t/\tau)$$

$$\ln(0.625) = -(2.0\,s)/\tau$$

$$-0.47 = -(2.0\,s)/\tau$$

$$0.47\,\tau = (2.0\,s)$$

$$\tau = 4.26\,s$$

$$\tau = RC \longrightarrow C = \frac{\tau}{R} = \frac{4.26\,s}{500\,\Omega} = 0.00851\,F$$

$$= 8.51\,mF = 8510\,\mu F$$

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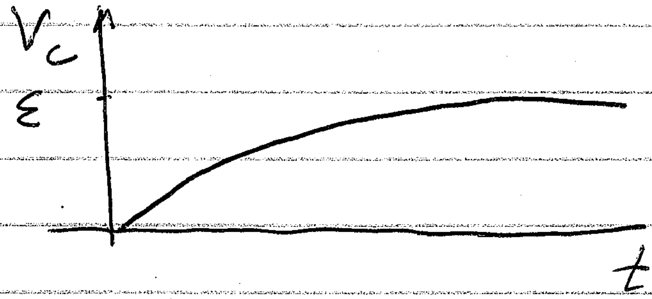
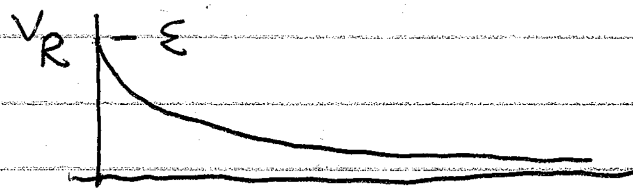
Charging:

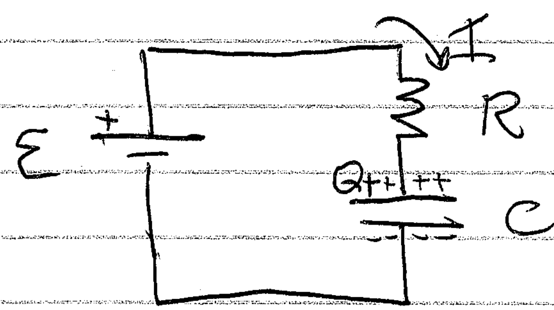


V_c grows



V_R decays

$V_c + V_R = \mathcal{E}$ = Battery voltage



$$V_c + \mathcal{E}e^{-t/\tau} = \mathcal{E}$$

$$V_c = \mathcal{E} - \mathcal{E}e^{-t/\tau}$$
$$= V_f(1 - e^{-t/\tau})$$

$V_f = \mathcal{E}$

I "looks like" V_R

Q "looks like" V_C

$I = +dQ/dt$

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Temperature Coefficients

$\alpha \Delta T$ = relative change

For resistance $\frac{\Delta R}{R_0} = \alpha \Delta T$

$$\frac{R - R_0}{R_0} = \alpha (T - T_0)$$

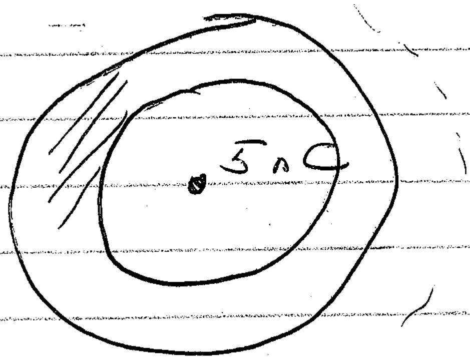
$$R = R_0 + R_0 \alpha (T - T_0)$$
$$= R_0 (1 + \alpha (T - T_0))$$

$\alpha = 0.005$ means 0.5% increase per degree

$\alpha = \oplus$ = metals
$\alpha = \ominus$ = semiconductor

(6)

Gauss's Law



metal shell w/ $Q_{net} = 8\,nC$

Small r: Inside shell
only 5nC matters

$$E = \frac{k(5\,nC)}{r^2}$$

$$\Phi = 4\pi k Q_{enc} = 4\pi(9\times10^9\ Nm^2/C^2)(5\,nC)$$
$$= 180\pi\ (N/C)\,m^2$$

Large r: Outside shell:

$$\Phi = 4\pi k(5+8\ nC) = 468\pi\ (N/C)\,m^2$$

$$E = \frac{k(13\,nC)}{r^2}$$

Middle r: Embedded in metal.

$$E = 0$$
$$\Phi = EA = 0$$
$$Q = 0 = +5\,nC + Q_{inner}$$
$$Q_{inner} = -5\,nC$$
$$Q_{net} = Q_{in} + Q_{out}$$
$$8\,nC = -5\,nC + Q_{out}$$
$$13\,nC = Q_{out}$$