① Phys 2426 2017-10-05 Lec 10

Math Background for Magnetism

Vector Cross Product

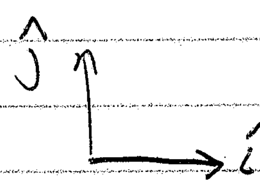
$$\vec{A} \times \vec{B} = (AB \sin\theta)\ (\text{Dir by RHR})$$

θ = angle between $\vec{A}$ and $\vec{B}$

$\sin\theta$ = max when $\theta = 90°$ i.e. $\vec{A} \perp \vec{B}$

Ex: $|\hat{x} \otimes \hat{y}| = |\hat{i} \otimes \hat{j}| = 1$

Right-Hand Rule (RHR) for cross products:



Index = right
Middle = Top
Thumb = Out of page

$\hat{i} \otimes \hat{j} = \hat{k}$

Try: $\hat{j} \otimes \hat{i} = -\hat{k}$ (Anti-commutative)

Unit vectors:

$\hat{i} \otimes \hat{j} = \hat{k}$ $\qquad$ $\hat{j} \otimes \hat{i} = -\hat{k}$

$\hat{j} \otimes \hat{k} = \hat{i}$ $\qquad$ $\hat{k} \otimes \hat{j} = -\hat{i}$

$\hat{k} \otimes \hat{i} = \hat{j}$ $\qquad$ $\hat{i} \otimes \hat{k} = -\hat{j}$

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$\vec{A} = 3\hat{i} + 2\hat{j}$

$\vec{B} = \quad 5\hat{j}$

$$\vec{A} \otimes \vec{B} = (3\hat{i} + 2\hat{j}) \otimes (5\hat{j})$$

$$= (3\hat{i} \otimes 5\hat{j}) + (2\hat{j} \otimes 5\hat{j})$$

$$= (3)(5)\hat{k} + 0$$

$$= 15\hat{k}$$

Note: $A_y = 2$ didn't matter
Parallel component doesn't matter

Matrix Method:

$$\vec{A} \otimes \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 0 \\ 0 & 5 & 0 \end{vmatrix}$$

$$= \hat{k}(3)(5)$$

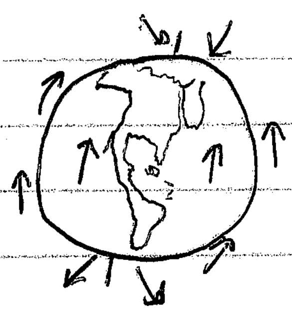
$$= 15\hat{k}$$

Formula: $\vec{A} \otimes \vec{B} = (A_y B_z - A_z B_y)\hat{i}$
$(A_z B_x - A_x B_z)\hat{j}$
$(A_x B_y - A_y B_x)\hat{k}$

③

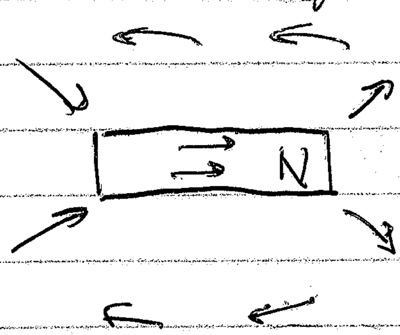
Magnetism - Magnetic Field ($\vec{B}$) is in teslas (T)
- Virtual "wind" of magnetic flux.
- How do we know B is there?
  - Makes compass needle align
  - Deflects charged particles
  - Generate Electric Current
  - Form Electromagnetic Waves

Earth's Magnetic Field



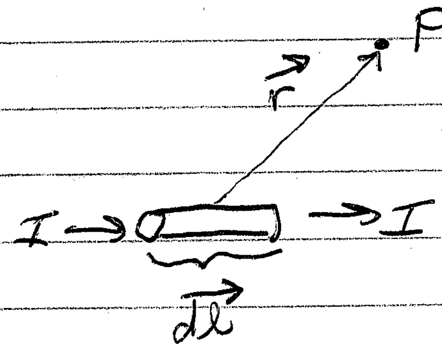
- Compasses point "north"
- $\vec{B}$ points away from geographic south.
- $B \sim 50\ \mu T$

Permanent Dipole Magnet



- N end points north if magnet is free to rotate.
- $\vec{B}$ points away from North of magnet

Magnetic Field of a current
Biot - Savart Law



$\vec{r}$ = from source to P

Magnetic Field of this segment of wire:

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I \vec{dl} \otimes \hat{r}}{r^2}$$

Little contribution toward $\vec{B}$

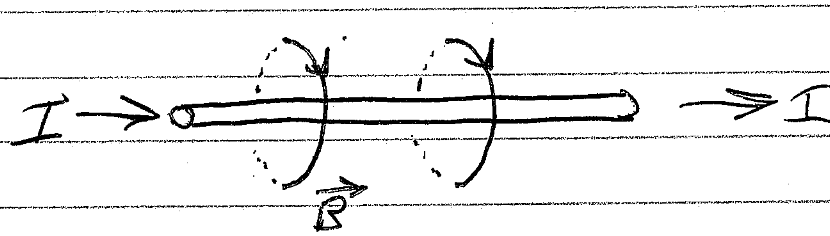
$$\mu_0 = 4\pi \times 10^{-7} \frac{T \cdot m}{A}$$

Ex: Above:

$$\vec{dl} = \sim \hat{i}$$
$$\hat{r} = \sim \hat{i} + \sim \hat{j}$$

$$\vec{dl} \otimes \hat{r} = (\sim \hat{i}) \otimes (\sim \hat{i} + \sim \hat{j})$$
$$= \sim \hat{k}$$

This $\vec{B}$ points out of page.



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Case 1: Long Straight Wire

$$B = \frac{\mu_0 I}{2\pi r}$$

r = dist from wire center to point P.

Ex: $I = 2.0\ A$

$r = 2.0\ cm$

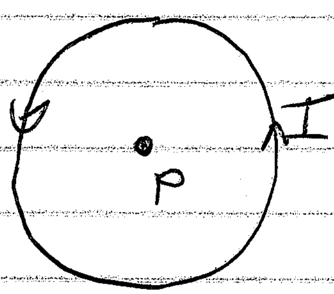
$$B = \frac{4\pi \times 10^{-7}\ (2.0)}{2\pi\ (0.02)}$$

cm to m

$$= 2 \times 10^{-5}\ T$$

$$= 20\ \mu T$$

Case 2: Loop or Coil @ Center



$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{d\ell} \otimes \hat{r}}{r^2}$$

$$= \frac{\mu_0 I \hat{k}}{4\pi} \int \frac{d\ell}{r^2}$$

$$= \frac{\mu_0 I}{4\pi} \frac{2\pi r}{r^2}$$

Single Loop: $\vec{B} = \frac{\mu_0 I}{2 r} \hat{k}$

Coil: $\vec{B} = \frac{\mu_0 N I}{2R} \hat{k}$