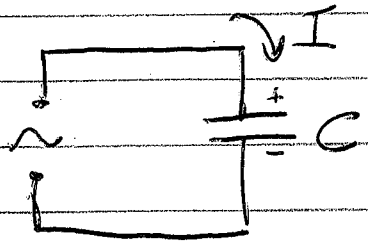
Phys 2426 2017-10-19 Lec 14

Capacitors in AC

What we know:

- Store Charge $Q = CV$
- Take time to charge or discharge $\tau = RC$
- Store Energy $\text{Energy} = \frac{1}{2}CV^2$

What happens in AC?



$$Q = CV$$

Time derivative →

$$\dot{Q} = C\dot{V}$$

$$I = C\frac{dV}{dt}$$

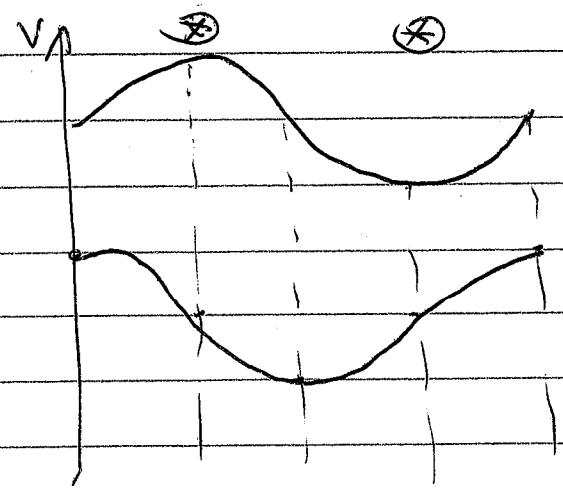
$$V = V_{max}\sin(2\pi f t)$$

If $V = V_{max}\sin(2\pi f t)$

$$\dot{V} = V_{max}\cos(2\pi f t)\,2\pi f$$

$$I = V_{max}\,2\pi f C\cos(2\pi f t)$$

⊛ = High Energy



$$P = VI$$

$$= V_{max}I_{max}\sin(\,)\cos(\,)$$

$$= V_{max}I_{max}\frac{1}{2}\sin(2(2\pi f t))$$

Average $P = 0$

$$V = I R$$

(2)

AC Ohm's Law for Cap:

$$I_{max} = V_{max}\, 2\pi f C$$

$$V_{max} = I_{max} \left(\frac{1}{2\pi f C}\right)$$

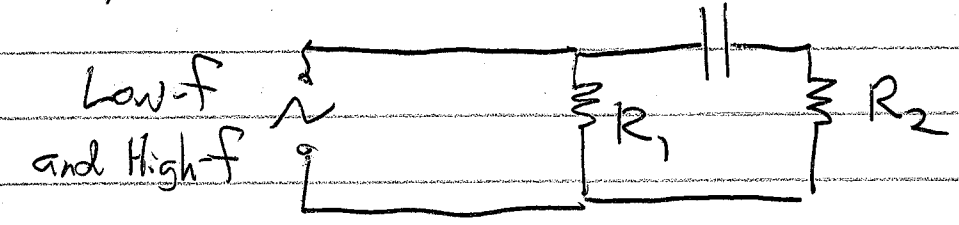
$$V_{max} = I_{max}\, X_C$$

Reactance (X_C) is in ohms (Ω)

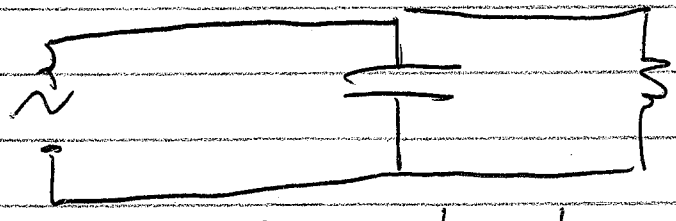
Capacitors have high reactance when:

- Low C
- Low f (DC, is $f=0$)

Capacitors "block" DC and low-f.



R_2 gets more high-f



R_1 gets low-f

(3)

Inductors - Devices that "feel" their own magnetic field.

Ex: Solenoid Coil $\quad B = \frac{\mu_0 N I}{l}$

Faraday's Law: $\quad \mathcal{E} = -\frac{d\Phi_B}{dt}$

Flux of coil's field: $\quad \Phi_B = B \cdot A$ (one loop)

$\Phi_B = NBA$ (whole coil)

$$\Phi_B = N\left(\frac{\mu_0 N I}{l}\right) A$$

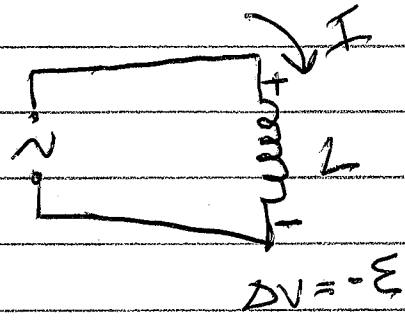
$$\Phi_B = \left(\frac{\mu_0 N^2 A}{l}\right) I = L I$$

Inductance (L) is measured in henrries (H)

Take derivative: $\quad \mathcal{E} = L \dot{I}$

The EMF of L fights $\dot{I}$

As a passive device, we drop the $\ominus$ sign.



(4)

If $V = V_{max} \sin(2\pi f t)$

Guess $I = -I_{max} \cos(2\pi f t)$

$$\dot{I} = I_{max} \sin(2\pi f t)\, 2\pi f$$

$$V = I_{max}\, 2\pi f L \sin(2\pi f t)$$

$$V_{max} = I_{max}\, 2\pi f L$$

$$V_{max} = I_{max}\, X_L$$

For the inductor:

Low $f \Rightarrow$ Low $X_L \rightarrow$ Easy current flow.

Power of Inductor:

$P = VI =$ ~~~ $\sin() \cos()$

$=$ ——— $\frac{1}{2} \sin(2$ ~~~ $)$

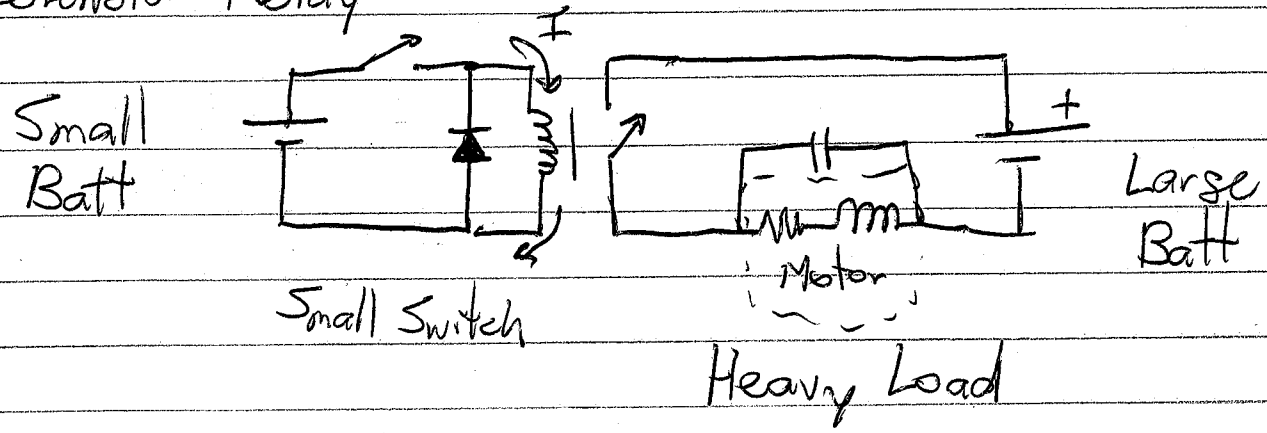
$P_{avg} = 0$

Energy of Inductor $\quad$ Energy $= \frac{1}{2} L I^2$

Capacitor Energy $= \frac{1}{2} C V^2$

(5)

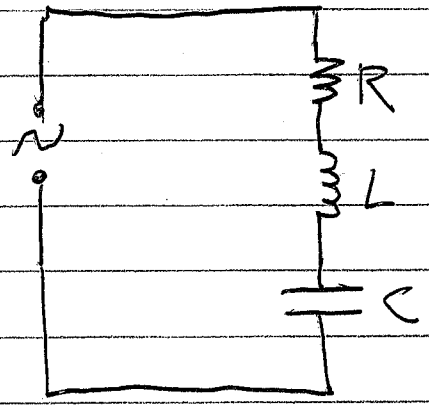
Solenoid Relay:



When you disconnect a coil, $|\dot{I}|$ is huge.
This generates large EMF.
This wrecks switches.

Protection options: 1. Diode
2. Capacitor

Series AC Circuit Intro



V_{max}
f

$$V_R = I_{max} R \sin(2\pi f t)$$

$$V_L = L\dot{I} = I_{max} 2\pi f L \cos(2\pi f t)$$

$$V_C = \frac{1}{C}\int I = I_{max} \frac{1}{2\pi f C}\left(-\cos(2\pi f t)\right)$$

Propose $I = I_{max} \sin(2\pi f t)$

$$V_{tot} = V_R + V_L + V_C$$