Phys 2426 2017-10-24 Lec 15

AC Ohm's Law $V_{rms} = I_{rms} Z$

Impedance (Z) is a generalized resistance used for AC.

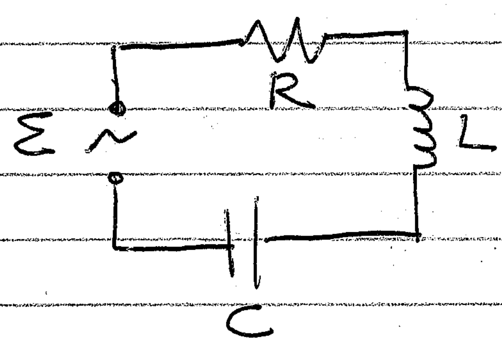
Resistor : $Z = R$

Inductor : $Z = X_L = 2\pi f L$

Capacitor : $Z = X_C = 1/(2\pi f C)$

Series AC Circuit:

- Same Current (I) everywhere. (Both $I(t)$ and I_{rms})
- Voltage Adds, but only as $V(t)$.



$$\mathcal{E} = V_R + V_L + V_C$$
(As functions of time)

$$\mathcal{E} = V_R \cos\theta + V_L \sin\theta - V_C \sin\theta$$
(For $I = I_{max}\cos\theta$)

$\uparrow \theta = 2\pi f t$

equivalent
$$\mathcal{E} = V_R \cos(2\pi f t) + (V_L - V_C)\sin(2\pi f t)$$

$$\mathcal{E} = \mathcal{E}_{max} \cos(2\pi f t + \phi)$$

$$\mathcal{E}_{max}^2 = V_R^2 + (V_L - V_C)^2$$

$$\tan\phi = \frac{V_L - V_C}{V_R}$$

②

Dealing with AC Circuits:

- Think of R as the x-direction
- X_L is the +y-dir
- X_C is the −y-direction

Series: Z's add as vectors.
Problem: Can't divide by vectors.
Solution: Use complex numbers = phasors

Series Result: Total x: R
Total y: $X_L - X_C = X$

Total Magnitude: $Z = \sqrt{R^2 + X^2}$

③

Ex: $R = 1\ k\Omega$ $L = 1.0\ H$ $C = 1.0\ \mu F$

In series

$V_{Tot} = 140 \sin(500\,t)$

$= V_{max} \sin(2\pi f t)$

$I_{rms} = ?$

$V_{max} = (140\ V)$

$2\pi f = (500\ s^{-1})$

$V_{rms} = \frac{140\ V}{\sqrt{2}} = 99\ V$

$f = \frac{500}{2\pi} = 79.6\ Hz$

Want to use :

$V_{rms} = I_{rms} Z$

Need $X_L = 2\pi f L = 500\Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi f\ 1\times10^{-6}}$

$= 2000\ \Omega$

$X = X_L - X_C = -1500\ \Omega$

$(99v) = I_{rms}(1803\Omega)$

$\boxed{I_{rms} = 0.0549\ A}$

$\tan\phi = \left(\frac{-1500}{1000}\right)$

$Z = \sqrt{1000^2 + 1500^2}$

$= 1803\ \Omega$

Power in AC — only the resistor.

$P = V_R I_R = I^2 R = (0.0549A)^2(1000\Omega)$

$= 3.02\ W$

Compare to

$V_{Tot} I = (99v)(0.0549\ A) = 5.4\ VA$

$\cos\phi = \frac{3.02}{5.4}$

(4)

Because X_L and X_C subtract, they can cancel!

Cause	Effect
$X_L = X_C$	$X = 0$
$2\pi f L = \frac{1}{2\pi f C}$	$Z = \sqrt{R^2 + X^2} = \sqrt{R^2} = R$
$2\pi f = \frac{1}{\sqrt{LC}}$	(As small as possible)
$f_R = \frac{1}{2\pi\sqrt{LC}}$	$V_{rms} = I_{rms} R$
Resonance freq.	For const V_{rms}, this freq gives large I_{rms}

This is a resonant circuit.

Above example: $f_R = \frac{1}{2\pi\sqrt{(1.0)(1\times10^{-6})}} = 159\ Hz$

At resonance: $X_L = 2\pi f L = 2\pi(159)(1.0) = 999\,\Omega$

$X_C = 1/2\pi f C = 999\ \Omega$

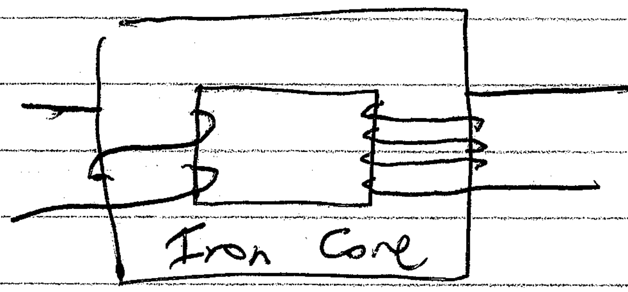
coincidence!

Impedance $Z = R = 1000\ \Omega$

$I_{rms} = \frac{99\,V}{1000\,\Omega} = 0.099\ A$

Transformers - Use AC to generate voltage & current

Two coils w/ interconnected B fields.



- Force AC Current thru "primary"
- This causes AC B oscillations
- This causes oscillating Φ_B
- This generates oscillating $\mathcal{E}_p$ and $\mathcal{E}_s$.

Flux per Loop: $\Phi_1 = \iint \vec{B} \cdot d\vec{A} = BA$

Take ratios of secondary to primary:

$$\frac{\Phi_s}{\Phi_p} = \frac{N_s \Phi_1}{N_p \Phi_1} = \frac{N_s \, d\Phi_1/dt}{N_p \, d\Phi_1/dt} = \boxed{\frac{N_s}{N_p} = \frac{\mathcal{E}_s}{\mathcal{E}_p}}$$

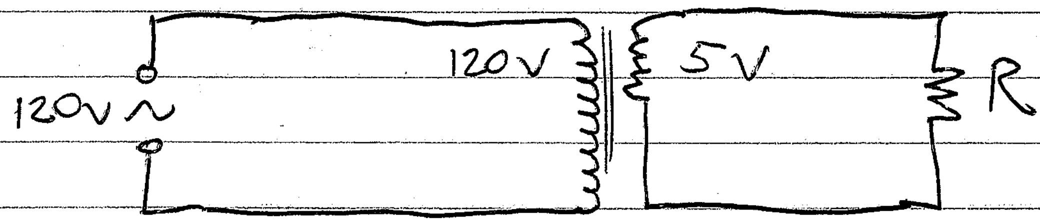
Power Transfer:

$$\epsilon \, \mathcal{E}_p I_p = \mathcal{E}_s I_s$$

Efficiency

6

Ex: Cell phone charger transformer



If $N_s = 100$, what is N_p?

$$\frac{\mathcal{E}_s}{\mathcal{E}_p} = \frac{N_s}{N_p} \qquad \frac{5\,V}{120\,V} = \frac{100}{N_p}$$

$$N_p = 2400$$

If $I_s = 1.5\,A$ and $\epsilon = 1.0$

$$\mathcal{E}_p I_p = \mathcal{E}_s I_s$$

$$(120) I_p = (5)(1.5) \rightarrow I_p = 0.0625\,A$$

Transmission Lines are resistors.

$$P_{lost} = I^2 R_{wire}$$

Why did any sparks shoot to helicopter?