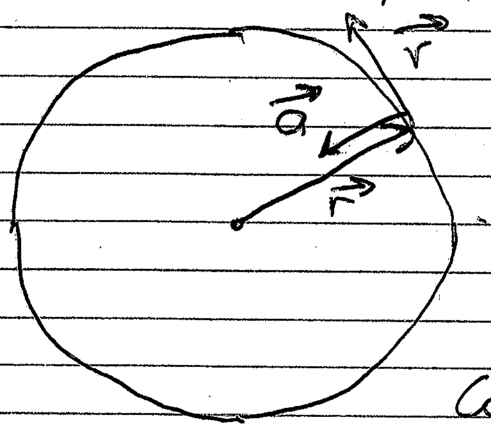
Phys 2426 2017-11-02 Lec 18

Review From Phys 2425 ~ Circular Motion



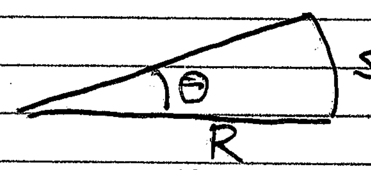
$$\vec{r} = R\,\hat{r}$$

$$\vec{a} = \frac{v^2}{R}(-\hat{r})$$

Compare: $\vec{a} = -\frac{v^2}{R^2}\vec{r}$

Solution for uniform motion:

$$v = \text{const} = \frac{\Delta s}{\Delta t}$$



$$\omega = \frac{v}{R} = \text{const}$$

$$s = R\theta$$

$$v = \dot{s} = R\dot{\theta} = R\omega$$

Angular speed

$$\theta = \omega t$$

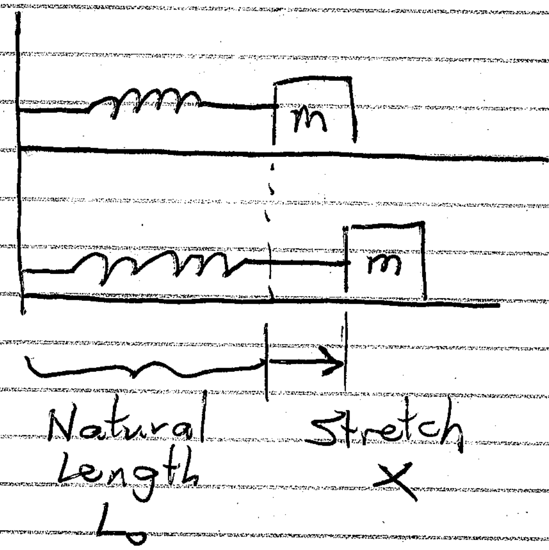
x-component:

$$x = R\cos\theta$$
$$= R\cos(\omega t)$$
$$v_x = R(-\sin(\omega t))\,\omega$$
$$= -R\omega\sin(\omega t)$$
$$a_x = -R\omega\cos(\omega t)\,\omega$$
$$= -\omega^2 R\cos(\omega t)$$

In ~~a~~ uniform circular motion, $a \propto -r$ and both can be described by $\sin(\omega t)$ or $\cos(\omega t)$ if $\omega = \frac{v}{R}$

②

A spring creates a force proportional to its stretch



m $F_s = 0$

$F_s = -kx$

Plug into Newton's 2nd Law

$$F_{net} = ma$$
$$-kx = ma$$
$$-\frac{k}{m}x = a$$

Looks like $a = -\omega^2 x$

Solutions: $x = A\cos(\omega t)$ or $x = B\sin(\omega t)$

A $\omega = \sqrt{k/m}$

k = stiffness or strength

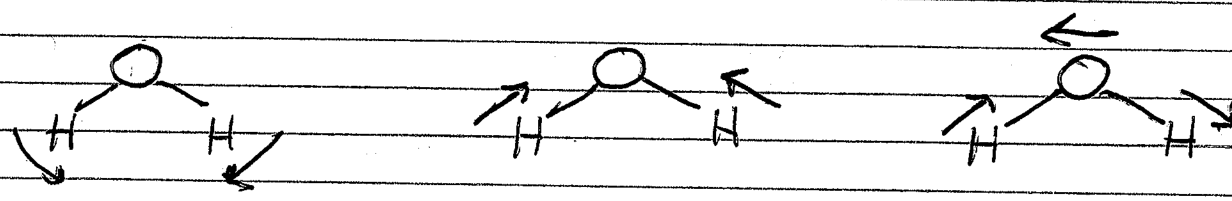
m = mass that moves

Alternate description $\omega = 2\pi f$ $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$$T = \frac{1}{f}$$

General Properties:
- Equilibrium value
- Repetitive behavior
- Restoring "force"
- Inertia – overshoot equilibrium
- Energy – usually tradeoff between two forms

Ex: Mass & Spring
Diving Board
Pendulum – small amplitude
Molecules



Bending Breathing

Nuclei in magnetic fields NMR, MRI
AC Voltage
Electromagnetic Fields
Pieces of Waves

(4)

Review: Energy of a spring:

Slowly compress spring

$$W = \int \vec{F}_A \cdot \vec{dx} = \int kx\, dx$$

Applied Force ($\vec{F}_A$); Displacement Step ($\vec{dx}$)

$F_A = -F_s = -(-kx)$

$$W = \int_0^{x_f} kx = \frac{1}{2} k x_f^2$$

I did work and now the spring has that energy.

We say $U_s = \frac{1}{2} k x^2$ is potential energy stored in the Spring

Energy of Oscillation

$$x = x_{max} \sin(2\pi f t)$$

Energy of spring: $\frac{1}{2} k x^2 = \left(\frac{1}{2} k x_{max}^2\right) \sin^2(\sim)$

max spring energy

$$v = v_{max} \cos(2\pi f t)$$

$$= 2\pi f\, x_{max} \cos(2\pi f t)$$

Kinetic Energy: $\frac{1}{2} m v^2 = \left(\frac{1}{2} m v_{max}^2\right) \cos^2(\sim)$

max Kinetic energy

$$\frac{1}{2} k x_{max}^2 = \frac{1}{2} m v_{max}^2$$ Cons. Energy

③

$$\frac{k}{m} x_{max}^2 = v_{max}^2$$

$$\sqrt{\frac{k}{m}}\, x_{max} = v_{max} = 2\pi f\, x_{max}$$

$$\sqrt{\frac{k}{m}} = 2\pi f$$

Ex: Car Suspension

$F_s = -kx$ Unloaded $|F_s| = k x_1$

Loaded $|F_{s2}| = k x_2$

100 kg $\rightarrow$ 1 cm

$\Delta F_s = k \, \Delta X$

Weight of Load response to Load

$\Delta F = 980\ N = k\,(0.01\ m)$

$98000\ N/m = k$

Natural oscillation $m = 2000\ kg$

$$f = \frac{1}{2\pi}\sqrt{\frac{98000}{2000}} = 1.11\ Hz$$