Phys 2426 2017-07-12 Lec 6

RC Circuits - Charging and Discharging

Common traits: Capacitor $C = 250\ \mu F$

V_c varies

Q varies

I_c varies

Resistor (in series w/ Cap) $R = 11\ k\Omega$

$V_R =$ varies

$I_R = I_C$

In both: $I = I_0\, e^{-t/\tau}$ Decaying Exponential

$\tau = RC$

Ex: $\tau = (11 \times 10^{3}\ \Omega)(250 \times 10^{-6}\ F)$

$= 2.75\ s$

Can express t relative to τ.

Ex: $t = 5.0\ s$

The time is 1.82 time constants,

$$\frac{t}{\tau} = \frac{5.0\ s}{2.75\ s} = 1.82$$

② If $I = I_0 e^{-t/\tau}$ $\qquad \dfrac{I}{I_0} = e^{-t/\tau}$

is the relative current.

t/τ	$e^{-t/\tau}$
0	1
1	0.37
⋮	
5	0.01 = 1% left 99% finished

Five time constants gets you 99% completed.

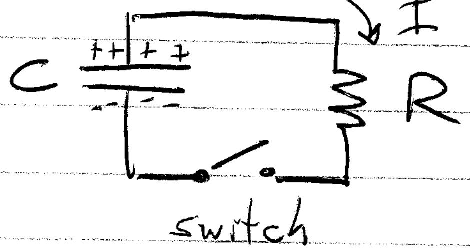
Note: $\ln\left(e^{-t/\tau}\right) = -t/\tau$

Ex: $\ln(0.5) = -0.69$

At $0.69\,\tau$, you're half way.

③

Discharging Circuit



At $t=0$ $Q=Q_0$ and the switch gets closed.

Kirchoff's Voltage Law: CW Loop

$$+V_C - V_R = 0$$

$$\frac{Q}{C} - IR = 0$$

Cap: $Q = CV$

$V = Q/C$

$$I = \frac{Q}{RC}$$

$$I = \frac{-dQ}{dt}$$

$$\frac{-dQ}{dt} = \frac{Q}{RC}$$

$$\frac{dQ}{dt} = \frac{-1}{RC} Q$$

Guess: $Q_0 e^{-t/\tau}$

$$\frac{dQ}{dt} = Q_0 e^{-t/\tau} \frac{-1}{\tau}$$

$$\frac{-1}{\tau} Q_0 e^{-t/\tau} = \frac{-1}{RC} Q_0 e^{-t/\tau}$$

We now know $\tau = RC$

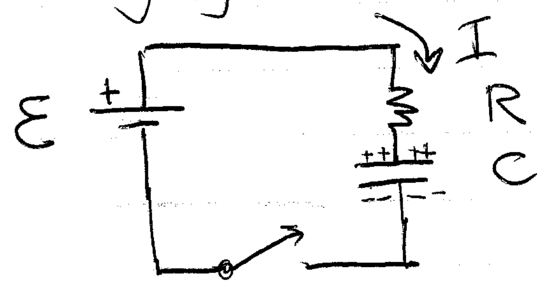
Q_0 could be anything.

$$I = \frac{-dQ}{dt} = \frac{Q_0}{\tau} e^{-t/\tau} = I_0 e^{-t/\tau}$$

$$V = IR = I_0 R e^{-t/\tau} = V_0 e^{-t/\tau}$$

(4)

Charging Circuit



Initially: $V_c = 0$
$Q = 0$

KVL: CW

$$V_{Batt} - V_R - V_c = 0$$

$$\mathcal{E} - IR - \frac{Q}{C} = 0$$

$I = \frac{dQ}{dt}$

$$\frac{d}{dt} \Rightarrow \quad 0 - R\frac{dI}{dt} - \frac{1}{C}\frac{dQ}{dt} = 0$$

$$-R\frac{dI}{dt} - \frac{1}{C} I = 0$$

$$\frac{dI}{dt} + \frac{1}{RC} I = 0$$

$$\frac{dI}{dt} = \frac{-1}{RC} I$$

$I = I_0 e^{-t/\tau}$ w/ $\tau = RC$

$$Q = \int_0^t I(t)\, dt = \int_0^t I_0 e^{-t/\tau} dt$$

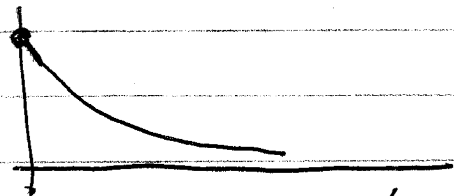
$$= I_0 e^{-t/\tau} (-\tau) \Big]_0^t$$

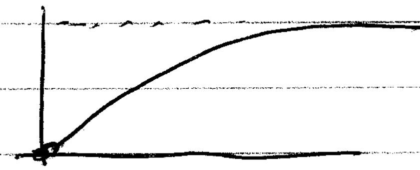
$$= -\tau I_0 \left[e^{-t/\tau} - 1\right] = \tau I_0 \left(1 - e^{-t/\tau}\right)$$

$$= Q_{max} \left(1 - e^{-t/\tau}\right)$$

Charging Capacitor

$$I = I_0 \, e^{-t/\tau} \qquad Q = Q_{max}\left(1 - e^{-t/\tau}\right)$$





$$V_R = IR = V_{max}\, e^{-t/\tau} \qquad V_c = \frac{Q}{C} = V_{max}\left(1 - e^{-t/\tau}\right)$$

$$V_R + V_c = V_{max} = \mathcal{E}$$

Ex: $C = 250\ \mu F$ $\quad \tau = RC = 2.75\ s$

$R = 11\ k\Omega$

$\mathcal{E} = 12\ V$

What is V_c after $5.0\ s$?

$$V_c = \mathcal{E}\left(1 - e^{-t/\tau}\right)$$

$$= (12v)\left(1 - e^{-1.82}\right)$$

$$= (12v)(1 - 0.162) \qquad 16.2\%\ \text{left}$$

$$= 10.1\ V$$

(6) What R if $V_c = 5.0\,v$ @ $t = 5.0\,s$

$$V_c = \mathcal{E}\left(1 - e^{-t/\tau}\right)$$

$$5.0 = 12\left(1 - e^{-t/\tau}\right)$$

$$\frac{5}{12} = 1 - e^{-t/\tau}$$

$$e^{-t/\tau} = 1 - \frac{5}{12} = \frac{7}{12} = 0.5833$$

$$-t/\tau = \ln(0.5833) = -0.539$$

$$t = 0.539\,\tau$$

Now plug in known t:

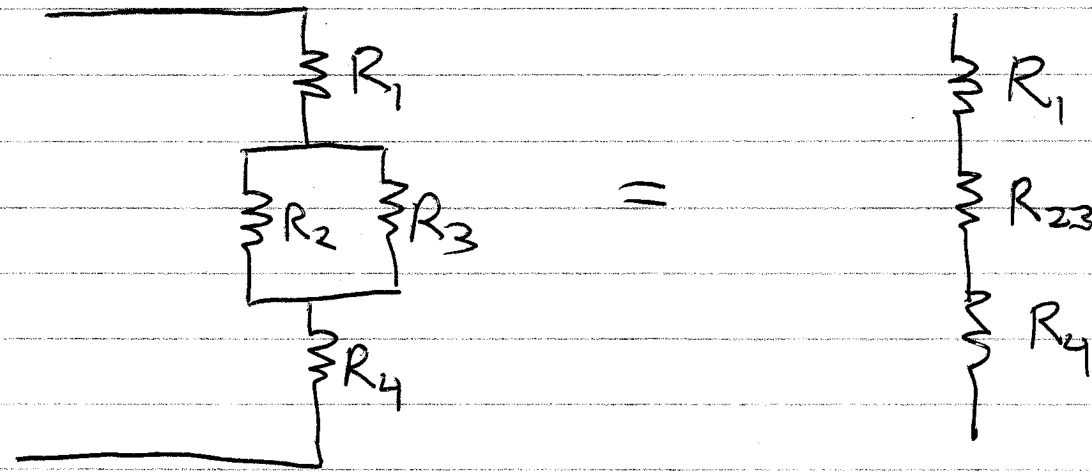
$$\tau = \frac{5.0}{0.539} = 9.28\,s$$

$$\tau = RC$$

$$R = \frac{\tau}{C} = \frac{9.28}{250 \times 10^{-6}} = 37100\,\Omega$$

$$= 37.1\,k\Omega$$

⑦



Parallel Equiv: $R_{23} = \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$

$V_2 = V_3 = V_{23}$

When substituting a parallel equiv.,
the voltage transfers back.

	V	I	R	P
R_1				
R_2				
R_3				
R_4				
Overall				
R_{23}				