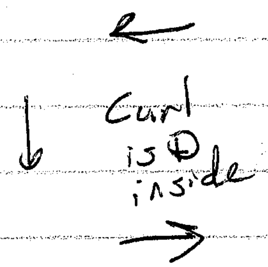
Phys 2426 2017-07-18 Lec 9

Calculating Static Magnetic Fields

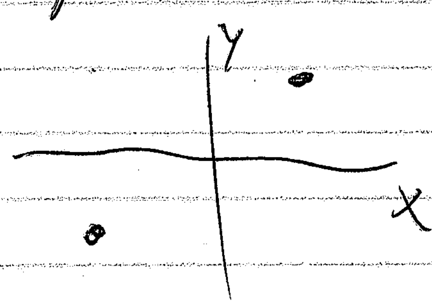
$$\nabla \otimes \vec{B} = \frac{4\pi k}{c^2} \vec{J}$$

$\vec{J}$ ← current density in A/m^2

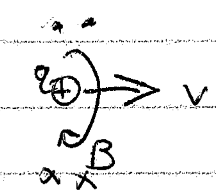
∇⊗ ← Curl operation describes net circulation



Somewhere in between, there is "curl". Curl is 1^{st}-derivative, Perpendicular to vector.

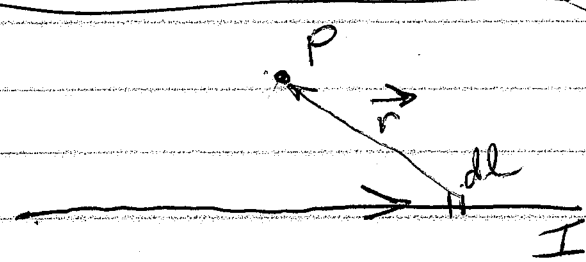


- Current causes $\vec{B}$ to curl. Current "stirs" B.
- Biot-Savart Law
  - solve for a "blip" of current, integrate to add solns.



$$\vec{B} = \frac{\mu_0}{4\pi} \int_{wire} \frac{I \vec{d\ell} \otimes \hat{r}}{r^2}$$

$\vec{r}$ points from wire to field point



$\vec{d\ell} = dx\,\hat{x}$

$\vec{r} = -x\hat{x} + y\hat{y}$

$\frac{\hat{r}}{r^2} = \frac{\vec{r}}{r^3}$

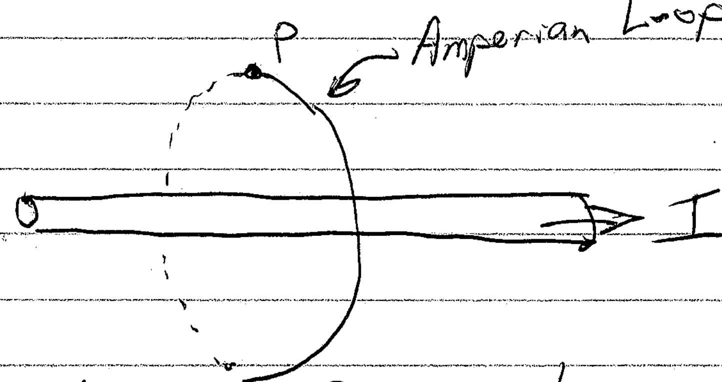
Green's Function Method

(2)

Other Method – Stoke's Theorem

Becomes Ampere's Law

$$\oint_{loop} \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$



Along this loop: $B = \text{const}$

$\hat{B} = d\hat{l}$ (same dir)

$$\oint \vec{B} \cdot d\vec{l} = Bl = B 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

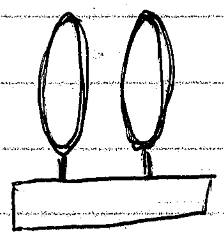
Ex: $I = 2.0\,A$ $\quad r = 2\,cm$

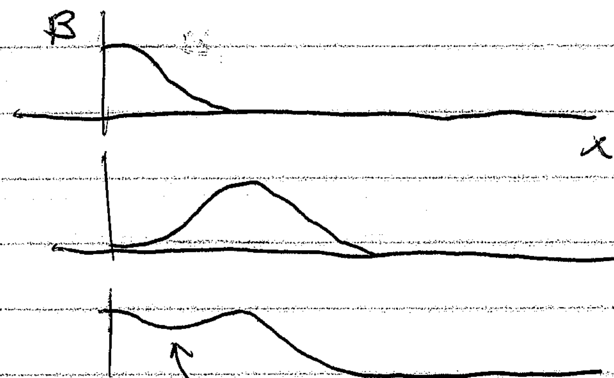
$$B = \frac{(4\pi \times 10^{-7}\ T\cdot m/A)(2A)}{2\pi (0.02\ m)} = 2\times10^{-5}\ T = 20\ \mu T$$

Loop or coil ("Hoop") @ center $B = \frac{\mu_0 N I}{2R}$

- No π in denom. N in numer.
- Still not uniform.

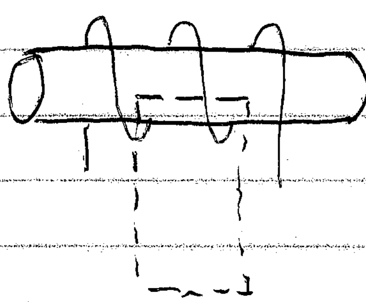
Helmholtz Coils - Two hoops spaced R apart.





- Fairly uniform $\vec{B}$
- Hollow area
- Access from all sides.

Solenoid Coil - Many parallel Loops



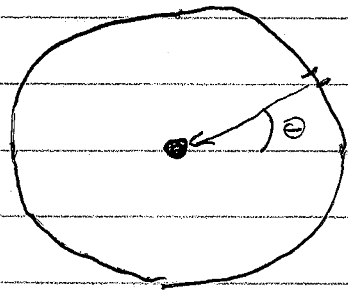
Amperian Loop is rectangle.
One side is in the coil, parallel to axis.
Other sides have
$B = 0$ (bottom)
$B \perp \vec{dl}$ (sides)

$$\oint \vec{B} \cdot \vec{dl} = Bl = \mu_0 I_{enc} = \mu_0 N_{enc} I$$

$$B = \frac{\mu_0 N I}{l}$$

Any N/l is ok. often, turns dens $= n = N/l$

Biot-Savart for Loop



$$\hat{r} = -\cos\theta\,\hat{x} - \sin\theta\,\hat{y}$$

$$r = R$$

$$\vec{dl} = R\,d\theta\,\hat{\theta}$$

$$\vec{dl} \otimes \hat{r} = dl\,\hat{z}$$

$$\frac{\mu_0}{4\pi}\oint \frac{I\vec{dl} \otimes \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\int \frac{dl\,\hat{z}}{R^2}$$

$$= \frac{\mu_0 I}{4\pi R^2}\hat{z}\int dl = \frac{\mu_0 I}{4\pi R^2}\hat{z}\;2\pi R$$

$$= \frac{\mu_0 I}{2R}\hat{z}$$

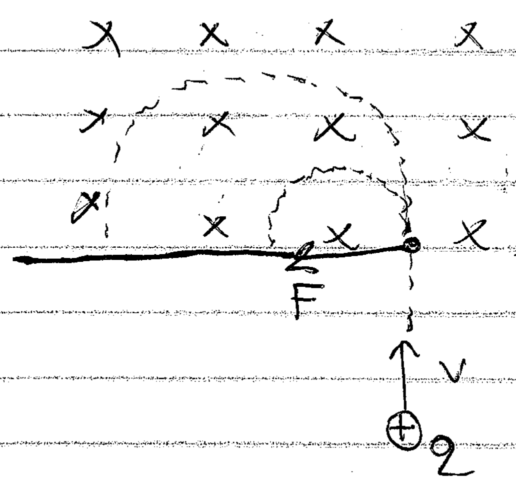
Effects of Magnetic Fields

- Force on moving charges.

$$\vec{F}_B = q\vec{v} \otimes \vec{B} \qquad \text{magnitude } F_B = q v_\perp B$$

Ex: Mass Spectrometer.

$\hat{B} = -\hat{z}$ = "into page"



$$F = qvB = ma = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

Typical Mass spec: $q = +e = +1.6 \times 10^{-19}\,C$
B = (given)
v = (given)

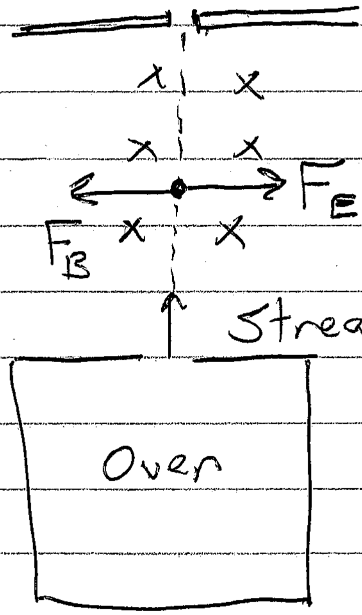
$r \propto$ mass, separates light from heavy.

How do we give them v?
Method 1: Accelerating voltage

$$q\Delta V = \text{Energy} = \frac{1}{2}mv^2$$

(ΔV ← voltage; v ← speed)

Method 2: Heat, Ionize, select.



Apply $\vec{F_E}$ to cancel $\vec{F_B}$

$$qE = qvB$$

Stream w/ various $\vec{v}$, steal electrons.

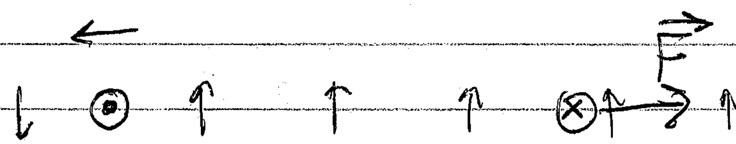
$$E = vB \quad \text{or} \quad v = \frac{E}{B}$$

Velocity Selector

Force on a current

$$\vec{F} = I\vec{l} \otimes \vec{B} \qquad \text{often} \qquad F = IlB$$

$$F/l = IB$$



I_1 = "source" = 3A

I_2 = "target" = 5A

$r = 7\text{ cm}$

- Calculate B @ I_2 as if I_2 isn't there.

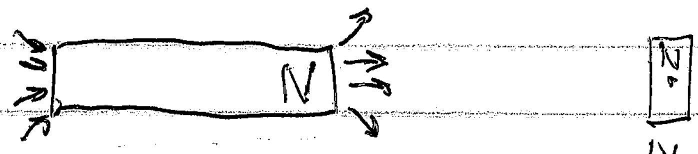
$$B = \frac{\mu_0 I_1}{2\pi r} = \frac{(4\pi \times 10^{-7})(3)}{2\pi(0.07)} = 8.57\ \mu T$$

-

$$\frac{F}{l} = I_2 B = (5A)(8.57\ \mu T) = 42.9\ \mu N/m$$

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Torques & Forces on magnets



Strong, Fixed.

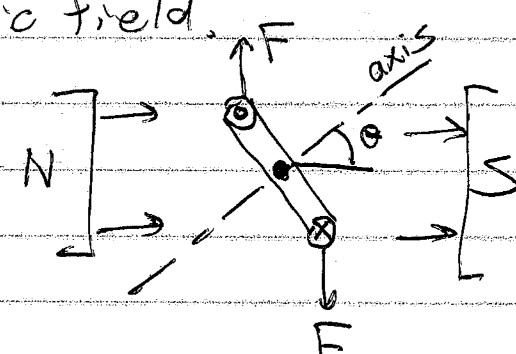
Needle is
Movable
Spinable

- Needle feels torque
- Attraction to strong field.

The torque is how we make motors.

$$\tau = NBAI\sin\theta$$

Torque on a coil in a magnetic field.



- N = loops in coil
- B = field
- A = Area of coil
- I = current
- θ = angle between coil axis and B

Torque tries to line up axis of coil w/ $\vec{B}$

Strong torque when 90° "off"