Lec 1 – Charges and E-Fields

Monday, July 8, 2019 11:27 AM

PHYS-2426, University Physics II
Blackboard: https://bb9.tamucc.edu
Dr. Spirko

Why study Electricity and Magnetism (E&M)?

- Our society is electric
  - Energy
  - Information: Communication, Computation, Storage
- Basis of Light and Radio Waves
- Basis of Chemistry
- Practice with math and learning new things.

Electrostatics

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Study of stationary charges. What are charges?

All stuff is made of atoms. Atoms are made of charged particles:
Protons, electrons, and neutrons.

Charge is a measure of "how electric" something is.
"A charge" is a particle or object that has charge.

Elementary Charge

Proton: $q_p = 1.6 \times 10^{-19}\ C = +e$

Variable for charge

Unit is coulomb.

Electron: $q_e = -1.6 \times 10^{-19}\ C = -e$

In solid materials, the protons are stuck in place. Only the electrons can move.
Most electric effects are because of electron movement.

Electric effects

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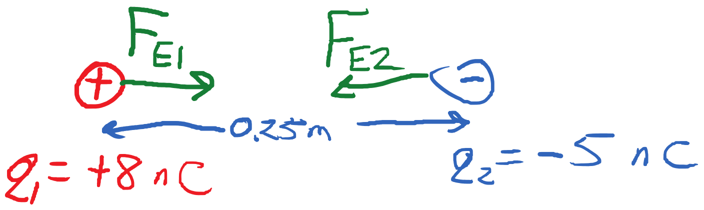
Electric force is called Coulomb force.
Easiest case is for two small (diameter) charges.



Direction:

- Opposites Attract

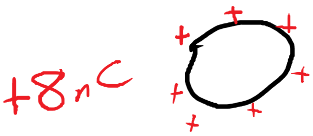
$$k = 9\times10^9 \frac{Nm^2}{C^2}$$

Magnitude: $$F_E = k\frac{|q_1|\,|q_2|}{r_{12}^{\;2}}$$

$$F_{E1} = \left(9\times10^9 \frac{Nm^2}{C^2}\right)\frac{(8\times10^{-9} C)(5\times10^{-9} C)}{(0.25 m)^2} = 5.76\times10^{-6} N$$
$$= 5.76\ \mu N$$

Usually, objects that have nC or micro-C charges are metal.
How does metal carry its charge?
Metals are conductors. They allow electrons to move around.
The electrons forming the nC of charges are all repelled.

$$-5\ nC = N_e(-1.6\times10^{-19}\ C)$$
$$3.13\times10^{+10} = N_e$$



Metals carry their charge on the outside. The inside (the "bulk") is neutral.

What happens if we connect these two objects with a small wire?

- Both sets of charges repel each other. Effectively, the charges combine, then split evenly between the objects. (Really what happens is the two objects get the same Voltage, but we'll get to that.)

Now, $q_1 = 1.5\,nC$ $q_2 = 1.5\,nC$

Can recalculate the force, which is now repulsive.

Charge distributions

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Point Charges $Q_{net} = \Sigma q_i$

Line Charges $Q_{net} = \int \lambda \, dl$ ← Length piece

↳ λ = charge per length

Surface Charges $Q_{net} = \iint \sigma \, dA$

↳ σ = surface charge density

Volume Charges $Q_{net} = \iiint \rho \, dV$

↳ ρ = volume charge density

E-Field instead of Force

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If we wanted to calculate the total electric force on some charge, Q, we would do:

"Hat" = unit vector = direction

$$\vec{F}_{net} = \sum_i \frac{kQq_i}{r_i^2}\hat{r}_i = Q\left[\sum_i \frac{kq_i}{r_i^2}\hat{r}_i\right]$$

This tells me the electric force has two contributions:

- Q, a property of my object
- [(stuff)], a property of the environment

This environmental contribution is a lot like *g* in $F_g = m\,g$

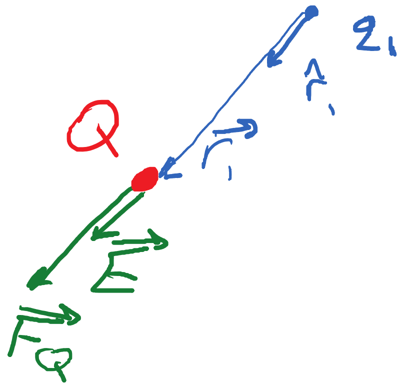
$$\vec{F}_E = Q\,\vec{E}$$

$\vec{E}$ = electric field

The electric field due to a point charge is:

$$\vec{E} = \frac{kq_1}{r_1^2}\hat{r}_1$$

$$\vec{F}_Q = Q\vec{E} = \frac{kQq_1}{r_1^2}\hat{r}_1$$



Electric field is the force per coulomb of test charge, felt by a test charge.

E is in N/C

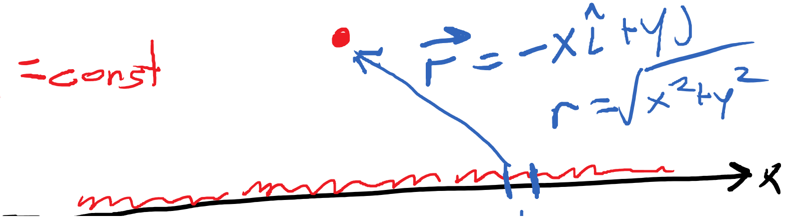
$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

$$\hat{r} = \frac{\vec{r}}{r}$$

Other sources of electric field:

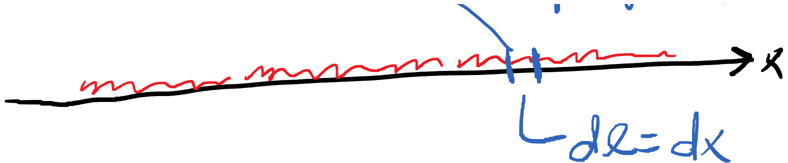
λ: $\vec{E} = \int \frac{k\lambda\, d\ell}{r^2}\hat{r}$

Uniform infinite λ: λ = const



$$\vec{r} = -x\hat{i} + y\hat{j}$$

$$r = \sqrt{x^2+y^2}$$



$$\vec{E} = \int \frac{k\lambda\,dx}{(x^2+y^2)} \frac{(-x\hat{i}+y\hat{j})}{\sqrt{x^2+y^2}} = k\lambda y\hat{j}\int \frac{dx}{(x^2+y^2)^{3/2}}$$

(Due to Fire Alarm, we will pick up here Tuesday.)

Lec 2 - Gauss's Law

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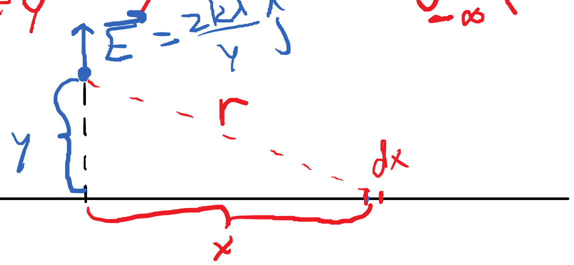
$$\vec{E} = \int_{-\infty}^{\infty} \frac{k\overbrace{(\lambda\, dx)}^{dq=\lambda dx}}{(x^2+y^2)} \frac{(\overbrace{-x\hat{i}}^{\text{odd function}}+\overbrace{y\hat{j}}^{\text{even function}})}{\sqrt{x^2+y^2}} = k\lambda y\hat{j}\overbrace{\int_{-\infty}^{\infty}\frac{dx}{(x^2+y^2)^{3/2}}}^{\text{math integral}=\frac{2}{y^2}}$$



$\vec{r} = -x\hat{i} + y\hat{j}$

$\hat{r} = \frac{-x\hat{i}+y\hat{j}}{\sqrt{x^2+y^2}}$

Result: $E = \frac{2k\lambda}{y}$ y = distance from line charge

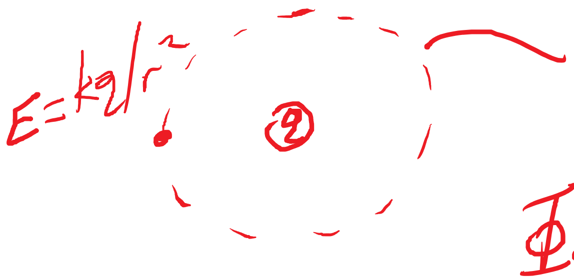
It would be nice if there was an easier method.
The above method is called the Green's Function method.

The easier method is based on Stoke's Theorem.
In application to Physics II, it's called Gauss's Law.

Every point charge "emits" a particular amount of electric flux.
Electric field is electric flux per unit area.

$$E = \frac{\Phi_E}{A} = \frac{kq}{r^2}$$

How much flux goes thru a sphere surrounding q?



Area = $4\pi r^2$
Flux = Φ_E

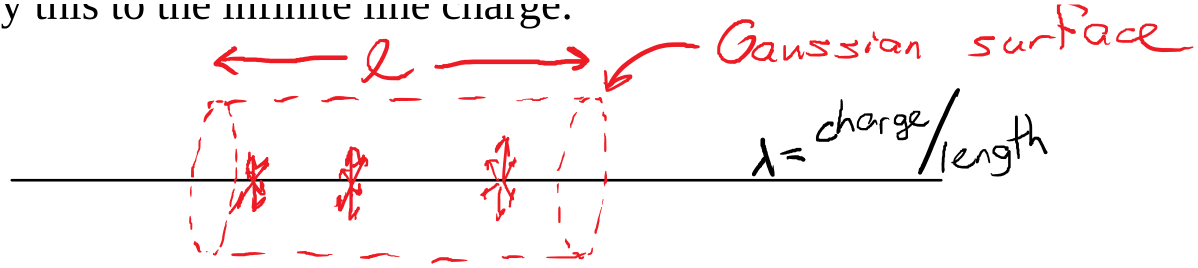
$$\frac{\Phi_E}{4\pi r^2} = \frac{kq}{r^2}$$

Gauss's Law: $\Phi_E = 4\pi k q$

Apply this to the infinite line charge:

G surface

Apply this to the infinite line charge.



Our Gaussian surface is a cylinder. Only the round part actually "catches" any of the flux.

$A = \ell 2\pi r$

Inside the cylinder: $q_{enc} = \lambda \ell$

Flux generated inside: $\Phi = 4\pi k q_{enc} = 4\pi k \lambda \ell$

Elec Field $E = \frac{\Phi_E}{A} = \frac{4\pi k \lambda \ell}{2\pi r \ell}$

$$E = \frac{2k\lambda}{r}$$

Slab symmetry

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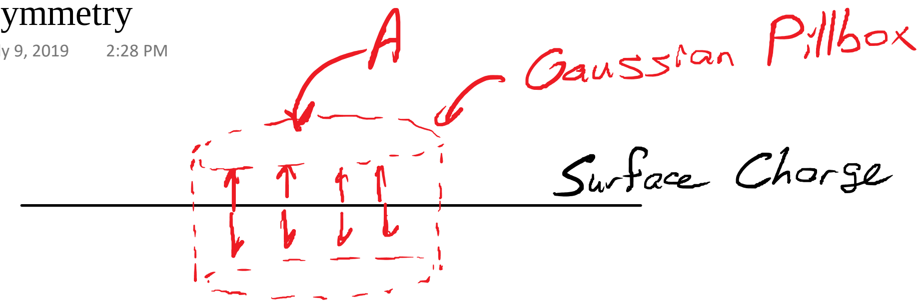

Surface Charge $\sigma = \frac{\text{charge}}{\text{Area}}$

Flux is emitted up and down, perpendicular to the surface.
Only the top and bottom of the pillbox "catch" flux.
The flux can be emitted both ways. Two cases:

- Equally - thin insulating surface
- One-way - surface of a metal conductor

Flux Emitted Both ways

$$E = \frac{\Phi_E}{A_{surf}} = \frac{4\pi k \sigma A}{2A}$$

$$E = 2\pi k \sigma$$

- Contribution To total E

Flux Emitted one way

$$E = \frac{4\pi k \sigma A}{A}$$

$$E = 4\pi k \sigma$$

- Value of total E
- $E = 0$ on other surface.

There is another constant used in electrostatics.

- Coulomb constant: $k = 9 \times 10^{9}\ Nm^2/C^2$
- Permittivity of free space: $\varepsilon_0 = 8.85 \times 10^{-12}\ \frac{C^2}{Nm^2}$

$$\frac{1}{k\,\varepsilon_0} = 12.6 = 4\pi$$

$$k = \frac{1}{4\pi\varepsilon_0}$$

$$4\pi k = \frac{1}{\varepsilon_0}$$

Gauss's Law: $\Phi_E = 4\pi k\, q_{enc} = \frac{q_{enc}}{\varepsilon_0}$

Surface Charge Field: $E = 2\pi k \sigma = \frac{\sigma}{2\varepsilon}$

Surface Charge Field: $E = 2\pi k\sigma = \frac{\sigma}{2\varepsilon_0}$

$E = 4\pi k\sigma = \sigma/\varepsilon_0$

Spherical Symmetry

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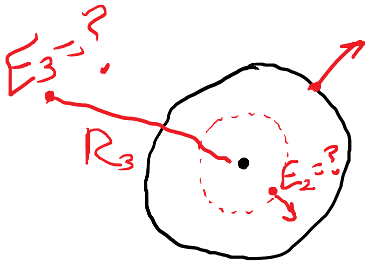
A uniformly charged insulating sphere has an electric field of 10000 N/C at the surface. How much charge exists at half of the radius?

$\rho = \frac{\text{Charge}}{\text{Volume}}$



$E_1 = 10000\ N/C = \frac{\Phi_1}{A_1}$

$V = \frac{4}{3}\pi r^3$

$E_2 = \frac{\Phi_2}{A_2} =$

$A = 4\pi r^2$

$\frac{A_2}{A_1} = \frac{4\pi r_2^2}{4\pi r_1^2}$

$r_2 = \frac{r_1}{2}$

$A_2 = \frac{A_1}{4}$

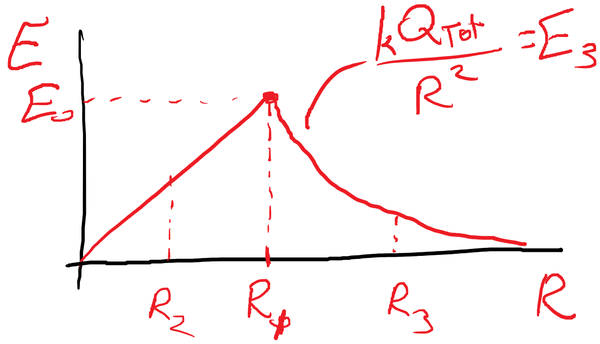
$\frac{\Phi_2}{\Phi_1} = \frac{1}{8}$

$\Phi = 4\pi k q$

$q = \rho V$

$E_2 = \frac{\Phi_2}{A_2} = \frac{\Phi_1/8}{A_1/4} = E_1\left(\frac{4}{8}\right) = \frac{E_1}{2}$

$E_2 = 5000\ N/C$



$\frac{kQ_{Tot}}{R^2} = E_3$

Outside the sphere, the flux "caught" by our Gaussian surface can't change. (R_3)

Electric Potential

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Work is defined as: $W = \int \vec{F} \cdot d\vec{\ell} = \vec{F}_{avg} \cdot \Delta\vec{\ell}$

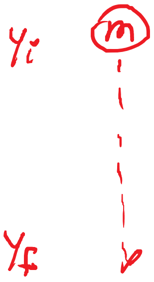
If you push something, and it moves in the direction you pushed it, you have given it energy.

Work against gravity. When you lift something, you give the object energy, and gravity takes that energy away. Of course, gravity is perfectly willing to give that energy back.

Work due to gravity when falling:

$W = F_g h = mgh$

$F_{gy} = -mg$



$U_g = mgy$

$\Delta U_g = mg\Delta y$

$W = -\Delta U_g$

$w = mgh$

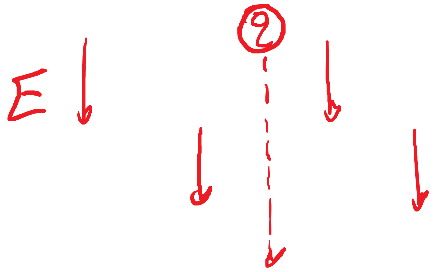
The mass doesn't "want" to be at a high location.
It naturally will feel a force pushing it toward low places.
The force of gravity points toward low potential energy.

$$F_{gy} = -\frac{dU_g}{dy}$$

There is a derivative relationship between potential energy and force.

Now let's do this with electricity.
There is an electric potential energy.

$$F_{Ey} = -\frac{dU_E}{dy}$$



$$-\int F_{ey}\, dy = +\Delta U_E$$

$$-\int q E_y\, dy = \Delta U_E$$

$\vec{F}_E = q\vec{E}$

$F_{Ey} = qE_y$

$$q\Big(\underbrace{-\int E_y\, dy}\Big) = \Delta U_E$$

Energy per unit charge

$q\Delta V = \Delta U_E$

$\Delta V = \frac{\Delta U_E}{q}$ in $\frac{J}{C}$

$\Delta V = -\int E_y dy$ in $(N/C)(m)$

$-\frac{dV}{dy} = E_y$ in V/m

ΔV in volts $= V$

$E_y = -\frac{\Delta V}{\Delta y}$

Mass	⊕ Charge	⊖ Charge
$\vec{F_g} = m\vec{g}$	$\vec{F_E} = q\vec{E}$	
Force same dir as field	same as $\vec{E}$	opposite $\vec{E}$
$U_g = mgy$	$U_E = qV$	
Tend to Low y. Tend to Low U_g	Tend to Low V	Tend to high V
	Tend to Low U_E	

In lab, we will accelerate electrons using a voltage of 250 V.

$q = -e = -1.6\times10^{-19}$ C

$\Delta U_E = q\Delta V = (-e)(250\text{ V}) = (-1.6\times10^{-19}\text{C})(250\text{ V})$

≈ -250 eV $= -4\times10^{-17}$ J

$m_e = 9.11\times10^{-31}$ kg

$K = \frac{1}{2}mv^2$

speed

$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(4\times10^{-17})}{9.11\times10^{-31}}}$

$= 9.4\times10^{6}$ m/s

speed $= 9.4 \times 10^6$ m/s

Voltage of a charged sphere

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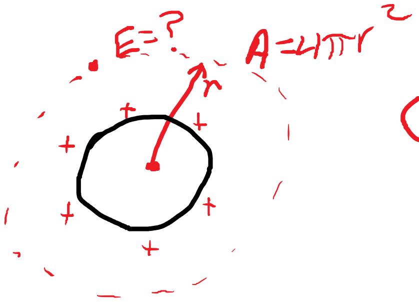
What is the maximum charge we could put on a BB?
How much voltage would that take?
How much energy would it take?

Diameter = 3 mm



Gauss's Law:

$$E = \frac{\Phi_E}{A} = \frac{4\pi k q_{enc}}{A}$$

$$E = \frac{4\pi k q_{enc}}{4\pi r^2} = \frac{k q_{enc}}{r^2}$$

(A spherical charge "looks like" a point charge, as long as we're on the outside.)

The maximum electric field for air is 1,000,000 V/m. (dielectric strength)

$$10^6 = \frac{(9\times10^9)\, q}{(0.0015)^2}$$

$10^6 = 1E6$

$= 1\times10^6$

$2.5\times10^{-10}\,C = q$

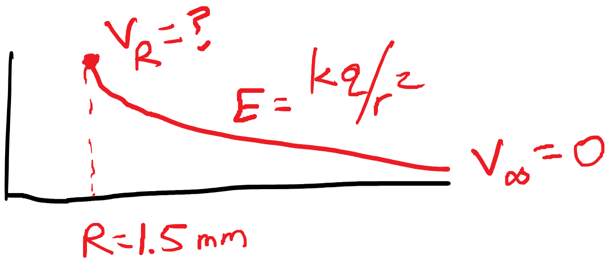
$0.25\,nC = q$

Our BB could hold a maximum of 0.25 nC.

So what voltage is necessary to make this happen?

$$E = \frac{-dV}{dx}$$

$$\Delta V = -\int E\,dx$$



$\int^{\infty} kq$

$$\Delta V = V_\infty - V_R = -\int_R^\infty \frac{kq}{r^2}\,dr$$

$$\int \frac{1}{x^2}\,dx = \frac{-1}{x}$$

$$= -\left[\frac{-kq}{r}\right]_R^\infty$$

$$= \left[\frac{kq}{r}\right]_R^\infty$$

$$V_\infty - V_R = \frac{kq}{\infty} - \frac{kq}{R}$$

$$V_R = \frac{kq}{R}$$

Looks like voltage near point charge.

$$V_R = \frac{(9\times10^{9})(0.25\times10^{-9})}{0.00015} = 1500\ \text{V}$$

It takes a lot of voltage to put a tiny bit of charge on the BB.

How much energy does it take to make this happen?
It's true that U = q V.
We cannot multiply q = 0.25 nC by V = 1500 V.
Need to add up the total energy of every little charge that gets added to the BB.

$$\text{Energy} = \sum_i q_i V_i = \int V\,dq$$

└ V proportional to q.

$$\text{Energy} = \int_0^{q_{max}} \frac{kq}{R}\,dq = \frac{k}{R}\int_0^{q_{max}} q\,dq$$

$$= \frac{k}{R}\,\frac{1}{2}q_{max}^2 = \frac{1}{2}\left(\frac{kq_{max}}{R}\right)q_{max}$$

$$= \frac{k}{R}\frac{1}{2}q_{max}^2 = \frac{1}{2}\left(\frac{kq_{max}}{R}\right)q_{max}$$

$$= \frac{1}{2}V_{max}\,q_{max}$$

$$= \frac{1}{2}(1500\,V)(0.25\,nC) = 1.88\times10^{-7}\,J$$

$$= 0.188\,\mu J$$

A BB could be used as a charge-storing, energy-storing device. But, static charges tend to dissipate.

Little metal objects are not a good charge-storing, energy-storing mechanism. Can we do better?

Capacitors

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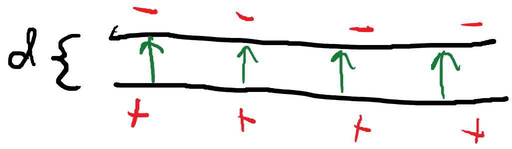
A capacitor is an electrical device made of two metal objects held very near each other. These are called "plates".

We store exactly opposite charges on each plate.



$E = 4\pi k\sigma$ $\sigma = \frac{Q}{A}$

$\Delta V = Ed = 4\pi k\sigma d$

$\frac{1}{4\pi\varepsilon_0} = k$

$\Delta V = \frac{4\pi k Q d}{A}$

$\frac{1}{4\pi k} = \varepsilon_0$

$\frac{A}{4\pi k d}\Delta V = Q$

$\frac{\varepsilon_0 A}{d}\Delta V = Q$

Example capacitor, made of aluminum foil and plastic wrap:

$A = 1\,m^2$

$d = 0.1\,mm = 10^{-4}\,m$

$\frac{\varepsilon_0 A}{d} = C = \frac{(8.85\times10^{-12})(1)}{(1\times10^{-4})}$

Variable = Capacitance

$= 8.85\times10^{-8}$

The same 1500 V (from the BB example) would store how much charge?

$Q = CV = (8.85\times10^{-8})(1500)$

$= 1.33\times10^{-4}\ C$

Unit = coulomb

$= 133\ \mu C$

$= 133000\ nC$

With this simple lousy capacitor, we can store a half million times as much charge as the BB.

Above:

$C = 88.5\ nF$

With this simple lousy capacitor, we can store a half million times as much charge as the BB.

Above:
$C = 88.5 \text{ nF}$

The unit of capacitance (C) is the farad (F).

Common: $C = 1000 \mu F$

How much energy does our capacitor store?

Self-interaction energy

$$\text{Energy} = \frac{1}{2} Q V$$
$$= \frac{1}{2} C V^2 = \frac{1}{2}(8.85 \times 10^{-8})(1500)^2$$
$$= 0.1 \text{ J}$$

Electric Current

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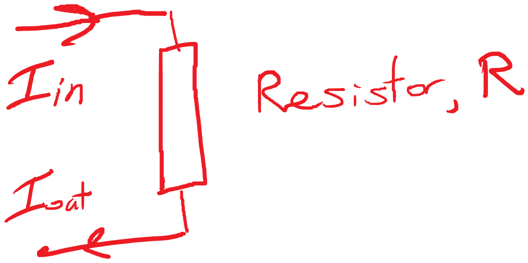
Current (I) is the rate of flow of charge, measured in amperes (aka amps, A).

$$I = \frac{dq}{dt}$$

This is somewhat misleading, as usually, there isn't some charge *q* that is changing. A current is just a contribution toward the change of charge on some object.



We've seen that only tiny amounts of charge can accumulate on small objects. The current that flows can be on the order of 1.0 A = 1.0 C/s, and it can flow continuously for a long time.

Kirchoff's Current Law:

$$I_{in} = I_{out}$$

$$\frac{dq}{dt} = I_{in} - I_{out} = 0$$

Since the input and output currents are equal, we say that the current "flows through" the resistor.

Every electrical device requires a minimum of 2 connections.

To **measure** current, we have to route the current through our meter.

Build the circuit first, then break it and put the meter in place.

- **Series** is when two devices are connected so that all current going through one device MUST go through the other device.
- When in series, two devices have the same current passing through.
- Since the meter and our device have the same current, the number on the meter is also the value for the device.

Voltage (V) is the motivating "force" making charges move.

- Voltage is like pressure. Voltage is like height.
- The current that flows depends on the voltage and the device.

Resistor

Incandescent Bulb

LED Bulb

Resistance is the basic relationship between current and voltage.

$$R = \frac{V}{I} \qquad V = IR \qquad \text{Ohm's Law}$$

Ex: A current of 0.1 A can be lethal.

To push this, a voltage of ~150 V is needed.

$$(150\ V) = (0.1 A)\ R$$

$$1500\ \Omega = R$$

Measuring Resistance and Voltage

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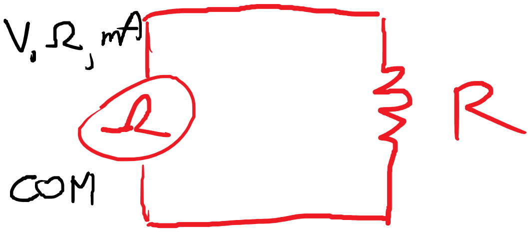
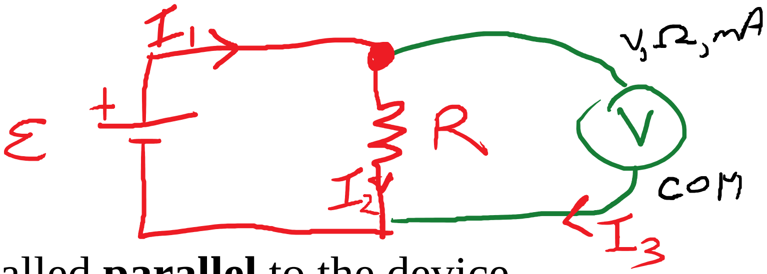
An ohmmeter measures resistance.

- It supplies voltage and measures current.
- It must be the ONLY voltage supply.



Measuring Voltage

- Voltage is always relative. A voltmeter actually compares the electric potential at two places in the circuit.
- For this reason, the meter is connected to two places:



The voltmeter connection is called **parallel** to the device.

- To be in parallel, two devices must be connected at both ends.
- The current path branches and joins.

Kirchoff's Current Law: $\sum I_{in} = \sum I_{out}$

Above: $I_1 = I_2 + I_3$

In a parallel circuit, the currents of the multiple branches add up to the overall current from the source.

Lec 4 - V, I, R, P

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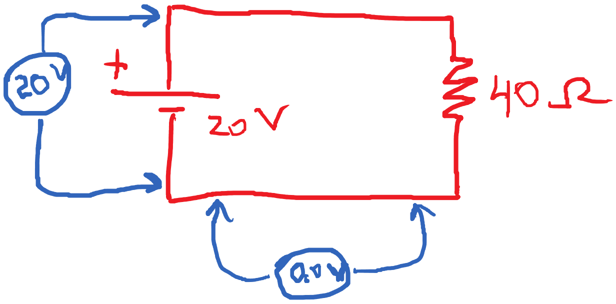


Battery

- Provides a (mostly) constant voltage called the EMF.
- Can provide (almost) any current.
- Converts chemical energy to electrical energy.

Wires

- Allow current to flow easily, with almost zero force.
- The force is electric, so zero force means zero electric field.
- Since electric field is the gradient of the voltage, there is no voltage change from one end of the wire to the other.

$$E = \frac{-dV}{dx}$$

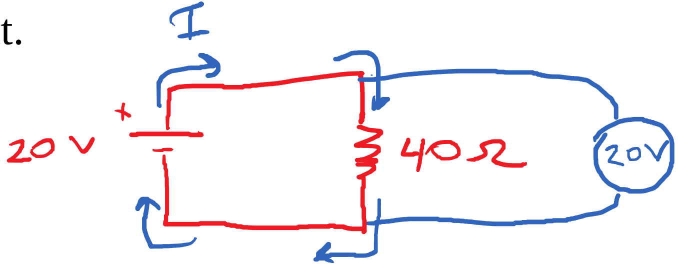
Resistor

- Has a (mostly) constant resistance.
- Current depends on applied voltage.
- Converts electrical energy to heat.

$$V = IR$$

$$(20\,V) = I\,(40\,\Omega)$$

$$0.5A \quad = I$$



Why could I take the 20 V from the battery and use it in the formula for the resistor?

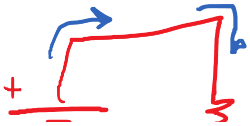
- The battery and resistor are connected at both ends.
- Parallel components have the same voltage.
- This is a form of Kirchoff's Voltage Law.

For any loop: $\Sigma \Delta V = 0$

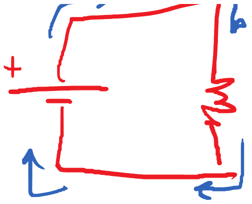
If I follow the loop of current:

$$(+20\ V) + \Delta V_R = 0$$

This resistor must cause Delta-V of -20 V.
When following the current, a resistor causes a "voltage drop" calculated from Ohm's Law.



"voltage drop" calculated from Ohm's Law.



This was also a **series circuit** because all of the current was directed through both components.

Power and Cost of Energy

Thursday, July 11, 2019 2:32 PM

Power is the rate of energy transfer.

$$P = \frac{\Delta Energy}{\Delta t} = \frac{\Delta Energy}{\Delta q} \frac{\Delta q}{\Delta t}$$

$$P = V I$$

For our light bulb: $P = (20\ V)(0.5\ A) = 10\ W$

Electrical energy is sold in different units: $1\ kWh = \left(1000\ \frac{J}{s}\right)(3600\ s)$

$$= 3.6 \times 10^6\ J$$

$$= 3.6\ MJ$$

$$Cost = (Rate)(Amount)$$

$$Rate \approx \$0.12/kWh$$

How much energy can we buy for $1.00?

$$Amount = \frac{Cost}{Rate} = \frac{\$1.00}{\$0.12/kWh} = 8.333\ kWh$$

$$= 8.333\ (3.6\ MJ) = 30\ MJ$$

Compare that to gasoline: $Rate = \frac{\$2.40}{33.7\ kWh} = \$0.071/kWh$

For how long can we run that light bulb for $1.00?

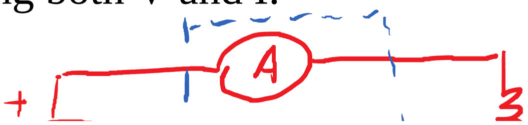
$$P = \frac{\Delta Energy}{\Delta t} \longrightarrow \Delta t = \frac{\Delta Energy}{P} = \frac{30\ MJ}{10\ W}$$

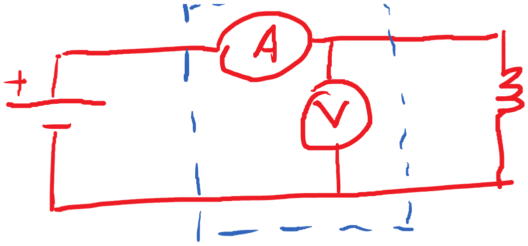
$$= 3\ Ms = 3 \times 10^6\ s$$

$$= 833\ hours$$

$$\sim 1\ month$$

Measuring Power requires measuring both V and I.



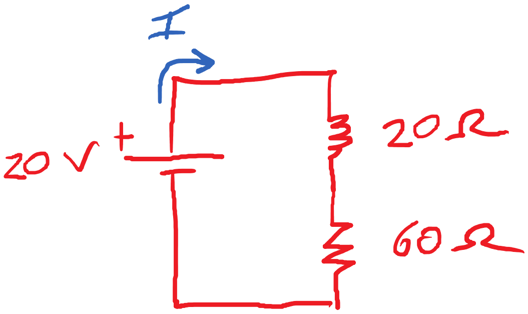

+
A
V

Series Circuits

Thursday, July 11, 2019 2:55 PM

A series circuit has just one current loop.

$I_1 = I_2 = \dots$



A series circuit obeys KVL.

$$\sum \Delta V = 0$$

$$(+20\,V) - I(20\,\Omega) - I(60\,\Omega) = 0$$

Each $\Delta V_R = -IR$

$$+20\,V = I(20\,\Omega + 60\,\Omega)$$

$$V_T = I R_{eq}$$

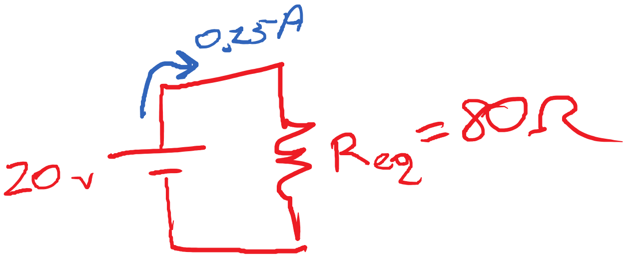
In a series circuit:

$I_1 = I_2 = \dots$ $V_T = V_1 + V_2 + \dots$ $R_{eq} = R_1 + R_2 + \dots$

In our example:

$$\frac{20\,V}{80\,\Omega} = I = 0.25\,A$$

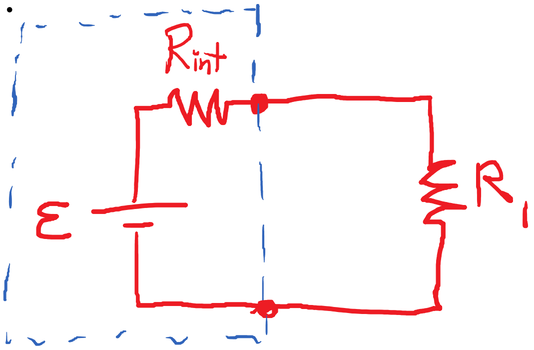
What we did is to build an equivalent circuit:



Internal Resistance

Thursday, July 11, 2019 3:09 PM

Batteries are not perfectly efficient.
The current must flow through the inefficiency.
The inefficiency is effectively in series with the EMF.
Often, it can be modeled as resistance.



$$V_T = V_1 + V_2$$

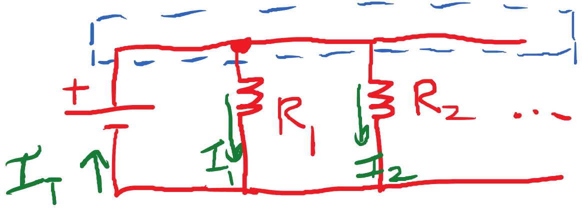
$$\underbrace{\mathcal{E}}_{\text{No-Load Voltage}} = \underbrace{V_1}_{\text{Loaded Voltage}} + \underbrace{I}_{\text{Current of Load}} R_{int}$$

Parallel Circuits

Thursday, July 11, 2019 3:16 PM

Components are in parallel if they are connected at both ends.

A circuit is in parallel if all components are in parallel.



K. Current Law: $\Sigma I_{in} = \Sigma I_{out}$

$$I_T = I_1 + I_2 + \dots$$

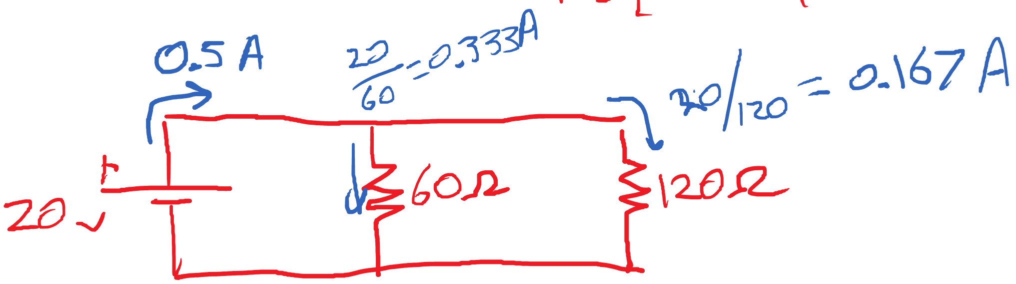
K. Voltage Law: $V_{Batt} = V_1 = V_2 = \dots$

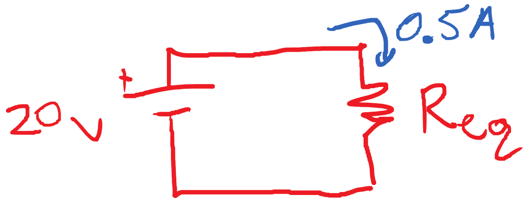
$$V_{Batt} = I_T R_{eq}$$

$$\frac{V_{Batt}}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \dots$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

Ex:





$$R_{eq} = \left(\frac{1}{60} + \frac{1}{120}\right)^{-1} = \left(\frac{3}{120}\right)^{-1} = 40\Omega$$

Power in the parallel circuit:

R_1: $P_1 = (20\text{ v})(0.333A) = 6.67\text{ W}$

$P_{Batt} = (20\text{ v})(0.5A) = 10\text{ W}$

R_2: $P_2 = (20\text{ v})(0.167A) = 3.33\text{ W}$

Short and Open Circuits

Thursday, July 11, 2019 3:31 PM

A wire is like a zero-ohm resistor.
A disconnection is like an infinity-ohm resistor.

Series:

$$R_T = R_1 + R_2 + \cancelto{0}{R_{wire}} + \dots$$

No problem

Ammeter

$$R_T = \cancelto{\infty}{(R_1 + R_2)} + R_{disconnect}$$

Open Circuit

Parallel:

$$\frac{1}{R_{eq}} = \cancelto{0}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} + \overset{\infty!}{\left(\frac{1}{R_{wire}}\right)}$$

Short Circuit

$$R_{eq} = R_{wire}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cancelto{0}{\frac{1}{R_{disconnect}}}$$

No problem

Voltmeter

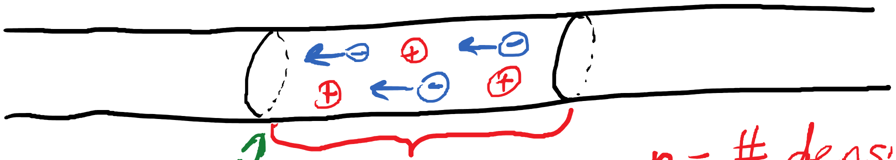
Lec 5 DC Circuit Analysis

Monday, July 15, 2019 1:50 PM

Exam 1 Thu. HW2 posted.

What happens in a wire, at a microscopic level?

- Many, many charges move slowly.



How much charge passes in Δt?

$$V = A\Delta x = A v_d \Delta t$$

n = # density = # movable charges per m^3.

q = charge of each movable charge

$$\Delta Q = \overset{\text{\# charges}}{N} q = (nV)q$$

$$n = \frac{N}{V} \qquad N = nV$$

V = volume

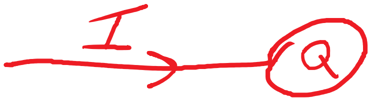
$$\Delta Q = n A v_d \Delta t \, q$$

$$I = \frac{\Delta Q}{\Delta t} = n A v_d q$$

v_d = drift velocity

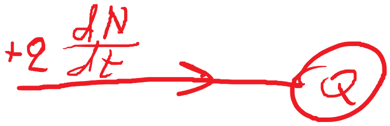
Note: because q = -e (i.e. negative), the direction of the current and the direction of the velocity are opposite.

Generic:



$$I = \frac{dQ}{dt}$$

+ charges



$$Q = Nq$$

$$\frac{dQ}{dt} = \frac{dN}{dt} q$$

- charges

$$Q = N(-q)$$

$$\frac{dQ}{dt} = -\frac{dN}{dt} q$$

Negative charges flowing outward causes Q to increase.
Current flowing inward causes Q to increase.
Negative charges flowing outward ***are*** a current flowing inward.
The current is opposite to the velocity of the charges if they're negative.

Numerical calculation for copper:

Density: $\rho = 8.92\ g/cm^3 = 8920\ kg/m^3$

$m = 63.5\ u = 1.06 \times 10^{-25}\ kg$

L unified atomic mass unit
~ proton mass

Density of Atoms: $n = \frac{\rho}{m} = 8.41 \times 10^{28}\ atoms/m^3$

2 mm wire $A = 3 \times 10^{-6}\ m^2$

$I = 10.0\ A$ $q = -1.6 \times 10^{-19}\ C$

$$I = nAv_d q \Longrightarrow v_d = \frac{I}{nAq}$$

$$\frac{10}{(8.41 \times 10^{28})(3 \times 10^{-6})(-1.6 \times 10^{-19})} = -2.5 \times 10^{-4}\ m/s$$

$$= -0.25\ mm/s$$

How can we reconcile this with our experience?
The wire is already filled with charges. We don't have to wait for them to move slowly.

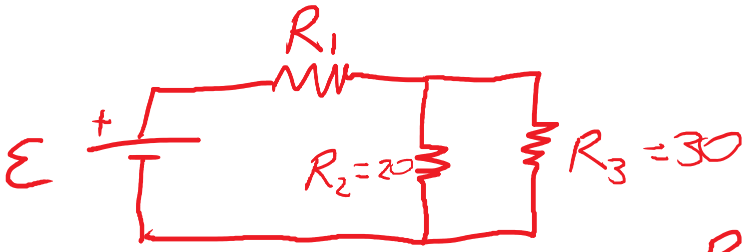
Circuit Analysis Techniques

Monday, July 15, 2019 2:28 PM

- Ohm's Law, Series Rules, Parallel Rules
- Equivalent Circuits
- Kirchoff's Laws

Equivalent circuits:

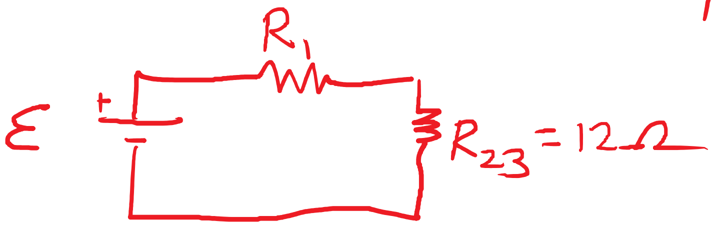
- Simplify a circuit by substituting out part of it with an equivalent series or parallel part.



Parallel: $\frac{1}{R_{eq}} = \frac{1}{R_2} + \frac{1}{R_3}$

$$R_{23} = \left(\frac{1}{20} + \frac{1}{30}\right)^{-1}$$

$$= \frac{60}{5} = 12\Omega$$



Eventually, you'll get to a simple, series, or parallel circuit that you can analyze directly. Some information can transfer back to the complicated circuit:

- Info about the unchanged parts.
- Voltage in a parallel substitution
- Current in a series substitution

$\mathcal{E}$, I_{batt} R_1, V_1, I_1

Example Combination Circuit

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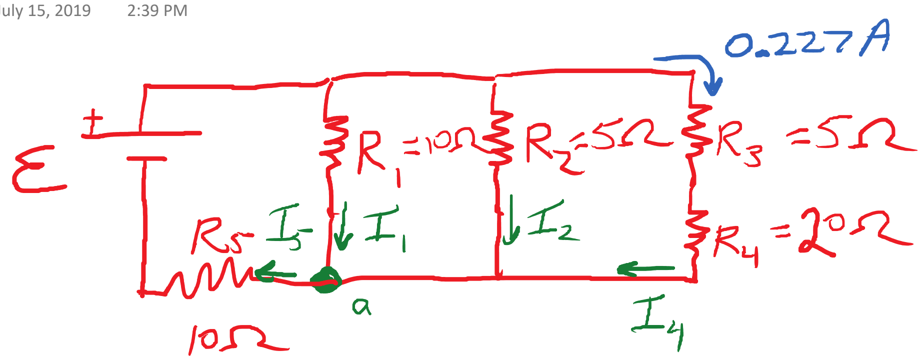
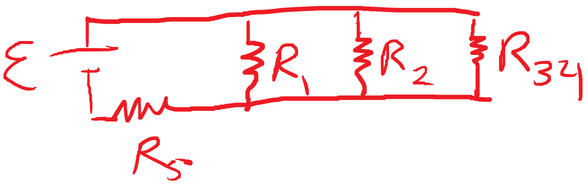


Desc	V	I	R
R_1	5.67	0.567	10Ω
R_2	5.67	1.134	5Ω
R_3		0.227	5Ω
R_4		0.227	20Ω
R_5		1.928	10Ω
Batt		1.928	12.94
$R_3+R_4=R_{34}$	5.67	0.227	25Ω
$R_1\Vert R_2\Vert R_{34}=R_{1234}$	5.67		2.94



$$\left(\frac{1}{10}+\frac{1}{5}+\frac{1}{25}\right)^{-1}=2.94$$

$$V_{34}=I_{34}R_{34}=5.67$$

At point a: $I_1+I_2+I_4=I_5$ (Kirchoff's Current Law)

$$0.567+1.134+0.227=I_5$$

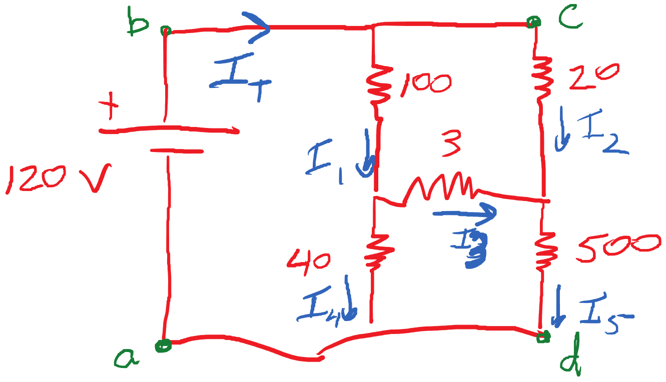
$$1.928=I_5$$

$R_{1234}+R_5=12.94$ = overall R_{eq}

$$\mathcal{E}=I_{batt}R_{eq}=(1.928)(12.94)=24.9\text{ V}$$

Kirchoff's Laws

Monday, July 15, 2019 3:03 PM



How much current flows through the 3 ohm resistor? What direction is it flowing?

No series or parallel equivalents.
Could use a Delta-Wye transformation.
Could use Kirchoff's Laws.

Assign a variable to each current in the circuit. Label each with an arrow.
Write out Kirchoff's Current Laws and Voltage Laws for the unknowns.

$I_T = I_1 + I_2$

$I_T = I_4 + I_5$

$I_1 = I_3 + I_4$

$I_2 + I_3 = I_5$ (Redundant?)

Kirchoff's Voltage Law: For any loop: Total $\Delta V = 0$

Loop abcda: Battery $a \to b$ is ⊖ to ⊕ $+120$ V

R_2, "with" I_2 : $-R_2 I_2$

R_5, "with" I_5 : $-R_5 I_5$

Outer Loop: $120 - 20 I_2 - 500 I_5 = 0$

Left Loop: $120 - 100 I_1 - 40 I_4 = 0$

Bottom Loop: $-3 I_3 - 500 I_5 + 40 I_4 = 0$

$I_1 = I_3 + I_4 \rightarrow a = c + d$

$I_2 + I_3 = I_5 \rightarrow b + c = e$


$a \approx 0.40104, \quad b \approx 1.7657, \quad c \approx -1.5964, \quad d \approx 1.9974, \quad e \approx 0.16937$

↑

I_3 came out negative

I3 has a magnitude of 1.60 A and points to the left.

Here, I solved it via Wolfram Alpha. Alternatives:

- Matrix method for linear simultaneous equations.
- Many TI Calculators can do it.
- Substitution and Elimination, by hand.

On the exam, we may have a 3 equations + 3 unknowns.

Multiple Battery Example

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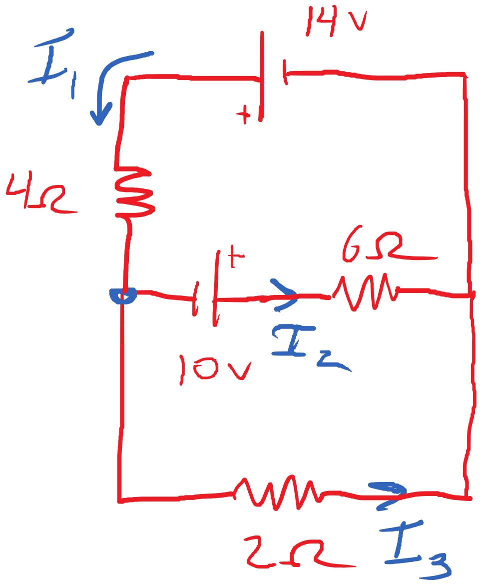


Current in each branch?

$$I_1 = I_2 + I_3$$

Top Loop: $14 - 4I_1 + 10 - 6I_2 = 0$

CCW $14 - 12 + 10 - 12 = 0$

Outer Loop: $14 - 4I_1 - 2I_3 = 0$

$14 - 12 - 2 = 0$

Solution:

$I_1 = 3.0$ A

$I_2 = 2.0$ A

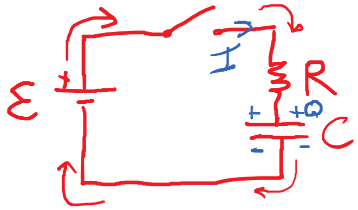
$I_3 = 1.0$ A

Lec 6 - RC Circuits

Tuesday, July 16, 2019 1:43 PM

The RC circuit has 3 components:

- EMF - Battery used to charge the capacitor, or even a wire used to discharge it.
- Resistor - Represents whatever resistance the current must push through.
- Capacitor - Builds up charge (Q) which is proportional to its voltage (V_C).



$$Q = CV_c$$

Kirchoff's Voltage Law: $+\mathcal{E} - IR - \frac{Q}{C} = 0$

The drawn current increases Q: $I = \frac{dQ}{dt}$

$$\mathcal{E} - \frac{dQ}{dt}R - \frac{Q}{C} = 0$$

$$-\frac{dQ}{dt}R - \frac{Q}{C} = -\mathcal{E}$$

$$\frac{dQ}{dt} + \frac{Q}{RC} = \frac{+\mathcal{E}}{R}$$

There is a special kind of function whose derivative is VERY closely related to the function.

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dt}e^{-t/\tau} = \frac{-1}{\tau}e^{-t/\tau}$$

Let $Q = Q_0 e^{-t/\tau}$ then $\frac{dQ}{dt} = \frac{-1}{\tau}Q$

$$\frac{-1}{\tau}Q + \frac{Q}{RC} \overset{?}{=} \mathcal{E}/R$$

$$\left(\frac{-1}{\tau} + \frac{1}{RC}\right)Q_0 e^{-t/\tau} \overset{?}{=} \mathcal{E}/R$$

The only way to make the LHS constant is for:

$$\frac{-1}{\tau} + \frac{1}{RC} = 0 \qquad \frac{1}{\tau} = \frac{1}{RC} \qquad \boxed{\tau = RC}$$

This ALMOST works. It makes the LHS = 0, which is a constant. But it's the wrong constant.

Apparently, our function was wrong in one slight way. Correct by adding a shift.

$$Q = Q_0 e^{-t/\tau} + b$$

$$Q' = Q_0 e^{-t/\tau}\left(\frac{-1}{\tau}\right)$$

$$\frac{dQ}{dt} + \frac{1}{RC}Q = \frac{\mathcal{E}}{R}$$

$$Q_0 e^{-t/\tau}\left(\frac{-1}{\tau}\right) + \frac{1}{RC}\left(Q_0 e^{-t/\tau} + b\right) = \frac{\mathcal{E}}{R}$$

If $\tau = RC$, the exponential terms cancel,

That leaves: $\frac{b}{RC} = \frac{\mathcal{E}}{R}$

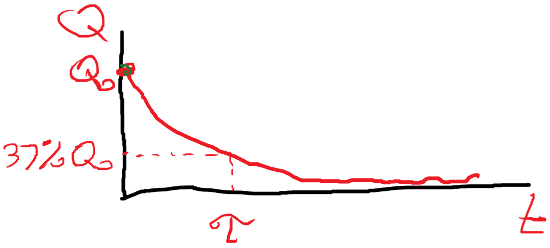
$$b = \mathcal{E}C$$

What does this function look like?

Simple case: EMF = 0 $b = 0$ $Q = Q_0 e^{-t/\tau}$

$e = 2.71818...$

This is "Exponential Decay".



	$-t/\tau$	$\exp(-t/\tau)$	
	0.0	1.0	
$t=\tau$	1.0	0.368	= 37%
	2.0	0.135	= 14%
	3.0	0.050	= 5%
	4.0	0.018	= 2%
	5.0	0.007	= 1%

When discharging a capacitor (EMF=0) the charge gets under 1% of the original (Q_0) just before 5 time constants, where the time constant is equal to R*C.

Hook a 500 micro-F capacitor up to a 2 k-ohm resistor. If the voltage starts at 15 V, what is the voltage after 3.0 s?

$\tau = RC$

$$= (2000\ \Omega)(500 \times 10^{-6} F)$$
$$= 1.0\ s$$

We already know how the charge of the capacitor behaves.

$$Q = Q_0 e^{-t/\tau}$$

$$\frac{Q}{C} = V_C = \frac{Q_0}{C} e^{-t/\tau} = V_0 e^{-t/\tau}$$

$$V_C = V_0 e^{-t/\tau}$$
$$= (15 v)\ e^{-3} = 0.75\ V$$

At what time would the voltage be equal to 10.0 V?

$$10 = 15\ e^{-t/\tau}$$

$$\frac{10}{15} = 0.667 = e^{-t/\tau}$$

Note: The exponential is "how much of the process remains to be completed".

$$\ln(0.667) = \ln\left(e^{-t/\tau}\right)$$

$$-0.405 = -t/\tau$$

$$0.405\ \tau = t$$

Here: $\tau = 1.0\ s$ $\quad t = 0.405\ s$

Get the exponential by itself then take the natural logarithm.

How we got here: We set EMF=0, which made b=0, which meant: $Q = Q_0 e^{-t/\tau}$

The Q_0 value is called the initial condition or the boundary value.

Charging a capacitor

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$$\mathcal{E} - IR - \frac{Q}{C} = 0$$

$$R\frac{dQ}{dt} + \frac{Q}{C} = \mathcal{E}$$

$$\frac{dQ}{dt} + \frac{Q}{RC} = \frac{\mathcal{E}}{R}$$

$$Q = ae^{-t/\tau} + b$$

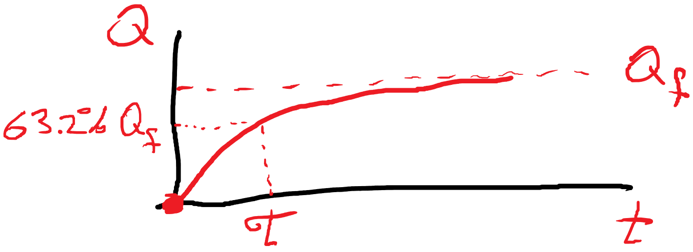
$$b = \mathcal{E}/C$$

Typically, we want Q(0) = 0 when charging.

$$0 = ae^{0} + b = a + b \qquad a = -b$$

Then $Q = b - be^{-t/\tau}$

$$Q = Q_f\left(1 - e^{-t/\tau}\right)$$



Exponentially approaching a limit.

What happens when t = tau?

$$e^{-t/\tau} = e^{-1.0} = 0.368$$

$$\left(1 - e^{-t/\tau}\right) = (1 - 0.368) = 0.632$$

RC Summary:

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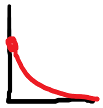
Charging from empty

$$Q = Q_f(1-e^{-t/\tau})$$

Discharging

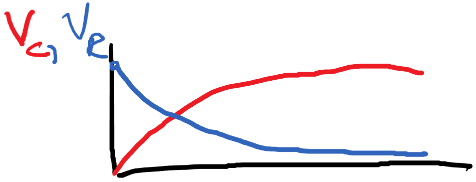
$$Q = Q_0 e^{-t/\tau}$$



$$I = I_0 e^{-t/\tau}$$

$$V_c = V_f(1-e^{-t/\tau})$$

$$V_c = V_0 e^{-t/\tau}$$



$$V_R = V_{max} e^{-t/\tau}$$

Ex: In an RC circuit with a 5 k-ohm resistor, the capacitor starts at 8.0 V. The current is measured to be 1 mA after 4.0 s. What is the capacitance? Strategy: Find C via tau = R*C. Since I know the current at a particular time, use the current function.

$V_0 = I_0 R$

$8.0 = I_0 (5000)$

$I_0 = 0.0016$

$I = I_0 e^{-t/\tau}$

$(0.001) = (0.0016) e^{-t/\tau}$

$\frac{1}{1.6} = e^{-t/\tau}$

$\ln\left(\frac{1}{1.6}\right) = -0.47 = -t/\tau$

$0.47\tau = t$

$\tau = 4/0.47 = 8.51\ s$

$\tau = RC$

$8.51 = 5000\ C$

$0.0017\ F = C$

$1.7\ mF = C$ (Never use millifarads)

$1700\ \mu F = C$

HW Example

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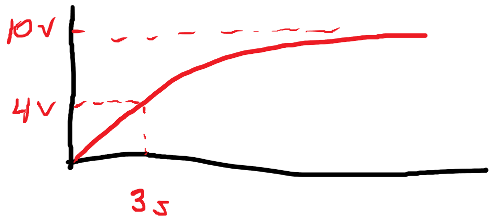
A 10.0 μF capacitor is charged by a 10.0 V battery through a resistance R. The capacitor reaches a potential difference of 4.00 V at a time 3.00 s after charging begins. Find R.



$$V = V_f\left(1 - e^{-t/\tau}\right)$$

40% done

60% remaining

$$e^{-t/\tau} = 0.6$$

$$-t/\tau = \ln(0.6)$$

$$t = -\ln(0.6)\,\tau$$

At t=6.0 s, what % of the final voltage have we reached? (64%)

$$e^{-t_2/\tau} = e^{-2t_1/\tau} = \left(e^{-t_1/\tau}\right)^2 = 0.6^2 = 0.36$$

100% − 36% = 64%

A capacitor is charged to 100 V and discharged through a resistor. At t=6 s, the voltage is 50 V. What is the voltage at t=12 s? (25 V)

$t_{1/2}$ = time to get to 50%

$$e^{-t_{1/2}/\tau} = 0.5$$

$$-t_{1/2}/\tau = -0.693$$

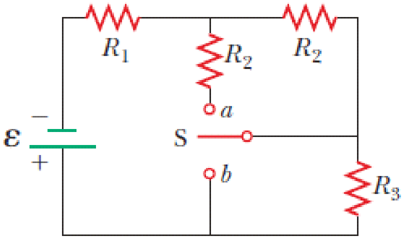
$$t_{1/2} = 0.693\,\tau$$

$$1.44\, t_{1/2} = \tau$$

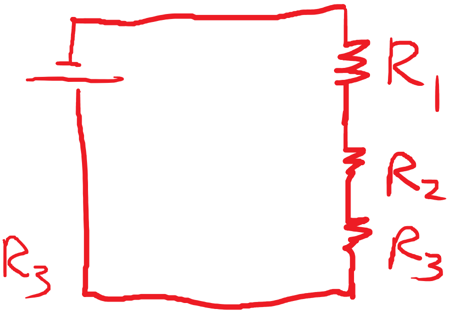
HW2-14

Tuesday, July 16, 2019 3:40 PM

A battery with $\mathcal{E}$ = 2.00 V and no internal resistance supplies current to the circuit shown in the figure below. When the double-throw switch *S* is open as shown in the figure, the current in the battery is 1.02 mA. When the switch is closed in position *a*, the current in the battery is 1.23 mA. When the switch is closed in position *b*, the current in the battery is 2.07 mA. Find the following resistances.

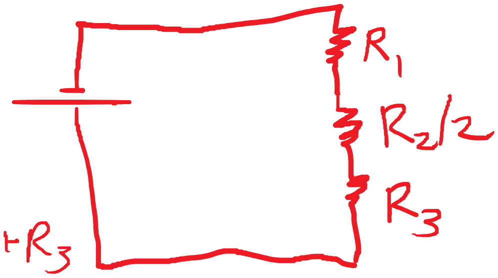


Open



$R_{eq} = R_1 + R_2 + R_3$

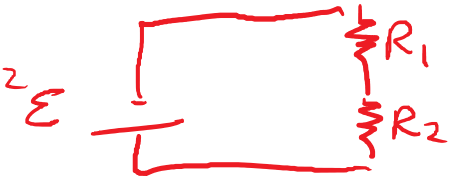
a:



$\left(\frac{1}{R_2}+\frac{1}{R_2}\right)^{-1} = \left(\frac{2}{R_2}\right)^{-1} = \frac{R_2}{2}$

$R_{eq} = R_1 + \frac{R_2}{2} + R_3$

b:



$\left(\frac{1}{0}+\frac{1}{R_3}\right)^{-1} = (\infty)^{-1} = 0$

$R_{eq} = R_1 + R_2$

The "middle R2" is "open" except when the switch is in position (a). The resistor R3 is "shorted" in position (b).

Lec 7 - Review

Wednesday, July 17, 2019 1:53 PM

Electrostatics

- Charge (q)
- E-Field (E) and Force (F)
- Potential (V) and P. Energy (U)
- Conductors and Insulators
- Capacitors - Parallel Plate Cap.
- Linear Accelerator
- Gauss's Law

DC Circuits

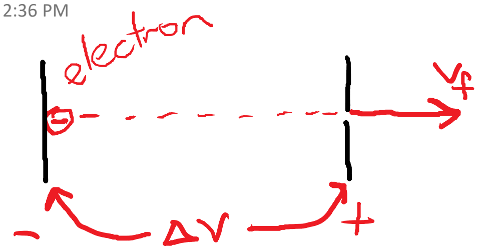
- Current (I), Voltage (V)
- Resistance (R), Ohm's Law
- Drift Velocity
- Series, Parallel
- Kirchoff's Laws
- Power, Energy, Cost
- RC Circuits

Resources:

- Equation Sheet
- Homework
- Textbook - Look for conceptual questions throughout the chapters.
- Old practice exams
- Prelab Practice, Lab Instructions

Linear Accelerator

Wednesday, July 17, 2019 2:36 PM



$$|\Delta U| = |\Delta K|$$

$$|q|\,\Delta V = \frac{1}{2} m v_f^2$$

velocity

In lab, you might accelerate the electron with 250 V.

$$(1.6\times10^{-19})(250) = \frac{1}{2}(9.11\times10^{-31})v_f^2$$

$$4\times10^{-17} =$$

$$v = \sqrt{\frac{2\,(4\times10^{-17})}{9.11\times10^{-31}}} = 9.4\times10^{6}\ \mathrm{m/s}$$

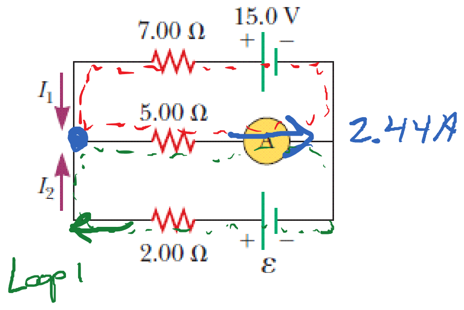
If we reduced the voltage to 2.5 V, what would happen to the speed?

(9.4×10^{5} m/s)

Kirchoff Example

Wednesday, July 17, 2019   2:55 PM

The ammeter shown in the figure below reads 2.44 A. Find the following.



$I_1 + I_2 = (2.44A)$

KVL: $+\mathcal{E} - 2I_2 - 5(2.44) = 0$

Loop 1:

↑negative because Loop 1 same direction as I_2.

Loop 2: $+15 - 7I_1 - 5(2.44) = 0$

$15 - 7I_1 = 5(2.44) = 12.2$

$15 - 12.2 = 7I_1$

$2.8 = 7I_1$

$0.4 = I_1$

$I_1 + I_2 = 2.44$ $I_2 = 2.04$ $\mathcal{E} - 2I_2 - 12.2 = 0$

Solve simultaneous equations using Cramer's Rule if necessary.

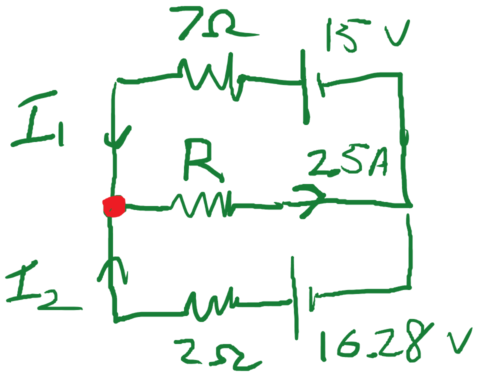
$\mathcal{E} - 4.08 - 12.2 = 0$

$\mathcal{E} = 16.28$ V

What if? What resistance could be used instead of 5.0 ohms to make the meter read 2.5 A?

$\boxed{\text{Ans } R = 4.84\Omega}$

$I_1 + I_2 = (2.5)$



Upper: $15 - 7I_1 - 2.5R = 0$

Lower: $16.28 - 2I_2 - 2.5R = 0$

$$\begin{aligned} I_1 &+ I_2 &+ 0 &= 2.5 \\ 7I_1 &+ 0 &+ 2.5R &= 15 \\ 0 &+ 2I_2 &+ 2.5R &= 16.28 \end{aligned}$$

$$\overset{?}{I_1} = \begin{vmatrix} 2.5 & 1 & 0 \\ 15 & 0 & 2.5 \\ 16.28 & 2 & 2.5 \end{vmatrix} \div \begin{vmatrix} 1 & 1 & 0 \\ ? & 0 & 2.5 \\ 0 & 2 & 2.5 \end{vmatrix}$$

Electrostatic levitation with surface charge

Wednesday, July 17, 2019 3:27 PM

$\ominus \quad q = -0.69 \mu C$

$E = 2\pi k \sigma$

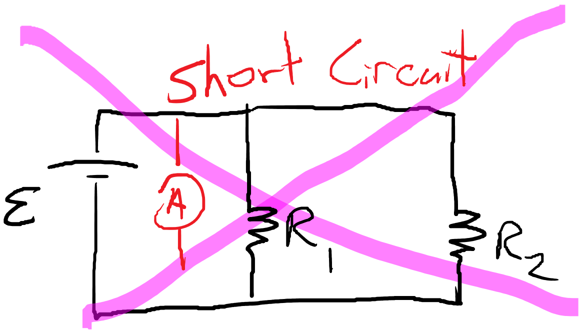
$\sigma = -?$

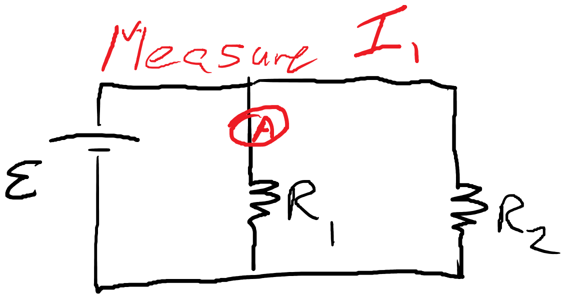
$F_E = F_g$

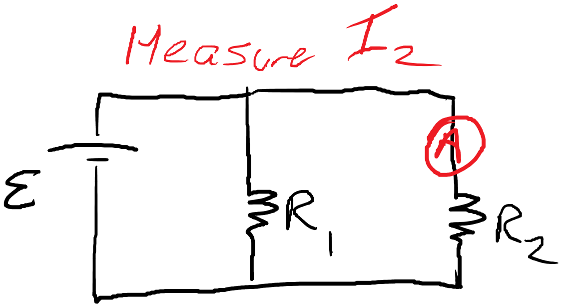
$qE = mg$

Ammeter in a parallel circuit

Wednesday, July 17, 2019 3:33 PM







Lec 10 - Magnetism

Tuesday, July 23, 2019 1:51 PM

Exam 1 Average: 65%
Use the Course Average to see where you stand.
Error in #11: "electric" is vague. Should say either "electric field" or "electric potential". Choice A or B will be accepted.

Equivalent to ohm (Ω)? Choices involve F or V

Relate R to C and R to V

$$\tau = RC \qquad V = IR$$

$$\frac{\tau}{C} = R \qquad \frac{V}{I} = R$$

$$\boxed{1\ \mathrm{s/F} = 1\ \Omega} \qquad 1\ \mathrm{V/A} = 1\ \Omega$$

Trends in resistance:

$$R = \frac{\rho \ell}{A} \qquad R = \frac{\rho \ell}{\pi r^2}$$

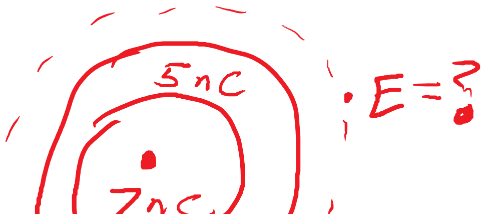
$$\frac{0.5}{(0.5)^2} = 2$$

$$\frac{R_2}{R_1} = \frac{\cancel{\rho} \ell_2 / (\cancel{\pi} r_2^2)}{\cancel{\rho} \ell_1 / (\cancel{\pi} r_1^2)}$$

$$= \frac{\ell_2}{\ell_1} \left(\frac{r_1}{r_2}\right)^2$$

$$= (0.5)\left(\frac{1}{0.5}\right)^2 = 2$$

Gauss's Law in Spherical Symmetry



$$E = \frac{\Phi_E}{A} = \frac{4\pi k q_{enc}}{4\pi r^2}$$

$$(\quad {}^{9})(12 \times 10^{-9})$$

$$E = \frac{kq_{enc}}{r^2} = \frac{(9\times10^9)(12\times10^{-9})}{r^2}$$

Magnetism

Tuesday, July 23, 2019 2:20 PM

Magnetism is a vector field (like E).
Unlike E, the Magnetic Field (B) always loops.

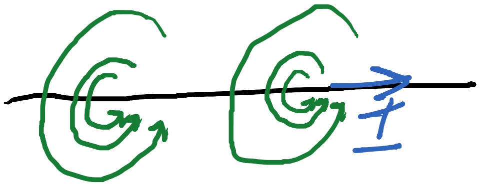
Magnetic fields can be created by:

- Magnetic materials
- Moving charges
- Electric currents
- Fluctuating electric fields

Effects of magnetic fields:

- Force on moving charges.
- Force on electric currents.
- Torque on magnetic dipoles.
- Attraction or repulsion of magnetic dipoles.
- Generate electric field and voltage.

Magnetic field due to current:



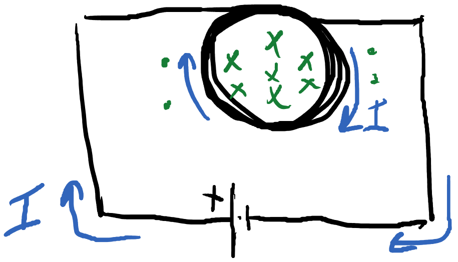
B points around I:

- Up, behind the current
- Out, above the current
- Down, in front.

$$B_{wire} = \frac{\mu_0 I}{2\pi R}$$

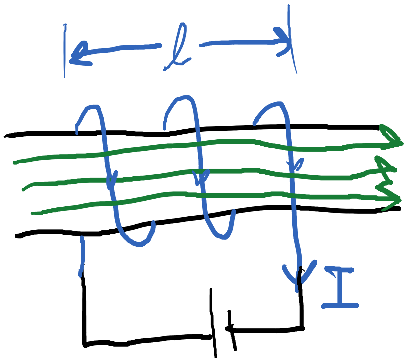
$$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$$

Magnetic field due to a loop or flat coil.

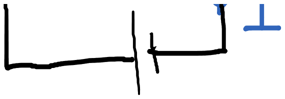


$$B_{coil} = \frac{\mu_0 I}{2R} N$$

Solenoid coil



$$B_{sol} = \frac{\mu_0 I N}{l}$$

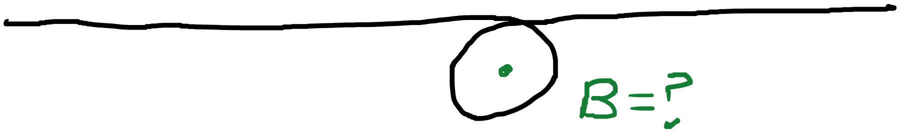


The three sources vary in uniformity of B:

- Wire: Strong near the wire, direction varies a lot.
- Loop: Strong inside, uniform direction, but that region is small.
- Solenoid: Same strength and direction everywhere inside the solenoid. Goes quickly to zero outside.

The solenoid is to magnetism as the parallel plate capacitor is to electricity.

If there are multiple sources of magnetism, their contributions can be added as vectors.



$$B = B_{wire} + B_{loop}$$

Ampere's Law

Tuesday, July 23, 2019 2:51 PM

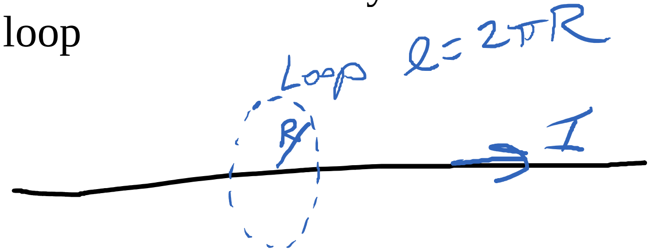
Ampere's Law summarizes the effect of current on a magnetic field.
It's a form of Stoke's Theorem from vector calculus.

$$\oint \vec{B} \cdot \vec{d\ell} = \mu_0 I_{enc}$$

$$B_{\parallel, avg} \cancel{\ell} = \mu_0 I_{enc} / \ell$$

The average parallel component of the magnetic field along a closed loop, is equal to mu_0 * the current enclosed by the loop divided by the length of the loop

Ex: For a long straight current:

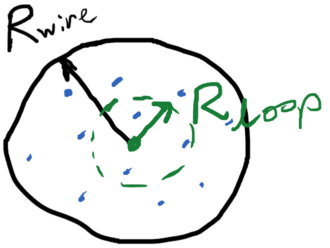


$\ell = 2\pi R$

Choose a loop that is a circle centered on the wire.
B is always parallel to the loop.
B is constant along the loop.

$$B_{\parallel avg} = B = \frac{\mu_0 I}{2\pi R}$$

Ex: Thick wire with uniformly distributed current.



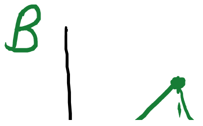
$$B = \frac{\mu_0 I_{enc}}{2\pi r} = \frac{\mu_0 I r^2}{2\pi r R^2}$$

$r = R_{loop}$ $R = R_{wire}$

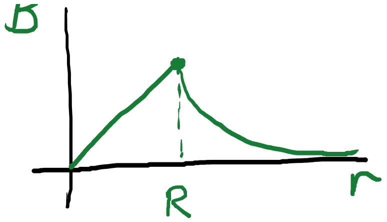
How much current is inside the dashed loop?

$$I_{enc} = I_{wire} \left(\frac{R_{loop}}{R_{wire}} \right)^2$$

Result: $B_{inside} = \frac{\mu_0 I r}{2\pi R^2}$



Result: $B_{inside} = \dfrac{\mu_0 I r}{2\pi R^2}$



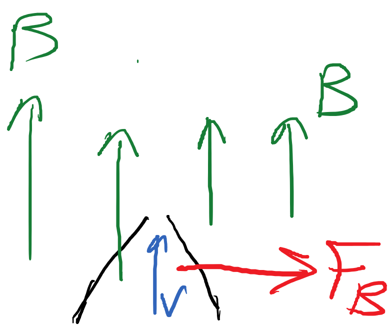
Outside, $I_{enc} = I$ $B_{outside} = \dfrac{\mu_0 I}{2\pi r}$

Magnetic Effects

Tuesday, July 23, 2019 3:08 PM

Force on a moving charge: To feel a force, a charged particle must be moving with a velocity component perpendicular to the magnetic field.

$v_\perp$



Particle driving down a road

$q = \oplus$

B = (up toward sky)

v = (into page)

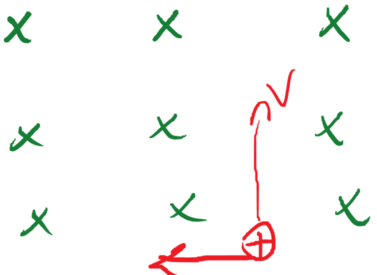
$$\vec{F}_B = q\vec{v} \otimes \vec{B}$$

If a positively charged car hits a magnetic field that points up toward the sky, the car is deflected rightward.

Drawing symbols for 3-D directions:

● = out

x = in



B = (in)

v = (top)

F_B = (left)

As this particle is deflected to the left (CCW), the magnetic force also bends CCW. The magnetic force is always perpendicular to the velocity and perpendicular to B.

A perpendicular force causes centripetal acceleration.

$$a = \frac{v^2}{R}$$

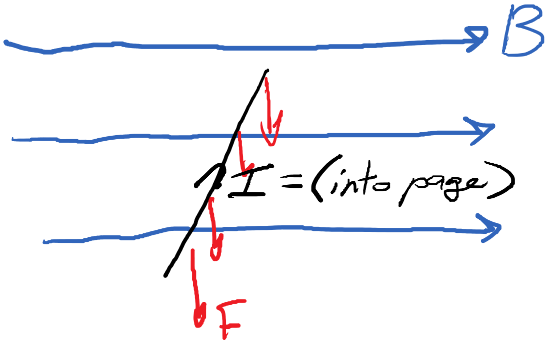
$$F = ma$$

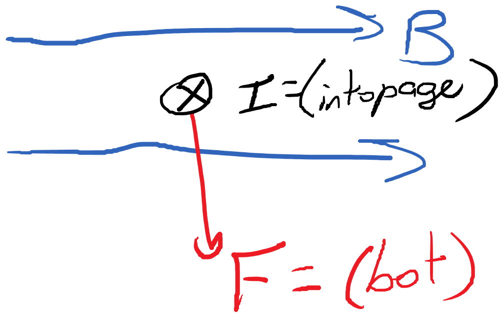
$$qv_\perp B = m\frac{v_\perp^2}{R} \Rightarrow R = \frac{mv_\perp}{qB}$$

This is the basis of the mass spectrometer.

Magnetic force on a current

Tuesday, July 23, 2019 3:30 PM





$$F_B = I \ell B_\perp$$

How much magnetism does it take to levitate a 24 AWG wire with max current flowing?

$I_{max} = 2.1\ A$

$\frac{m}{l} = 1.82\ \text{kg/km}$

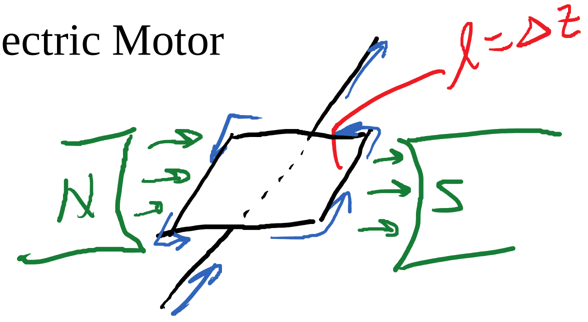
$$F_g = F_B$$

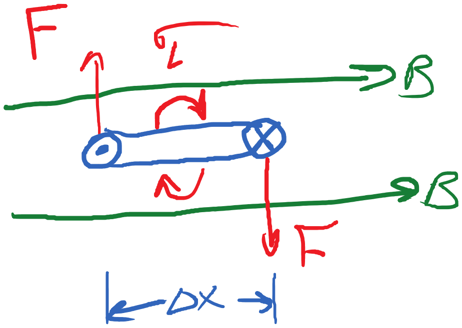
$$mg = I \ell B$$

$$\frac{m}{\ell}\frac{g}{I} = B = \frac{(1.82\times10^{-3}\ \text{kg/m})(9.8\ \text{N/kg})}{(2.1\ A)}$$

$$= 8.5\times10^{-3}\ T = 8500\ \mu T$$

Application: Electric Motor





$$\tau_1 = 2 F_B\, \Delta x/2$$

$$\tau_1 = I(\Delta z) B \Delta x$$

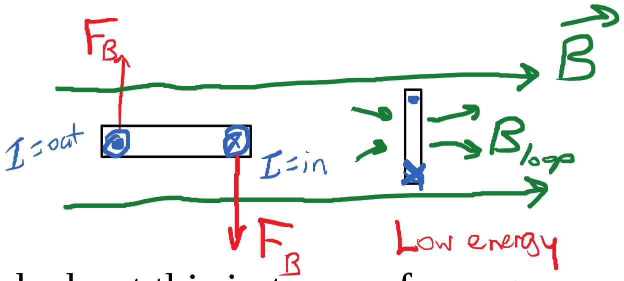
$$\tau = NBA\ I$$

Max torque on a coil in B.

Lec 11 - Magnetic Effects

Wednesday, July 24, 2019 11:45 AM

Current loop in a magnetic field feels a torque.



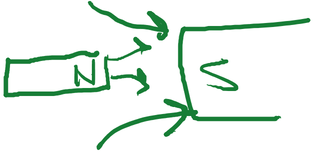
$\tau = NBAI$

I want to think about this in terms of energy.

The loop has a natural tendency to go toward the low-energy situation.

If the loop actually rotates to its most preferred position, this is the lowest energy state. (Above, when the inward current is at the bottom.)

- The current loop makes its own magnetic field!
- In the low energy state, which direction is the B of the loop?
- The loop's lowest energy state is when its magnetic field is aligned with the external magnetic field.
- This is what makes magnets stick together North-to-South



- This is what makes compasses point North.
- Technical term: Magnetic Moment

$\theta =$ angle between loop's B and external B

$$\text{Energy} = -NBAI\cos\theta$$

$\mu = NAI$

$$\text{Energy} = -\mu B \cos\theta$$

$\mu =$ magnetic moment

$\theta = 0 \rightarrow$ Low energy

$\theta = 180° \rightarrow$ High energy

$\mu = 9.28 \times 10^{-24}$ J/T for every electron

Every proton has a magnetic moment also.

An MRI works by tuning to the energy of "flipping" the magnetic moment of a proton.

This energy is why magnets are attracted to each other.

$E = -\mu B \cos\theta$

- Stronger B means lower Energy.

This energy is why magnets are attracted to each other.

$E = -\mu B \cos\theta$

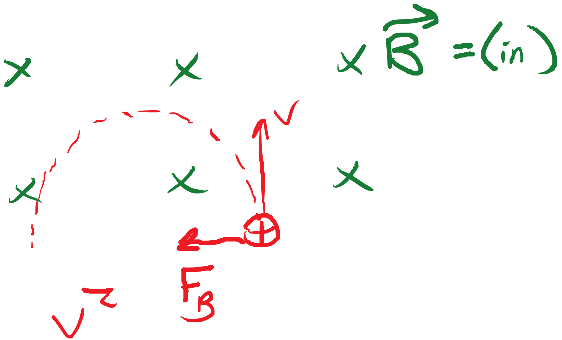
- Stronger B means lower Energy.
- Objects naturally seek out lower Energy.
- A properly oriented magnet will be attracted to a strong magnetic field.

Motion of moving charges

Wednesday, July 24, 2019 2:27 PM

B-only $F_B = q v_\perp B$

$\vec{F}_B \perp \vec{v}$ (perpendicular)

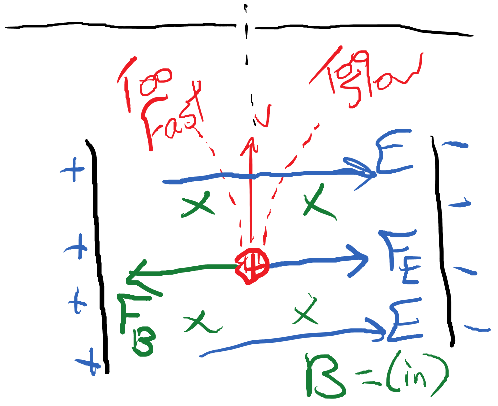


Perpendicular force causes centripetal accel. $a_c = \frac{v^2}{R}$

$$q v_\perp B = \frac{m v_\perp^2}{R} \longrightarrow R = \frac{m v_\perp}{q B}$$

This is the basis of the mass spectrometer.

B and E together.



Want $F_{net} = 0$

Set up $F_E =$ (right)

$$F_E = F_B$$

$$\cancel{q} E = \cancel{q} v B$$

$$\frac{E}{B} = v$$

What happens if the particles are going "too fast"? (F_B stronger)
Particles that go straight have v = E/B.

E-only (Accelerator)

How does the radius of curvature in the mass spec. depend on the mass, if the particles are accelerated with a fixed voltage?

$$q V_{acc} = \frac{1}{2} m v^2$$

$$= v$$

$$R = \frac{m v_\perp}{q B}$$

Hidden mass dependence

$$\sqrt{\frac{2qV_{acc}}{m}} = V_\perp$$

$$R = \frac{}{qB}$$

$$R = \sqrt{\frac{2qV_{acc}}{m}}\,\frac{\sqrt{m^2}}{qB}$$

$$= \frac{\sqrt{2qV_{acc}\,m}}{qB}$$

If the particles are accelerated by a voltage, the radius of curvature is only proportional to the square root of the mass.

Double $m \Rightarrow R$ increase by $\sqrt{2}$

Summary of mass spec feeding methods:

- Velocity selector: R proportional to m
- Accelerating Voltage: R proportional to sqrt(m)

Electromagnetic Induction

Wednesday, July 24, 2019 2:48 PM

Under the right conditions, magnetism can generate voltage.

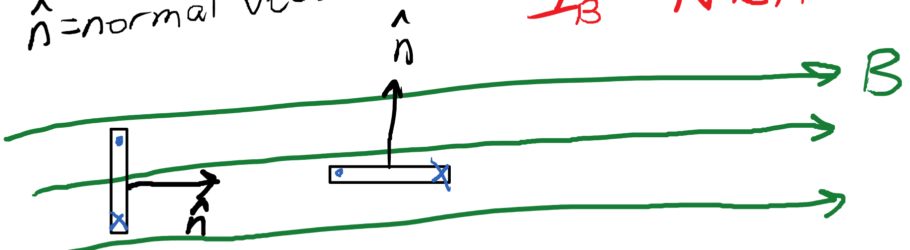
- Wire or coil moving through a magnetic field.
- Moving a magnet near a wire or coil.
- Changing the current in a coil.

All of these effects are summarized by Faraday's Law

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad \Phi_B = \iint \vec{B} \cdot d\vec{A}$$

$\hat{n}$ = normal vector

$$\Phi_B = NBA \cos\theta$$



$\theta = 0$ $\quad$ $\theta = 90°$

$\cos\theta = 1$ $\quad$ $\cos\theta = 0$

$\Phi_B = NBA$ $\quad$ $\Phi_B = 0$

Change any of the 4 parameters, and Phi_B changes, generating EMF.

Spinning Coil: $\frac{d\theta}{dt} = \omega \qquad \theta = \omega t$

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(NBA \cos\theta\right)$$

$$= -NBA \frac{d}{dt}(\cos\omega t)$$

$$= -NBA\left(-\sin\omega t\right)\omega$$

$$\mathcal{E} = \underbrace{NBA\omega}_{\text{Amplitude, Peak, Max}} \sin\theta \qquad \mathcal{E}_{max} = NBA\omega$$

Motor: $\tau_{max} = NBAI$

The motor and generator are actually the same thing.
The side-effect of a generator is:

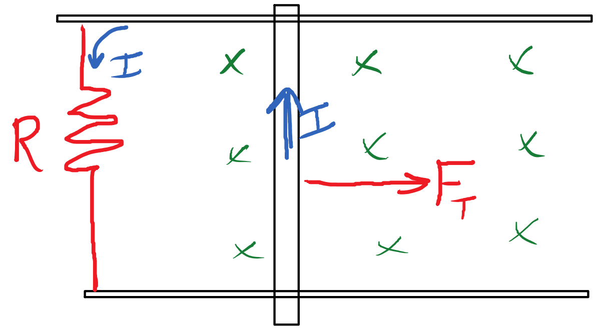
- Flowing current makes opposing torque. This makes it harder to spin the generator when it's being used heavily.

The side-effect of a motor is more subtle:

- When spinning freely, the motor generates Back-EMF. This is good for us. It means the motor doesn't draw as much current when it's freewheeling.
- When we make the motor do something, it slows down. There's less Back-EMF, and more current flows. This costs us money, but it makes the motor generate more torque.

Motional EMF: Changing the area

Wednesday, July 24, 2019 3:25 PM



Pulling on the bar makes it move.
The bar is made of charges.
+ charges are pushed upward.
In the bar, the net force on charges is zero.

$$F_E = F_B$$

$$\cancel{q}E = \cancel{q}vB$$

$$\frac{\Delta V}{\Delta y} = vB$$

$$\Delta V = IR \qquad E = \frac{\Delta V}{\Delta y}$$

$$\Delta V = vB\,\Delta y$$

That's the magnetic force argument. Here's the Faraday's Law argument:

$$A = \Delta y\,\Delta x \qquad \frac{dA}{dt} = \Delta y\,\frac{dx}{dt} = v\Delta y$$

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \frac{d}{dt}(NBA\cancel{\cos\theta}) = \cancel{N}B\frac{dA}{dt}$$

$$\mathcal{E} = Bv\Delta y$$

Ex: B = 3.0 T, R = 3.0 ohm, delta-y = 5 cm, v = 80 m/s

$$\left.\begin{aligned} \mathcal{E} &= (3)(80)(0.05) = 12\text{ V} \\ I &= \frac{V_R}{R} = \frac{12}{3} = 4.0\text{ A} \end{aligned}\right\} \begin{aligned} P &= (12\text{ V})(4\text{ A}) \\ &= 48\text{ W} \end{aligned}$$

How much Force? $F = I\ell B = (4.0)(0.05)(3.0) = 0.6\text{ N}$

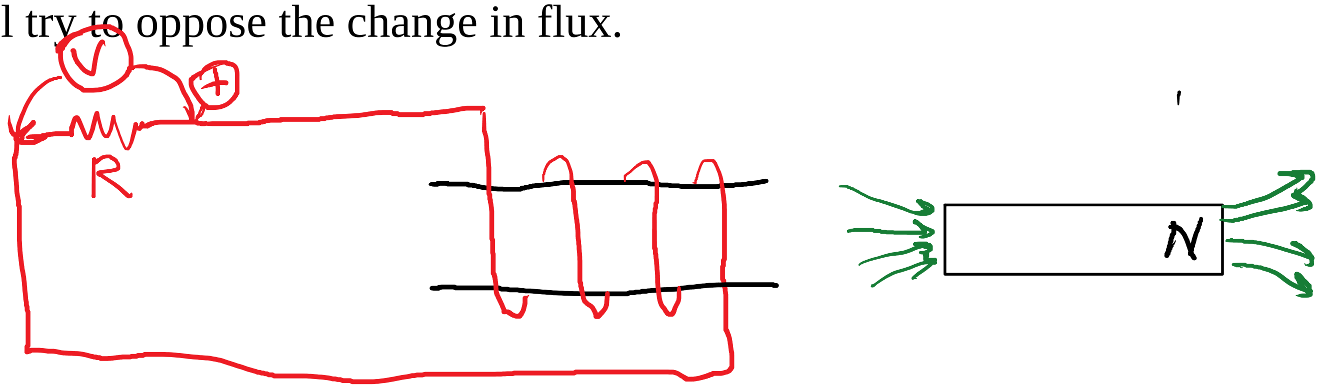
Work per time? $W = F\Delta x$

$$P = \frac{dW}{dt} = Fv = (0.6\text{ N})(80\text{ m/s}) = 48\text{ W}$$

Lenz's Law

Wednesday, July 24, 2019 3:40 PM

Electromagnetic effects tend to oppose the stimulus. Lenz's Law says that when flux changes, the induced EMF will try to oppose the change in flux.



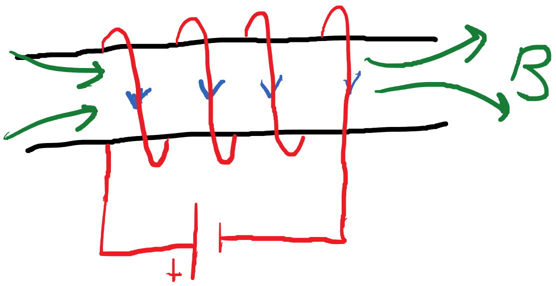
When the magnet is inserted, the flux increases. The magnetic field is pointing to the right. The EMF opposes that by trying to make a magnetic field that points to the left.

Lec 12 - Inductors and AC

Thursday, July 25, 2019 1:59 PM

Exam 2 Monday: Magnetism → AC

An inductor is any magnetic coil.



$$B = \mu I N / l$$

$$\Phi_B = NBA \cancelto{1}{\cos}\theta$$

$$\Phi_B = N\left(\frac{\mu I N}{l}\right)A$$

$$\Phi_B = \underbrace{\left(\frac{\mu N^2 A}{l}\right)}_{\text{Inductance}} I$$

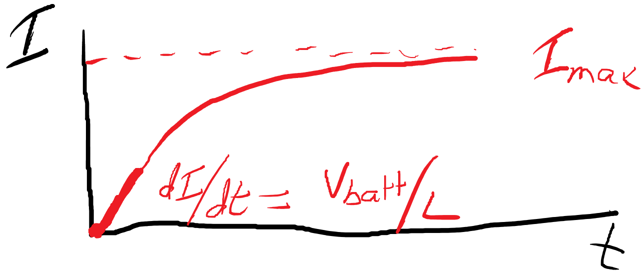
$$\Phi_B = LI$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

The inductor generates EMF that opposes change in current.
Since the EMF is negative, we can think of it as a "voltage drop".

$$V_L = L\frac{dI}{dt} \qquad V_R = RI$$

If you hook up an inductor to a battery, this happens:



$$I_{max} = V_{batt}/R$$

LR circuit

A "charged" inductor doesn't have Q, it has *I* flowing.
The flowing current causes *B*, which stores energy.

$$\text{Energy} = \tfrac{1}{2}LI^2$$

If you quickly disconnect an inductor, something interesting happens. The current must suddenly decrease.
This makes the inductor generate a large EMF.
This is called "inductive kick".
The inductive kick is also the release of the magnetic

energy.

AC Circuits

Thursday, July 25, 2019 2:19 PM

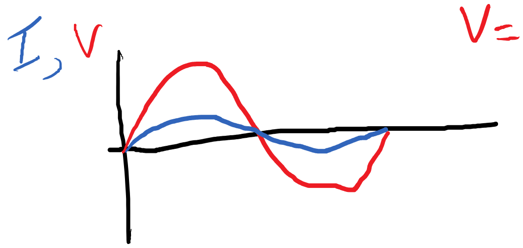
AC involves oscillating signals (V and I).

$$V = V_{max} \sin(2\pi f t)$$

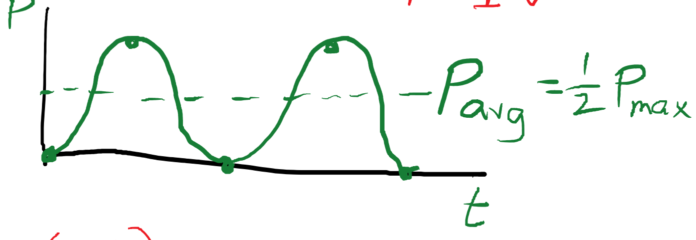
$\uparrow$ Peak or Amplitude $\uparrow$ Frequency

If an AC voltage is applied to a resistor, the power is constantly fluctuating.

$V = IR$



$P = IV$



$$\sin^2\theta = \frac{1}{2}(1 - \cos(2\theta))$$

$$P_{avg} = \frac{1}{2}(P_{max}) = \frac{1}{2}(I_{max}{}^2 R)$$

$$P_{avg} = \left(\frac{I_{max}}{\sqrt{2}}\right)^2 R$$

$$P_{avg} = I_{rms}{}^2 R \qquad I_{rms} = \frac{I_{max}}{\sqrt{2}}$$

Ex: V_RMS = 120 V $\quad V_{max} = \sqrt{2}(120\ V) = 170\ V$

In the US, we use f = 60 Hz. $\quad 2\pi f = 2\pi(60\ Hz) = 377\ s^{-1} = \omega$

Frequency = f in Hz or cycles/s

Angular Freq = ω in s^{-1} or rad/s

The radian is a "dimensionless unit" defined by $\quad s = R\theta$ Arc Length

$$(1 m) = (1 m)(1\ rad)$$

Our AC power as a function: $V = 170 \sin(377 t)$

Reactance and Impedance

Thursday, July 25, 2019 2:32 PM

AC Ohm's Law: $V_{rms} = I_{rms} Z$

Z = impedance = effective resistance for an AC component or circuit.

Resistor: Z = R

- The resistor opposes current.
- It's only "happy" (zero voltage) when the current is off.

$V_R = IR$

Inductor: V_L = L (dI/dt)

- Inductor opposes change in current.

$X_L = 2\pi f L$

Try $I = I_0 \cos(2\pi f t)$

$$\frac{dI}{dt} = I_0(-\sin(2\pi f t))(2\pi f)$$

$$V_L = L\frac{dI}{dt}$$

$$= \underbrace{2\pi f L I_0}_{}(-\sin(\ldots))$$

$$V_{max} = 2\pi f L I_{max}$$

$$V_{rms} = 2\pi f L I_{rms}$$

- X_L is the "reactance" and "impedance" of the inductor.
- The inductor can be "happy" in two ways:
    - ○ No RMS current
    - ○ Very low frequency.
- Inductors allow low-frequency current to pass thru.
- Inductors block high-frequency current.

Capacitors in AC Circuits

- The capacitor reacts to a constant current by "filling up" and opposing the current with voltage. The capacitor opposes constant current.
- If you apply a high-frequency AC voltage, then high-frequency current flows. The capacitor never gets a chance to "fill".

$$V_c = \frac{Q}{C} \qquad \frac{dV_c}{dt} = (I)\left(\frac{1}{C}\right)$$

$$V = V_{max} \sin(2\pi f t)$$

$$\frac{dV}{dt} = V_{max} \cos(2\pi f t)(2\pi f)$$

$$V_{max} \cos(2\pi f t)(2\pi f) = \frac{1}{C} I$$

$$\underbrace{V_{max}(2\pi f C)}_{I_{max}} \cos(\ldots) = I$$

$$V_{max}(2\pi f C) = I_{max}$$

$$V_{max} = \left(\frac{1}{2\pi f C}\right) I_{max}$$

$\ _{max} = (2\pi f C)\ _{max}$

$$X_C = \frac{1}{2\pi f C}$$

- The capacitor is "happy" (low voltage) when the frequency is very high.

Series AC Circuit:

- The same current exists in all components.
- Voltage adds at any instant in time, but the peak or RMS voltages don't add.
- If you know trig identities, you're good.
- The rules summarize to form:

$$V_{rms} = I_{rms} Z$$

$$Z = \sqrt{R^2 + X^2} \qquad X = X_L - X_C$$

Apply the P.S. voltage to the overall impedance.
Find the individual component voltages from the current.

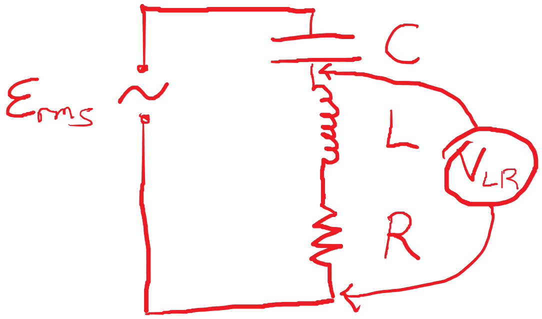
AC Example:

Thursday, July 25, 2019 2:57 PM

$R = 300\,\Omega$ $L = 0.3\text{ H}$ $C = 2.5\,\mu\text{F}$

$\mathcal{E}_{rms} = 120\text{ V}$ $f = 318.3\text{ Hz}$

$P_{avg} = ?$ $V_{LR} = ?$



$X_L = 2\pi f L = 2\pi(318.3)(0.3)$
$= 600\,\Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(318.3)(2.5\times10^{-6})}$
$= 200\,\Omega$

$X = X_L - X_C = 400\,\Omega$

$Z = \sqrt{R^2 + X^2}$
$= \sqrt{300^2 + 400^2}$
$= 500$

$V_{rms} = I_{rms} Z$
$(120\text{ V}) = I_{rms}(500\,\Omega)$
$0.24\text{ A} = I_{rms}$

Individual component voltages:

$V_R = (0.24)(300) = 72\text{ V}$
$V_L = (0.24)(600) = 144\text{ V}$
$V_C = (0.24)(200) = 48\text{ V}$

Here, V_L is bigger than the power supply voltage!
This effect is used in DC-DC "buck boost" converters.

$P = I_{rms}^2 R = (0.24\text{ A})^2(300\,\Omega) = 17.3\text{ W}$

$V_{LR} = I Z_{LR}$ $Z_{LR} = \sqrt{300^2 + 600^2} = 671\,\Omega$
$= (0.24)(671)$

$V_{LR} = 161\text{ V}$

Resonance

Thursday, July 25, 2019 3:14 PM

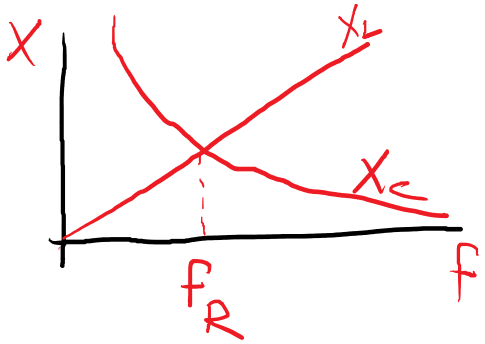
Since the inductive and capacitive reactance subtract, can they completely cancel each other?

$$X = X_L - X_C$$

$$X_L = X_C$$

$$2\pi f L = \frac{1}{2\pi f C}$$

$$f_R = \frac{1}{2\pi\sqrt{LC}}$$



Were we operating above or below resonance in the previous example? (Above)

$X_L = 600$ $X_C = 200$

What is f_R for our components?

$$f_R = \frac{1}{2\pi\sqrt{(0.3)(2.5\times10^{-5})}}$$

$$f_R = 183.8 \text{ Hz}$$

What does Z look like as a function of f?
Z is at a minimum when f = f_R.

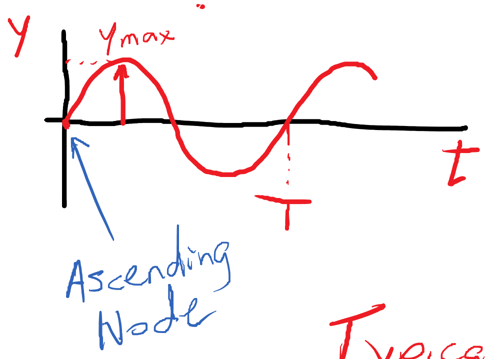
For the same applied voltage, Current is a maximum when f=f_R.
https://www.desmos.com/calculator/gr2njawhfx

At resonance, X = 0, so Z = R.

Lec 14 - Oscillations and Waves

An oscillation is a back-and-forth function caused by two things:

- A proportional restoring force. Tendency toward equilibrium.
- Inertia.



y_{max} = Amplitude

T = Period

$f = \frac{1}{T}$ = Frequency

Typical: $y = y_{max} \sin(2\pi f t + \phi) + C$

Phase shift
= Horizontal shift

Derivative: $v_y = 2\pi f y_{max} \cos(2\pi f t + \phi)$

2nd Derivative: $a_y = (2\pi f)^2 y_{max} \sin(2\pi f t + \phi)(-1)$

$$a_y + (2\pi f)^2 y = C' \qquad C' = (2\pi f)^2 C$$

$$\ddot{y} + \omega^2 y = C'$$

This is a standard non-homogeneous 2nd order linear differential equation.

Examples:

- Mass-and-Spring oscillator (simple harmonic oscillator)
- Pendulum
- Vibrating object or molecule
- x-component of circular motion
- Part of a wave
- LC series circuit (inductor + capacitor)

Mass-and-Spring oscillator

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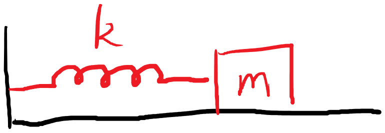

$F_s = -kx_s$

stretch

stiffness

$F_{net\,x} = m a_x$

$-kx = m a_x$

$0 = m a_x + kx$

C=0

$0 = \ddot{x} + \omega^2 x \leftarrow$ generic w/ C=0

$\omega^2 = \frac{k}{m}$ $\omega = \sqrt{\frac{k}{m}}$

ω = Angular Frequency $= 2\pi f$

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

So our shift is C=0, our frequency is set. There are 2 free parameters:

$x = x_{max} \sin(2\pi f t + \phi)$

We need 2 known conditions to set the 2 parameters.

- Typical: Initial position and velocity.

Energy in a SHO (simple harmonic oscillator):

$v_{max} = 2\pi f x_{max}$

$K = \frac{1}{2} m v^2$ $K_{max} = \frac{1}{2} m v_{max}^2 = \frac{1}{2}(2\pi f)^2 m x_{max}^2$

$U_s = \frac{1}{2} k x^2$ $U_{max} = \frac{1}{2} k x_{max}^2 \leftarrow$ equal!

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$2\pi f = \sqrt{\frac{k}{m}}$

$(2\pi f)^2 = k/m$

$K_{max} = \frac{1}{2}\left(\frac{k}{m}\right) m x_{max}^2$

$= \frac{1}{2} k x_{max}^2$

The energy of a perfect oscillator is constant.

Ex: Mass-and-spring oscillator

$$\left.\begin{array}{l} m = 0.25 \text{ kg} \\ k = 50 \text{ N/m} \end{array}\right\} f = \frac{1}{2\pi}\sqrt{\frac{50}{0.25}} = 2.25 \text{ Hz}$$

Prediction of model.

If I give the mass a displacement of 1 cm and a speed of 15 cm/s, what will the amplitude be?

$\llcorner y_o$ $\llcorner v_{yo}$

$$0.01 = y_{max} \sin(2\pi f t + \phi) = y_{max} \sin\phi$$

$$0.15 = 2\pi f\, y_{max} \cos(2\pi f t + \phi) = 2\pi f y_{max} \cos\phi$$

Energy: $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}(0.25)(0.15)^2 + \frac{1}{2}(50)(0.01)^2$
$= 0.00531 \text{ J}$

$$U_{max} = 0.00531 = \frac{1}{2}(50)\, x_{max}^2$$

$$\sqrt{\frac{2(0.00531)}{50}} = x_{max} = 0.0146 \text{ m}$$
$$= 1.46 \text{ cm}$$

When I let go @ 1 cm, with 15 cm/s of velocity, it has the same energy as being @ 1.46 cm with zero velocity.

Pendulum

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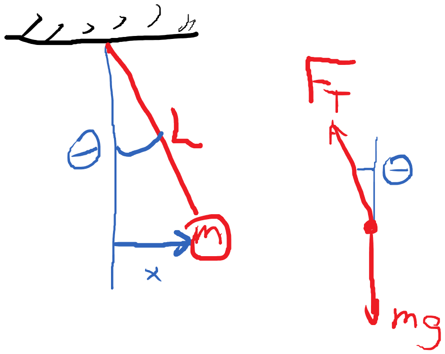

x: $-F_T \sin\theta = m a_x$

y: $F_T \cos\theta - mg = m a_y$

Small-angle approximation: $\cos\theta \approx 1$ $\sin\theta \approx \theta \approx \tan\theta$

Object is L*cos(theta) from ceiling. $y = \text{const}$ $a_y = 0$

$F_T = mg$

$-mg\theta = m\ddot{x}$ $a_x \equiv \ddot{x}$

$-\frac{mgx}{L} = m\ddot{x}$

$0 = \ddot{x} + \frac{g}{L}x$

$0 = \ddot{x} + \omega^2 x$

$\omega^2 = \frac{g}{L}$ $\omega = \sqrt{\frac{g}{L}}$ $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

As long as we keep theta < around 10 degrees,
this works okay. Must do the math in radians.

Ex: What length to achieve f = 2 Hz? (L = 6.2 cm)

Circular Motion

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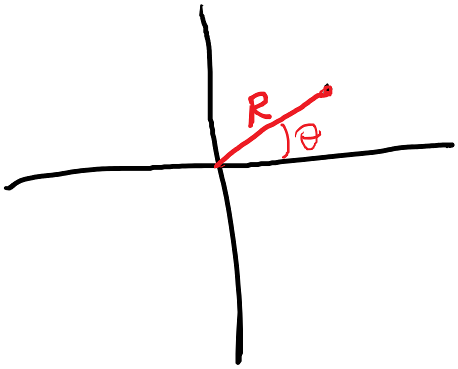


$x = R\cos\theta = R\cos(2\pi f t)$

$\theta = 2\pi f t$

θ increases w/ constant angular velocity

$x = R\sin\left(2\pi f t + \frac{\pi}{2}\right)$

From oscillations to Waves

Tuesday, July 30, 2019 3:05 PM

A wave is an organized disturbance in a set of coupled oscillators.

- A displacement of one oscillator exerts a force on its neighbors.
- The disturbance is often an oscillation.

An oscillation time causes a repeating pattern in space.

λ = wavelength
= length of repeating segment

Since the wave propagates with a velocity:

$$v = \frac{\lambda}{T} \qquad v = f\lambda$$

The propagation speed is also related to the medium:

Sound	Light	String	Water
$v = 340 \, m/s$	$v = 3 \times 10^8 \, m/s$	$v = \sqrt{F_T/\mu}$	$v \approx 1-10 \, m/s$

https://www.acs.psu.edu/drussell/Demos/waves/wavemotion.html

Generally, the oscillating disturbance has the same frequency all the way from source to observation.

Doppler Effect

Tuesday, July 30, 2019 3:24 PM

The Doppler Effect is a frequency shift caused by relative motion of a source and observer.

- Source motion directly affects the wavelength of the wave.
- Observer motion directly affects the observed period.

We're saved by the Binomial Approximation. $(1+x)^n \approx 1+nx$

True when $|x| \ll 1$

Ex: $\sqrt{1+x} \approx 1+\frac{1}{2}x$

$$\frac{1}{1+x} \approx 1-x$$

$$\frac{1}{(V_w - V_o)} \approx \frac{1}{V_w\left(1-\frac{V_o}{V_w}\right)} = \frac{1}{V_w}\left(1+\frac{V_o}{V_w}\right)$$

The net result of applying the true Doppler formulas (in the book) with the binomial approximation is this:

$f_o = f_s\cancel{\left(\frac{V_w - V_s}{V_w + V_o}\right)}$??

Frequency shift $\longrightarrow \Delta f$

Source freq $\longrightarrow f_s$

$$\frac{\Delta f}{f_s} = \frac{V_{rel}}{V_{wave}}$$

$V_{rel} \longleftarrow$ relative velocity of src and obs.

$V_{wave} \longleftarrow$ wave speed

Ex: What relative velocity will shift music by a half-step?

$$\left.\begin{aligned} f_s &= 440 \text{ Hz} \\ f_o &= 466 \text{ Hz} \end{aligned}\right\} \Delta f = 26 \text{ Hz} \qquad V_w = 340 \text{ m/s}$$

$$\frac{26}{440} = \frac{V_{rel}}{340} \Rightarrow V_{rel} = 20 \text{ m/s} = 45 \text{ mph}$$

One last thing about the Doppler Shift: Double when calculating doppler radar.

Lec 15 - Interference and Standing Waves

Wednesday, July 31, 2019 1:51 PM

Generic Oscillation: $y = A\sin(2\pi f t + \phi) + C$

Generic Pattern: $y = A\sin\left(\frac{2\pi x}{\lambda} + \phi\right) + C$

Generic Wave: $y = A\sin\left(\underbrace{\frac{2\pi x}{\lambda} - 2\pi f t}_{\frac{2\pi}{\lambda}(x - vt)} + \phi\right) + C$

It's the (x-vt) hiding inside that makes the wave go "forward".

When two sinewaves add, the result depends on the relative phase between them.
https://www.desmos.com/calculator/rvru8xzqtx

- Phase diff = 0, 2*pi, 4*pi, …: Constructive interference = addition
- Phase diff = pi, 3*pi, 5*pi, …: Destructive interference = cancellation.

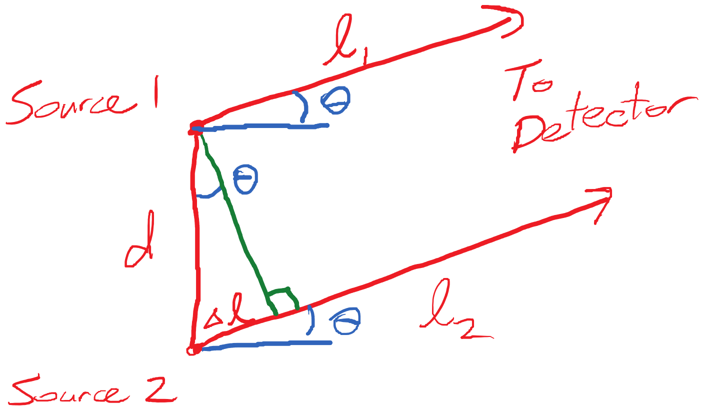
So how do we actually get this situation to happen?

- Need two synchronized sources of waves.
- Could manually "flip" one of the waves.
- Could introduce a time delay to one wave.
- Could make one wave travel further.

$\Delta t = \overset{\text{Integer = "order"}}{m} T$

$\Delta l = m\lambda$

It's difficult to measure the path length difference directly.



$\Delta l = l_2 - l_1$ (Useless)

$\Delta l = d\sin\theta$

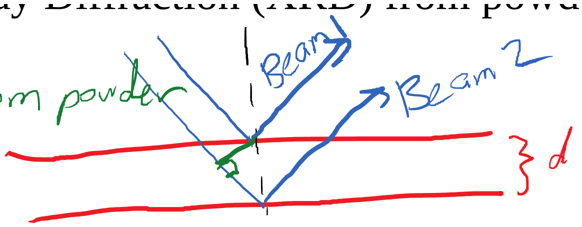
$m\lambda = d\sin\theta$

A similar situation is X-Ray Diffraction (XRD) from powders.

A similar situation is X-Ray Diffraction (XRD) from powders.

Diffraction from powder

$m\lambda = 2d\sin\theta$



Usually, we will encounter the diffraction grating.

$m\lambda = d\sin\theta$

How does theta depend lambda?

Bigger λ → Bigger $\sin\theta$

In the first quadrant, theta and sin(theta) trend together.

Exam 2 Tidbits

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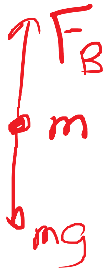
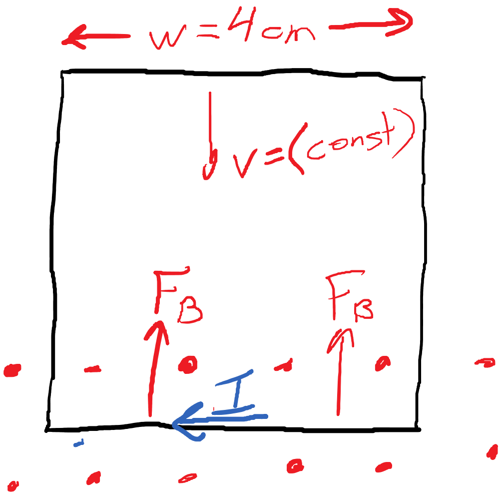


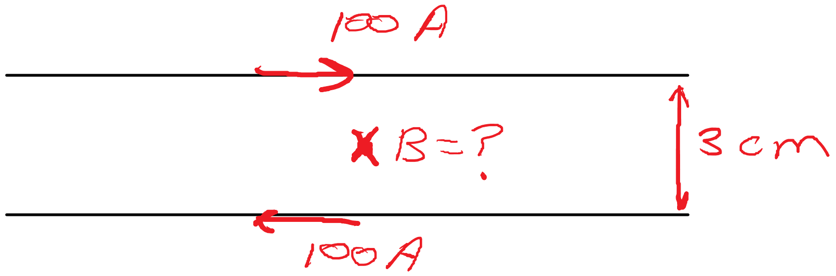
$$mg = I L_\perp B$$

$$(0.001)(9.8) = I(0.04)(0.8)$$



Key: const-v = zero accel.

Only Horizontal segment can have upward F_B.



$$B_{net} = \left(\frac{\mu_0(100\ A)}{2\pi(0.015\ m)}\right)2$$

$$r = \frac{mv_\perp}{qB} = \frac{(44)(1.66\times10^{-27})(6000)}{(1.6\times10^{-19})\ (0.25)} = 0.011\ m$$

$$\cancel{q}E = \cancel{q}v_\perp B$$

$$\frac{E}{B} = v_\perp$$

Standing Waves

Wednesday, July 31, 2019 3:04 PM

Standing waves are apparently stationary oscillations formed out of opposing travelling waves.

Generic standing wave function:

$$y = A\,\underbrace{\sin\left(\frac{2\pi x}{\lambda}\right)\sin(2\pi f t)}_{=\sin\left(\frac{2\pi x}{\lambda}-2\pi f t\right)+\sin\left(\frac{2\pi x}{\lambda}+2\pi f t\right)} + C$$

Standing waves are interesting because of two factors:

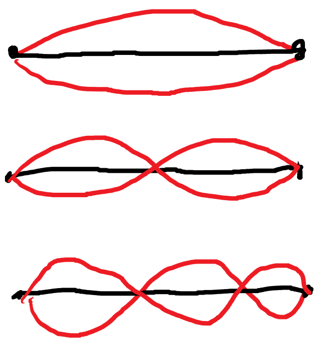
- The same wave is continually fed by input energy, and can grow to huge amplitudes.
- Only certain wavelengths are "allowed". Other wavelengths won't keep adding amplitude and will be comparatively weaker.

To hold standing waves, we build a "cavity", which is just a waving region with reflectors at each end.

Standing waves have nodes (zero points) that are $\lambda/2$ apart.

The "allowed" wavelengths will be those that have nodes at each end of a string, if the string is clamped at both ends.

$f = v/\lambda$



$\ell = \frac{\lambda}{2}$ $\quad \lambda = 2L$ $\quad f_1 = \frac{v}{2L}$

$\ell = \frac{2\lambda}{2}$ $\quad \lambda = \frac{2L}{2}$ $\quad f_2 = \frac{2v}{2L} = 2f_1$

$\ell = \frac{3\lambda}{2}$ $\quad \lambda = \frac{2L}{3}$ $\quad f_3 = \frac{3v}{2L} = 3f_1$

$\lambda = \frac{2L}{m}$ $\quad f_m = m f_1$

Even if we don't know the fundamental frequency, the spacing of harmonics can tell us the cavity size.

$$\Delta f = \frac{v}{2L}$$

Ex: $\quad \Delta f = 4000$ Hz

Ex: $9\Delta f = 4000$ Hz

$\Delta f = 444$ Hz

If this was a wind instrument, v=340 m/s.

$$L = \frac{v}{2\Delta f} = \frac{340}{2(444)}$$

$$= 0.383 \text{ m}$$

$$L = 38.3 \text{ cm}$$

In cavities with different ends, a different situation happens:

$$\lambda_1 = 4L \qquad \lambda_m = \frac{\lambda_1}{m} \quad (m = \text{odd})$$

$$f_m = m f_1 \quad (m = \text{odd}) \qquad f_3 = 3f_1 = \frac{3v}{\lambda_1} = \frac{3v}{4L}$$

Next overtone: $f_5 = \frac{5v}{4L}$

$$\Delta f = \frac{5v}{4L} - \frac{3v}{4L} = \frac{2v}{4L} = \frac{v}{2L}$$

Whether the ends are the same or different, the harmonic spacing of frequencies is the same.

Ex:

$f_1 = 31$

$f_2 = 61$

$f_3 = 91$

$f_4 = 124$

Estimate: $3\Delta f = (124 - 31)$

$3\Delta f = 93$

$\Delta f = 31$ Hz

$L = 1.0$ m

$$\Delta f = \frac{v}{2L}$$

$$2L\Delta f = v$$

$$2(1.0)(31) =$$

$$62 \text{ m/s} = v \quad (\text{Estimated})$$

Coffee cup observed frequencies: 55, 72, 89

17 17

$\Delta f = 17\ Hz$

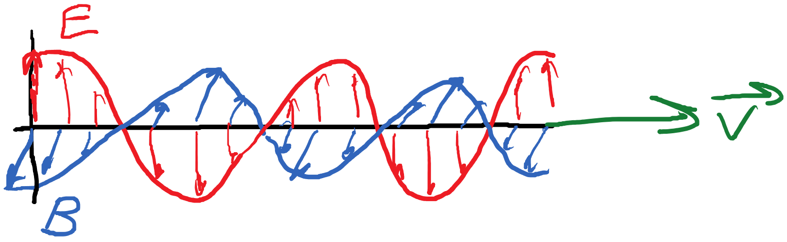
$\Delta f = \frac{v}{2L}$

$L = \frac{v}{2f} = \frac{340}{2(17)} = 10\ m$

Lec 16 - Intensity and Polarization

Thursday, August 1, 2019 11:51 AM

We already saw that fluctuating B-Fields make E-Fields.
It turns out that fluctuating E-Fields make B-Fields.
If an oscillating field of either type is stimulated, that will generate oscillations in the other field, forming an electromagnetic wave.



Both E and B are perpendicular to the "ray" or velocity.
The speed is always $v = c = 3 * 10^8$ m/s.
The amplitude is related to the intensity of the light.

$$I = \frac{1}{2c\mu_0} E_{max}^2$$

The frequency determines how the light interacts with matter.

- Radio Waves (low freq)
- Microwaves
- Infrared (heat radiation)
- Visible
- Ultraviolet
- X-Rays
- Gamma Rays

$\}$ non-Ionizing

$\}$ Ionizing Radiation = break bonds

$E_\gamma = hf$ — planck's Constant

$v = f\lambda$

Blue Green Red

Range of visible light: $\lambda = 400 \dots 750$ nm

$$f = \frac{v}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{400 \times 10^{-9} \text{ m}} = 7.5 \times 10^{14} \text{ Hz} = 750 \text{ THz}$$

$$f = \frac{3 \times 10^8}{750 \times 10^{-9}} = 4 \times 10^{14} \text{ Hz} = 400 \text{ THz}$$

$$f = 750 \dots 400 \text{ THz}$$

What is the wavelength of 100 MHz radio waves?

$$\lambda = \frac{3 \times 10^8}{100 \times 10^6} = 3 \text{ m}$$

A typical antenna is a "quarter-wave vertical".

Intensity and Power

Thursday, August 1, 2019 2:31 PM

Waves typically spread out as they propagate.
Intensity measures the power per unit area.
As a wave spreads, the power stays the same, but the area increases, so the intensity goes down.

Typical: $I \propto \frac{1}{R^2}$

If a point source emits in all directions:

$$P = IA$$

$$I = \frac{P}{A} = \frac{P}{4\pi R^2}$$

Ex: A light source has an intensity of 100 W/m^2 at 10 m, how bright is it at 20 m?

$R_2 = 2R_1$

$I \propto \frac{1}{R^2}$

$I_2 = \frac{1}{2^2} I_1$

I_2 = 25 W/m^2

If this was a perfect point source, what is the total power emitted?

$$P = IA = (100\ W/m^2)\, 4\pi\, (10\ m)^2 = 126000\ W$$

A typical light bulb is 100 W and is only 10% efficient, so that is 10 W of emitted light energy.

Our eyes (and ears) are sensitive to a HUGE "Dynamic Range" of intensities.

Intensity Level in Decibels

Thursday, August 1, 2019 2:41 PM

Intensity Level is a logarithmic scale for measuring intensities.

$$I = I_o 10^{\beta/10}$$

$$\beta = 10 \log_{10}\left(\frac{I}{I_o}\right)$$

β (dB)	$10^{\beta/10}$
0	1.0
3 dB	1.99 ≈ 2
-3 dB	0.501 ≈ 1/2
6 dB	3.98 ≈ 4
10 dB	10
20 dB	100

0 dB → no change in intensity

Adding dB corresponds to multiplying Intensity by a factor,

Radio signals (like WiFi) are often measured in dBm units.

$$P = (1\,mW)\,10^{\beta/10}$$

└ Reference is 1 mW.

Ex: $\beta = -45$ dBm

$$45\ dB \Rightarrow 10^{4.5} = 30000$$

$$P = \frac{1\,mW}{30000} = 3 \times 10^{-8}\ W = 30\ nW$$

Polarization and Polarizers

Thursday, August 1, 2019 3:04 PM

Polarization is the direction of the electric field of an EM wave.

Most light is unpolarized. The polarization direction changes randomly.

A polarizer is a filter that lets through light of only a certain polarization.

Unpolarized light: $I_{out} = \frac{1}{2} I_{in}$

Polarized light: $I_{out} = I_{in} \cos^2\theta$

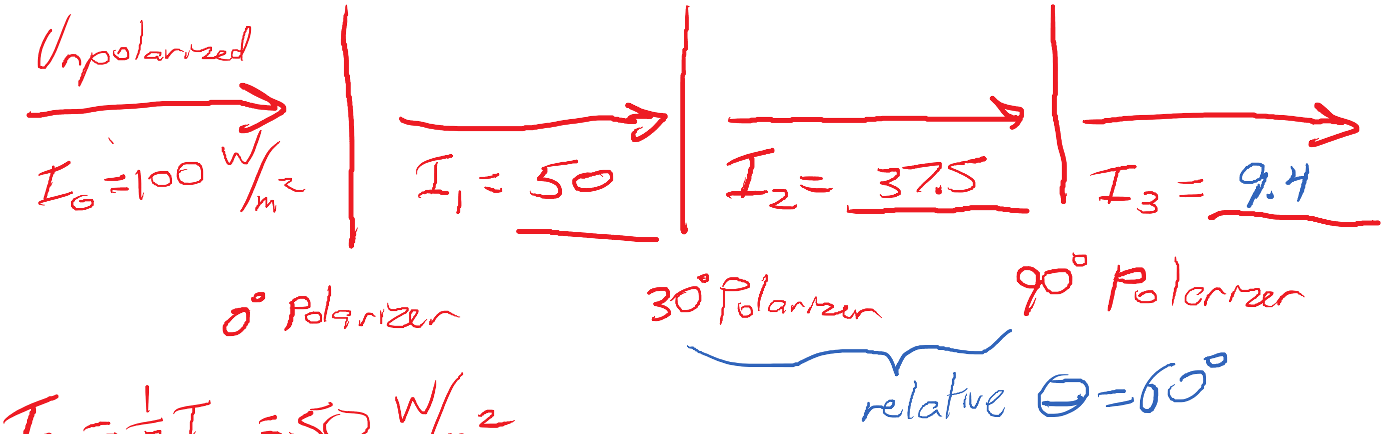
θ = relative twist

$\theta = 45^\circ \qquad \cos^2(45^\circ) = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$

Remember the photon idea? What if you hit a polarizer with a single photon?

- Either it gets through or it doesn't. The probability is cos^2(theta)
- The photon becomes polarized in the new direction.

Typical calculations involve a stack of polarizers.



$I_1 = \frac{1}{2} I_0 = 50$ W/m^2

$I_2 = I_1 \cos^2\theta = (50) \cos^2(30^\circ) = 37.5$

$I_3 = I_2 \cos^2\theta = (37.5) \cos^2(60^\circ) = 9.4$ W/m^2

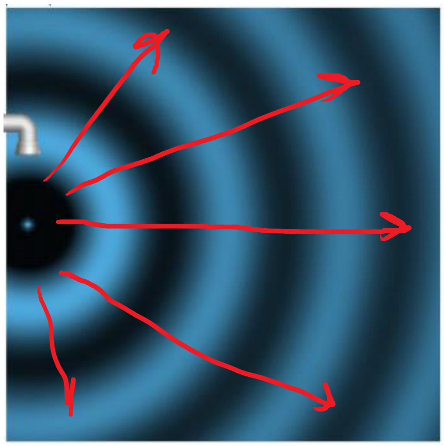
Weirdness of Polarizers

Thursday, August 1, 2019 11:52 AM

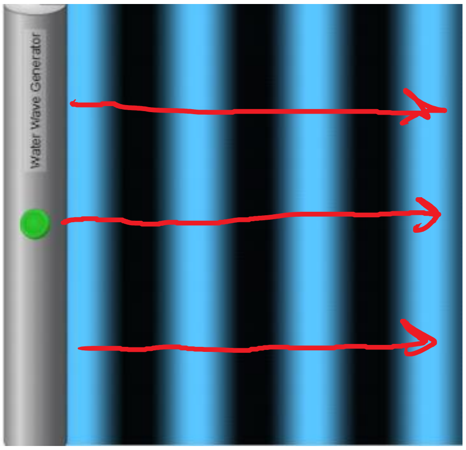
Bell's Theorem: The Quantum Venn Diagram Paradox

Lec 17 - Ray Optics

Monday, August 5, 2019   1:57 PM



Spherical Waves
Diverging Rays



Plane Waves
Parallel Rays

Rays are perpendicular to wave fronts.
When rays are more concentrated, the intensity is higher.

Generally, rays are associated with energy that moves in straight lines at a constant velocity.

- Speed of light: $c = 299792458\ m/s \approx 3\times10^{8}\ m/s$
- Speed in a material: $v = c/n$ n = Index of refraction

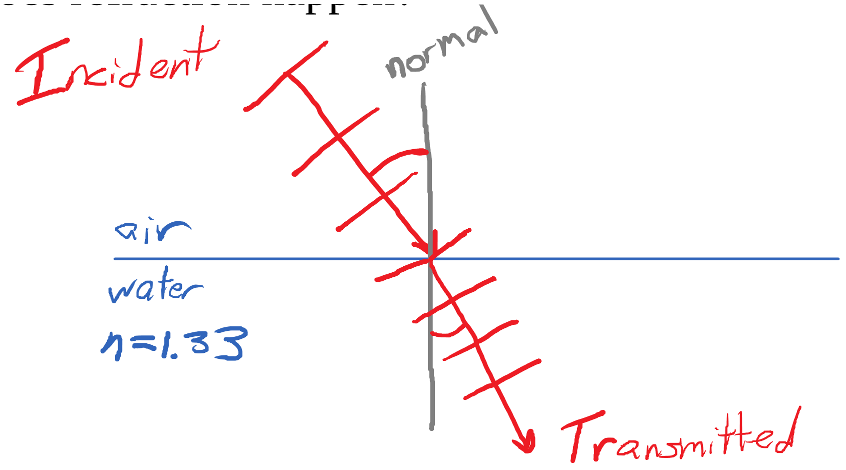
$n_{air} = 1.0004$ $n_{water} = 1.33$ $n_{glass} \approx 1.5$

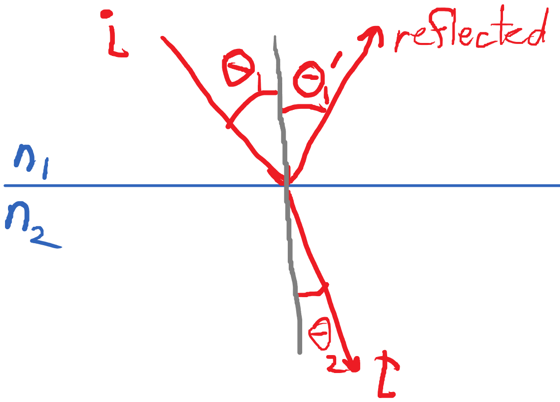
Other ways the straight line behavior changes:

- Diffraction: Bending because of going past a barrier.
- Absorption: When an object or material takes the energy from a ray.
- Scattering: Absorption and immediate re-emission in many directions.
- Reflection: Bouncing of a ray off of a surface.
- Refraction: Bending of a ray as it is transmitted through a surface into a new material.

Why does refraction happen?

Incident





$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\theta_1 = \theta_1'$

Ex: $\theta_1 = 50°$

$(1.0) \sin(50°) = (1.33) \sin\theta_2$

$0.576 = \sin\theta_2$

$35.2° = \theta_2$

What if the materials are reversed?

$n_1 = 1.33$ $\theta_1 = 50°$

$n_2 = 1.0$

$\frac{1.33 \sin(50°)}{1.0} = 1.02 = \sin\theta_2$

No valid θ_2!

If sin(theta2) > 1, there is no solution for the transmitted angle!
This is a situation called total internal reflection.
All of the light ray is reflected from the surface, getting stuck inside the material with n=n1.

The critical angle is the theta1 that separates refraction from reflection.
To find it, set theta2=90 degrees. $\sin\theta_2 = 1$

$n_1 \sin\theta_c = n_2$

$$\sin\theta_c = n_2/n_1$$

For a water-air interface:

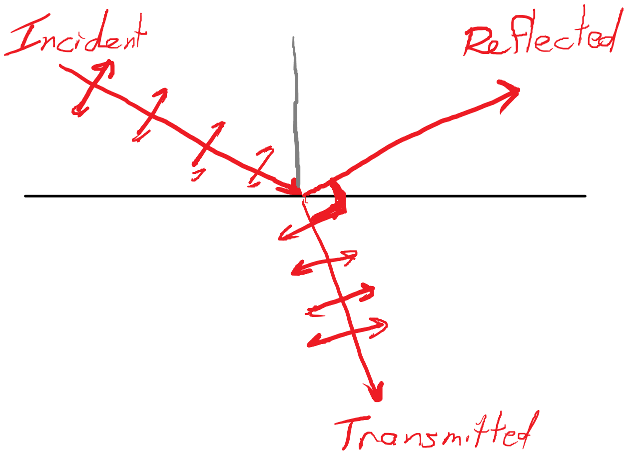
$$\sin\theta_c = \frac{1}{1.33} = 0.75$$

$$\theta_c = 48.6^\circ$$

Above, when theta1=50 deg, that was more than theta_C, so no light could be transmitted.

Polarization by Reflection - Brewster's Angle

Monday, August 5, 2019 2:50 PM



- It is the oscillations in the transmitted material that generate the reflected ray.
- To form a ray, oscillations must be perpendicular to the ray.
- If the transmitted ray oscillates along the direction of the reflected ray. It can't generate the perpendicular oscillations needed to form the reflected ray.
- If the transmitted and reflected rays are perpendicular, none of the reflected light can be polarized in the plane of incidence (plane of the drawing).
- The other polarization (perpendicular to the drawing) doesn't have this issue.
- So, at this special angle, all of the reflection is perpendicular to the plane of incidence.
- In the case of glare off horizontal surfaces, the glare is horizontally polarized.

Condition: $\theta_1 + \theta_2 = 90^\circ$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_1 \sin\theta_1 < n_2 \cos\theta_1$$

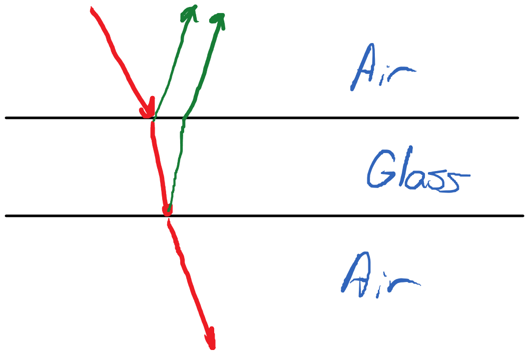
$$\tan\theta_1 = \frac{n_2}{n_1}$$

For the water-air interface: $n_2/n_1 = 1.33$ $\theta_1 < 53^\circ$

For one polarization, there is no reflection. This leads to the idea of a "Brewster Window" that doesn't have any glare.

Films and thin films

Monday, August 5, 2019 3:09 PM



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_2 \sin\theta_2 = n_3 \sin\theta_3$$

Since $n_3 = n_1 \rightarrow \theta_3 = \theta_1$

As long as the front and back surfaces are parallel, there is only a small shift of the ray to the side, not an overall bend.

Thin-Film Interference: The light that reflects off of the front and back surfaces has to travel different distances.
How much reflects? Regular glass in air reflects about 4%.
This is 25 times less than the original light. $\log(25) = 1.4$

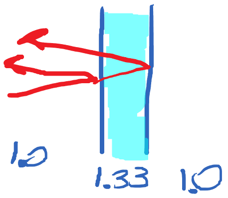
$\hookrightarrow \sim -14\,\text{dB}$

What are the conditions that determine interference?

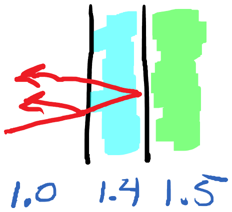
- Phase difference = 0, 2*pi, 4*pi = constructive
- Phase difference = pi, 3*pi, 5*pi = destructive

How can we get a phase difference?

- Reflection: There is a factor of pi introduced depending on whether the index of refraction is increasing or decreasing.



For a soap film, the two reflected beams have different phase differences caused by reflection. This is an overall phase difference of pi caused by reflection.



For an anti-reflective coating, both beams are reflected off of a higher-index material, and zero phase difference caused by the reflections.

What about the path length difference? Consider normal incidence only. The second beam must travel 2*thickness extra.

What about the path length difference? Consider normal incidence only.
The second beam must travel 2*thickness extra.

Thickness → $2d$ → $\Delta l = m\lambda$ ← Wavelength in film!

$m=$ integer → constructive

$$v = f\lambda$$

$$\frac{c}{n} = f\lambda$$

$$\lambda_0 = \frac{c}{f}$$

$$\frac{c}{f\,n} = \lambda$$

$$\frac{\lambda_0}{n} = \lambda$$

If there is a phase difference due to reflection, it flips the meanings of m=integer vs. m=half-integer.

$$2d = \frac{m\lambda_0}{n}$$

	No reflection phase	Reflection Phase
$m=$ integer	Constructive	Destructive
$m=$ integer $+\frac{1}{2}$	Destructive	Constructive

Thin-film interference doubles the reflection in cases of constructive interference but eliminates the reflection in cases of destructive.

How thick should an AR coating be if n_coating = 1.4 and we are targeting green 530 nm light?

No reflection Phase

$m=$ integer $+\frac{1}{2}$

Ex: $m=\frac{1}{2}$

$$2d = \frac{(\frac{1}{2})(530\text{ nm})}{1.4}$$

$$d = \frac{530\text{ nm}}{(2)(2)(1.4)} = 94.6\text{ nm}$$

Professional anti-reflective coatings are multiple layers to cover the visible spectrum.

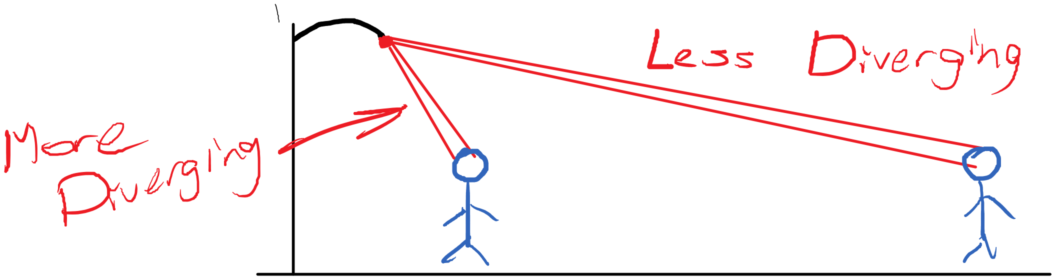
Review:

- Snell's Law of Refraction
- Total internal reflection and critical angle
- Brewster's Angle and polarization by reflection
- Thin-film interference

Lec 18 - Lenses and Optics Instruments

Tuesday, August 6, 2019 1:50 PM

Image Formation - We observe many rays of light coming from different point sources.

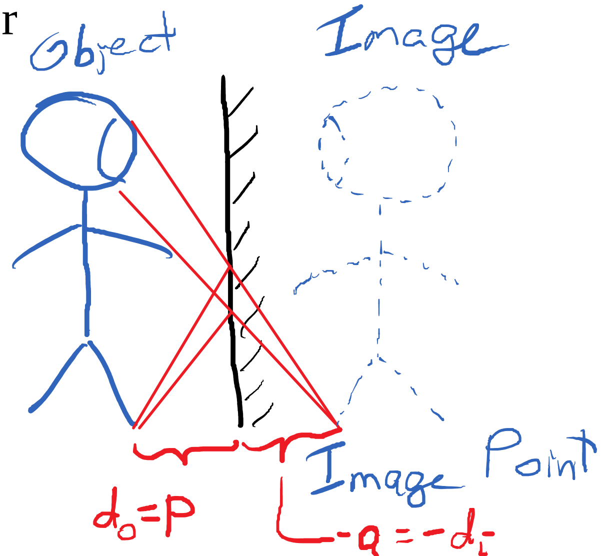


The divergence of the rays is inversely related to the distance to the source of the rays.

Everything we look at sends us diverging or parallel rays of light. We use the divergence to tell us the distance to the source.

Optics instruments (lenses/mirrors) redirect rays to fool our eyes.

Easiest case: Plane mirror



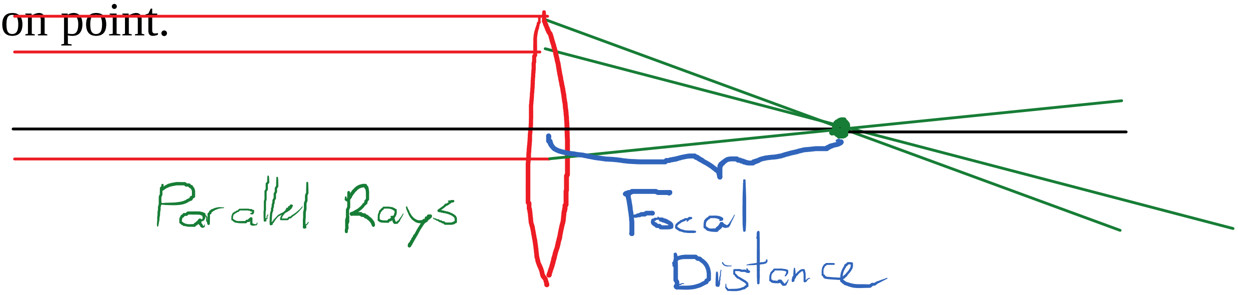
Since the rays don't actually touch the image point (no light inside the wall), it's a virtual image.
For a plane mirror, p = + and q is negative.

$q = -p$

What does a lens do?

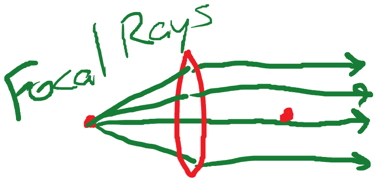
A converging lens takes incoming rays of light and increases their convergence (or decreases their divergence).

Easiest case: Incoming parallel rays are focused to a common point.



If the rays are parallel to the optic axis, they hit the focal point.

Second easiest case: A point source at the focal point will generate outgoing parallel rays. Usually this is drawn by placing the source on the other side. (the "other focal point")



Calculations:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

object location

q image location

focal length

$$m = \frac{-q}{p} = \frac{h_i}{h_o}$$

linear magnification

Converging Lens Ray Diagrams

Tuesday, August 6, 2019 1:58 PM

Far Object

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

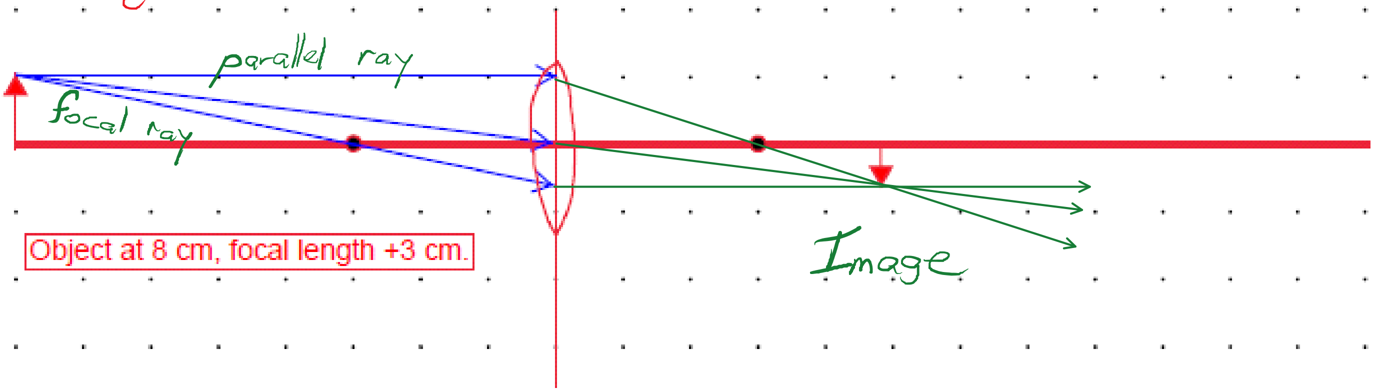
$$q = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1}$$

$$q = \left(\frac{1}{3} - \frac{1}{8}\right)^{-1} = 4.8 \text{ cm}$$

$$\frac{-q}{p} = \frac{-4.8}{8} = -0.6$$

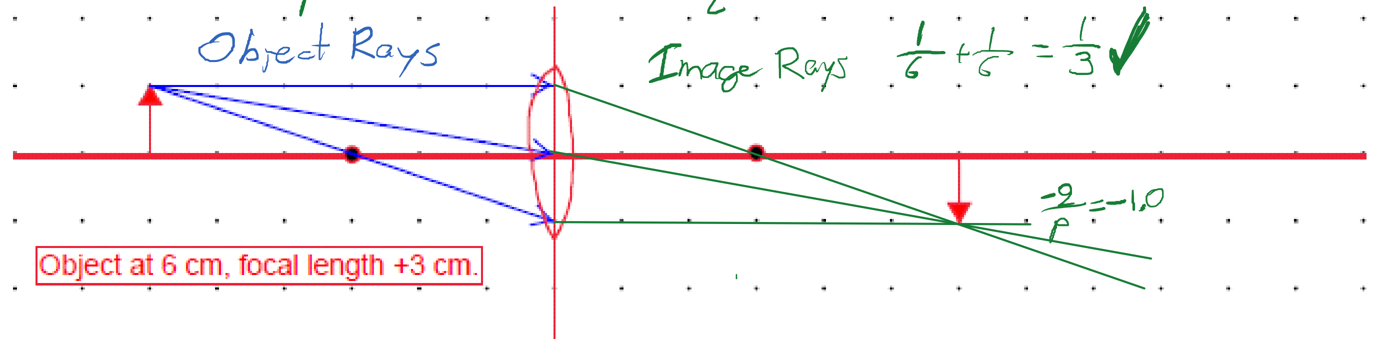
Flipped

reduced



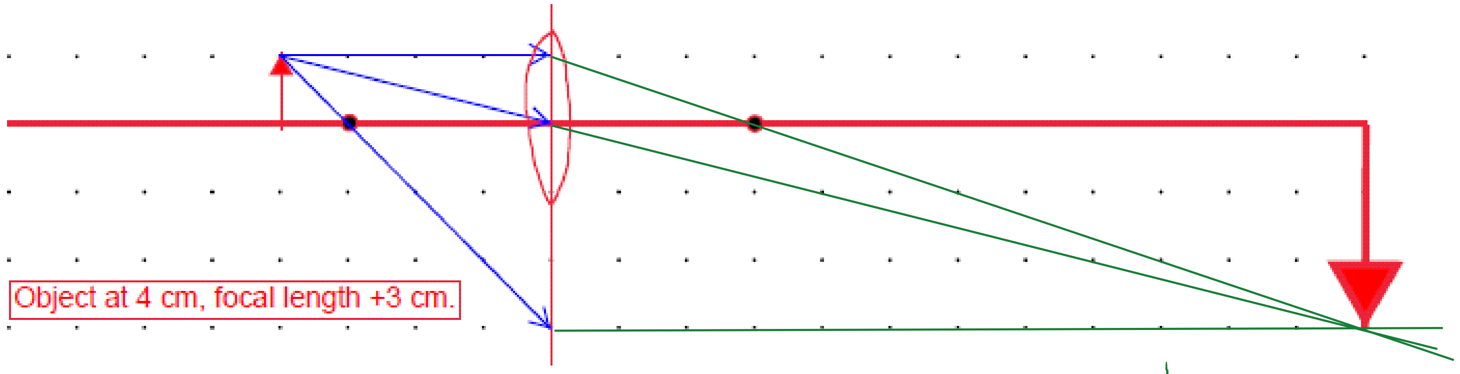
p = 6 cm

q = 6 cm

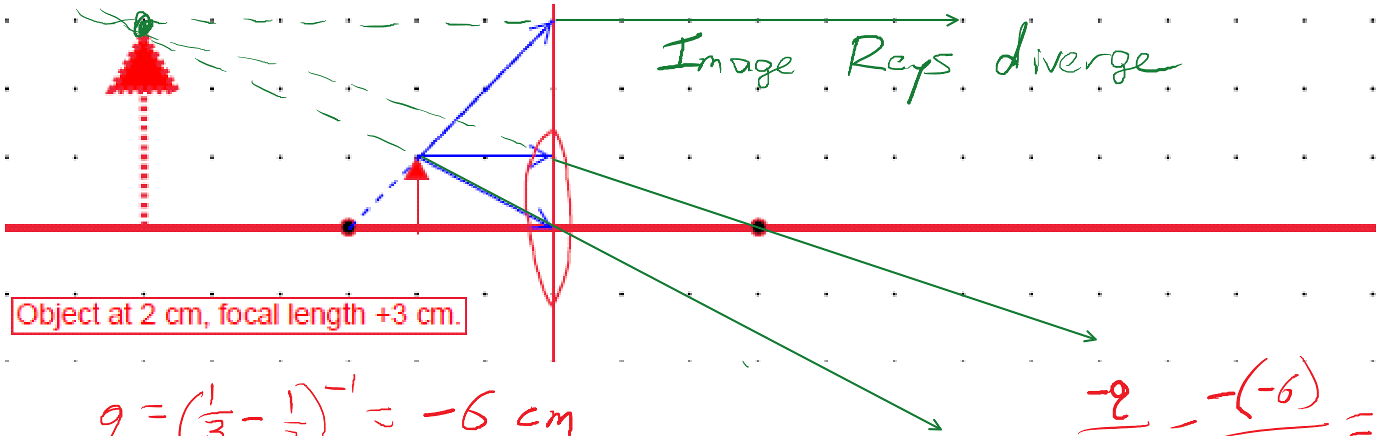


$$\frac{1}{6} + \frac{1}{6} = \frac{1}{3} \checkmark$$

$$\frac{-q}{p} = -1.0$$



$$\frac{1}{4} + \frac{1}{q} = \frac{1}{3} \Rightarrow q = \left(\frac{1}{3} - \frac{1}{4}\right)^{-1} = 12 \text{ cm}$$



$$q = \left(\frac{1}{3} - \frac{1}{2}\right)^{-1} = -6\ cm$$

$$\frac{-q}{p} = \frac{-(-6)}{2} = +3$$

Diverging Lens

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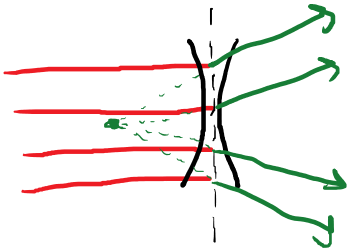
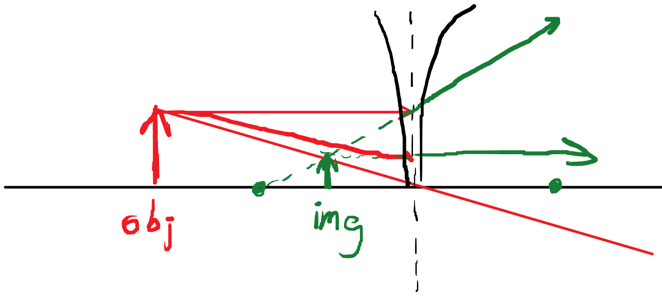
A diverging lens makes parallel rays diverge away from the focal point.



For an actual object, the rays hitting the lens are already diverging. The diverging lens makes them diverge more.



$p = 6\ \text{cm}$

$f = -3\ \text{cm}$

$$q = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \left(\frac{1}{-3} - \frac{1}{6}\right)^{-1}$$

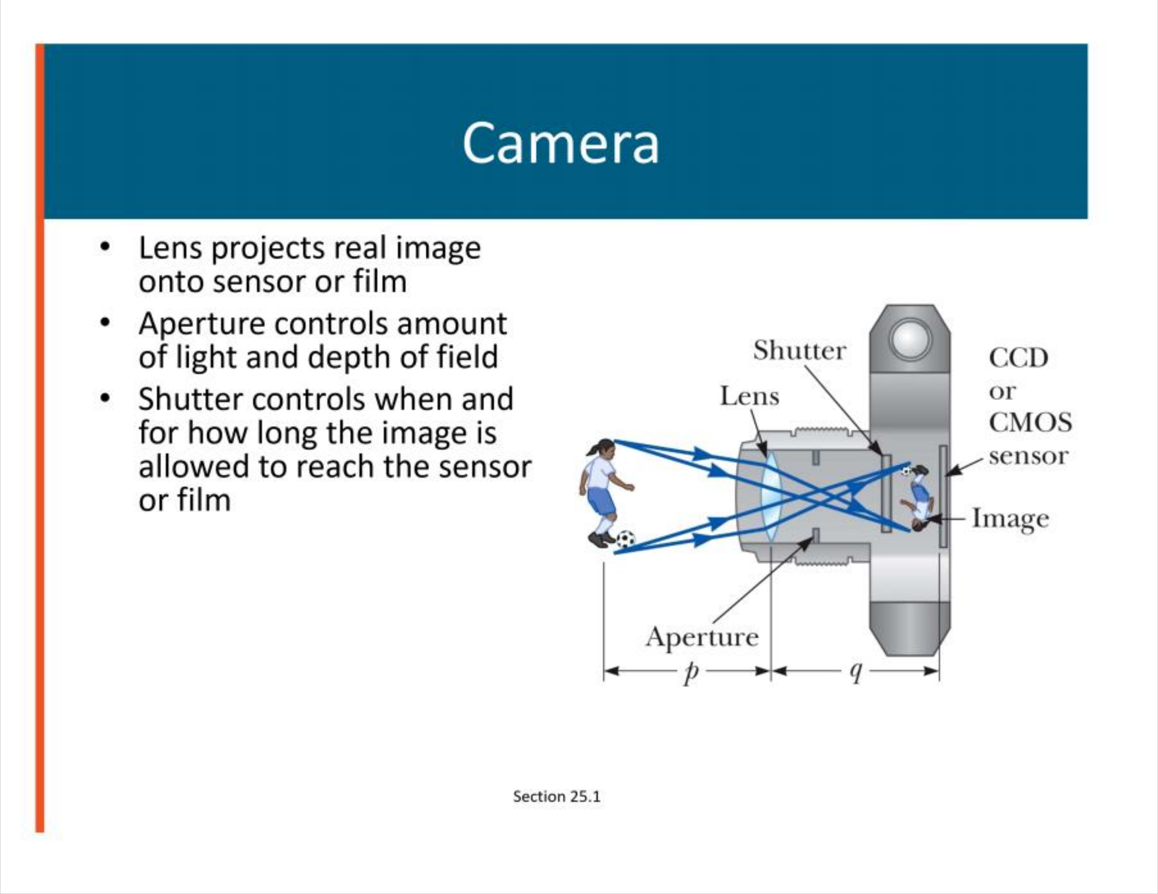
$$= -2\ \text{cm}$$

Since the image rays are always diverging, the image is always virtual. The image is also always reduced.

Optics Instruments

Tuesday, August 6, 2019 2:57 PM

Optics Instruments



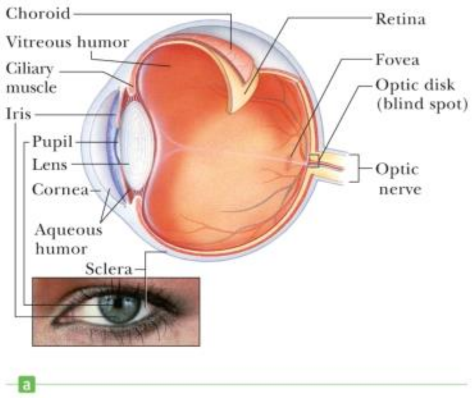
Camera

- Lens projects real image onto sensor or film
- Aperture controls amount of light and depth of field
- Shutter controls when and for how long the image is allowed to reach the sensor or film

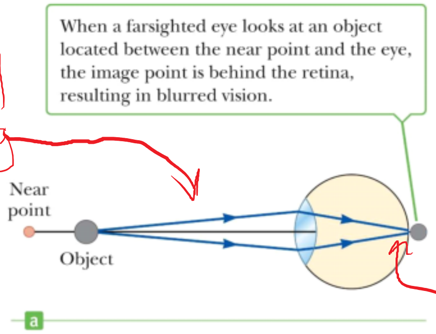
Section 25.1

The Eye

- The normal eye focuses light and produces a sharp image.
- Essential parts of the eye
  - Cornea – light passes through this transparent structure
  - Aqueous Humor – clear liquid behind the cornea



Farsightedness


When a farsighted eye looks at an object located between the near point and the eye, the image point is behind the retina, resulting in blurred vision.

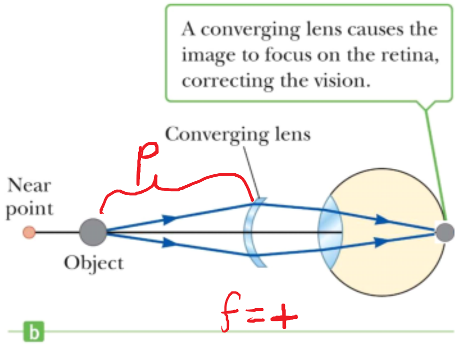
Strongly Diverging

Ex: n.p. = 40 cm

not quite converged

- Can usually see far away objects clearly, but not nearby objects
- Object is “too close”, so the image is “too far”.
- The image focuses behind the retina.

Correcting Farsightedness


A converging lens causes the image to focus on the retina, correcting the vision.

- Image Rays don't converge enough to focus, so make them converge more with a converging lens.
- The eye looks at a virtual image (not shown) at the near point.

Section 25.2

n.p. = 40 cm → q = −40

want p = 25 cm

object location

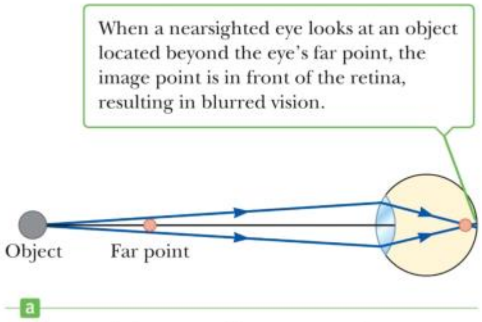
Always looking @ virtual image

$\left(\frac{1}{25} + \frac{1}{-40}\right)^{-1} = 66.7$ cm

Power $= \frac{1}{f} = \frac{1}{0.667} = +1.5$

↗ in meters

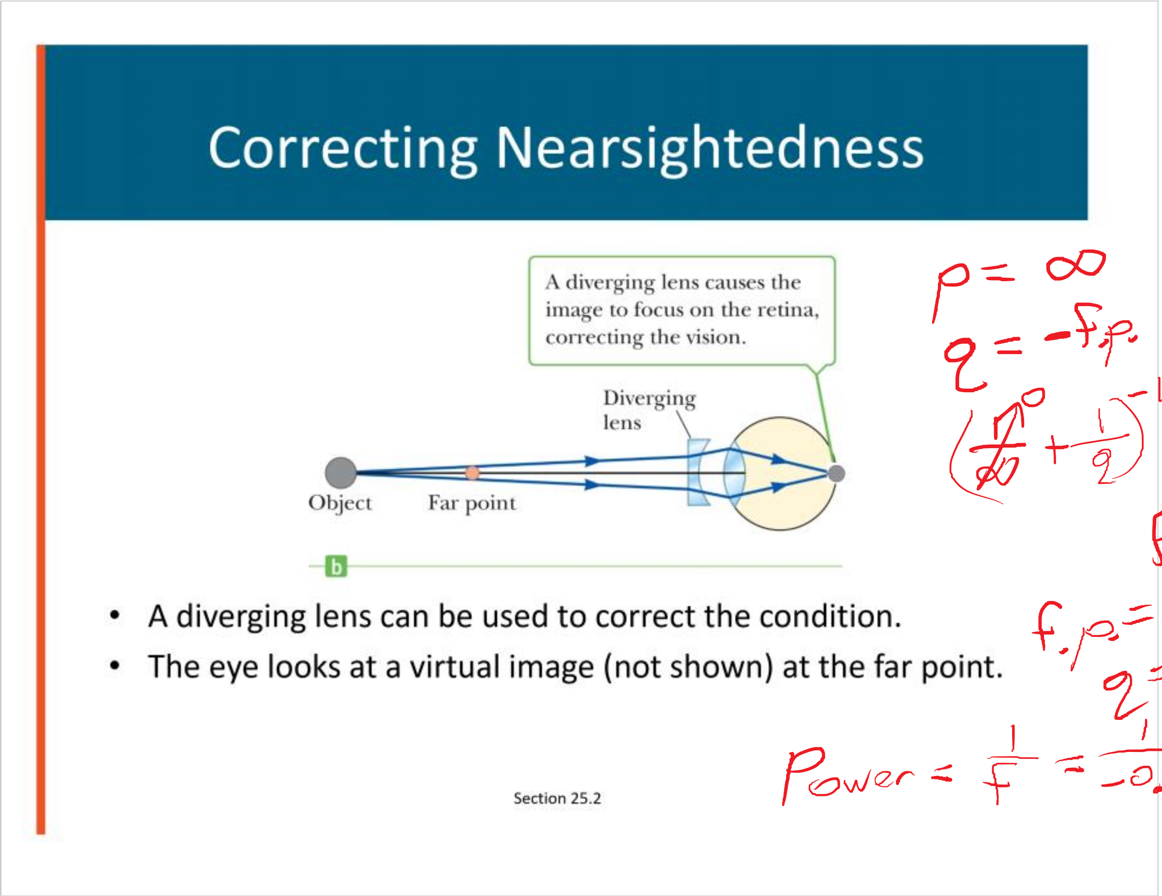
Nearsightedness


When a nearsighted eye looks at an object located beyond the eye's far point, the image point is in front of the retina, resulting in blurred vision.

- Can see nearby objects, but not far away objects.
- The object is "too far", so the image is "too close" to the lens.
- The image focuses in the front of the retina.

Section 25.2

Avg f.p. = 30 cm



Correcting Nearsightedness

- A diverging lens can be used to correct the condition.
- The eye looks at a virtual image (not shown) at the far point.

Section 25.2

$p = \infty$

$q = -f.p.$

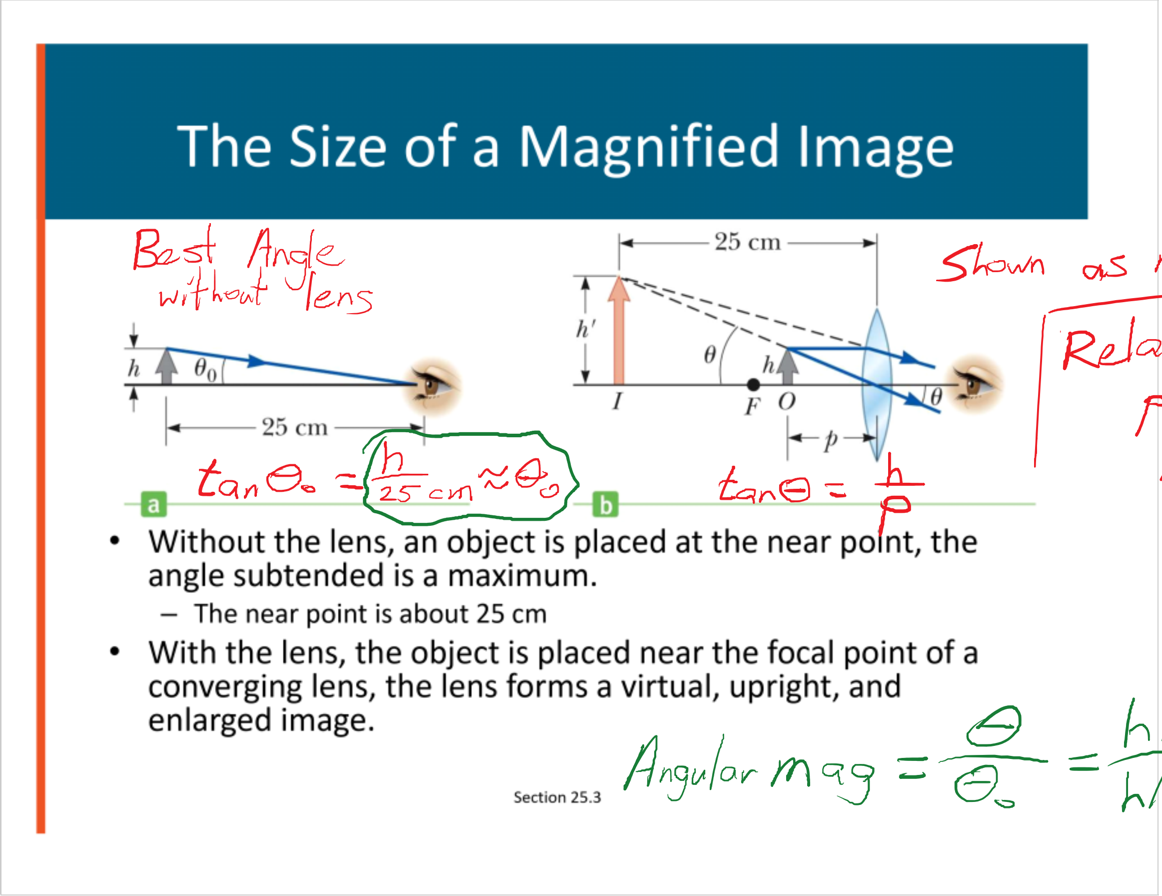
$\left(\frac{1}{\cancel{\infty}}^{0} + \frac{1}{q}\right)^{-1} = f$

$f = q$

$f.p. = 30\text{ cm}$

$q = f = -30\text{ cm}$

$\text{Power} = \frac{1}{f} = \frac{1}{-0.3} = -3.3$



The Size of a Magnified Image

Best Angle without lens

$\tan\theta_0 = \boxed{\frac{h}{25\text{ cm}} \approx \theta_0}$

$\tan\theta = \frac{h}{p}$

Shown as max mag.

Relaxed Viewing

$p = f$ $q = -\infty$

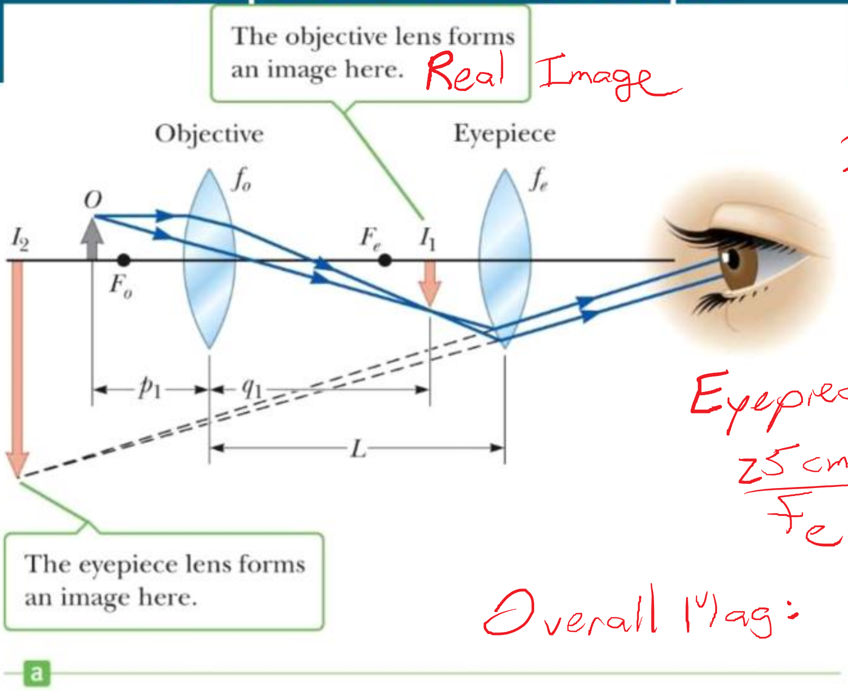
$\tan\theta = \boxed{\frac{h}{f} \approx \theta}$

- Without the lens, an object is placed at the near point, the angle subtended is a maximum.
  - The near point is about 25 cm
- With the lens, the object is placed near the focal point of a converging lens, the lens forms a virtual, upright, and enlarged image.

Section 25.3

$\text{Angular mag} = \frac{\theta}{\theta_0} = \frac{h/f}{h/25\text{ cm}} = \frac{25\text{ cm}}{f}$

Compound Microscope



The objective lens forms an image here. Real Image

The eyepiece lens forms an image here.

Initial Linear Mag.

$$\frac{-q_1}{p_1} \approx \frac{-L}{f_o}$$

Eyepiece Angular Mag

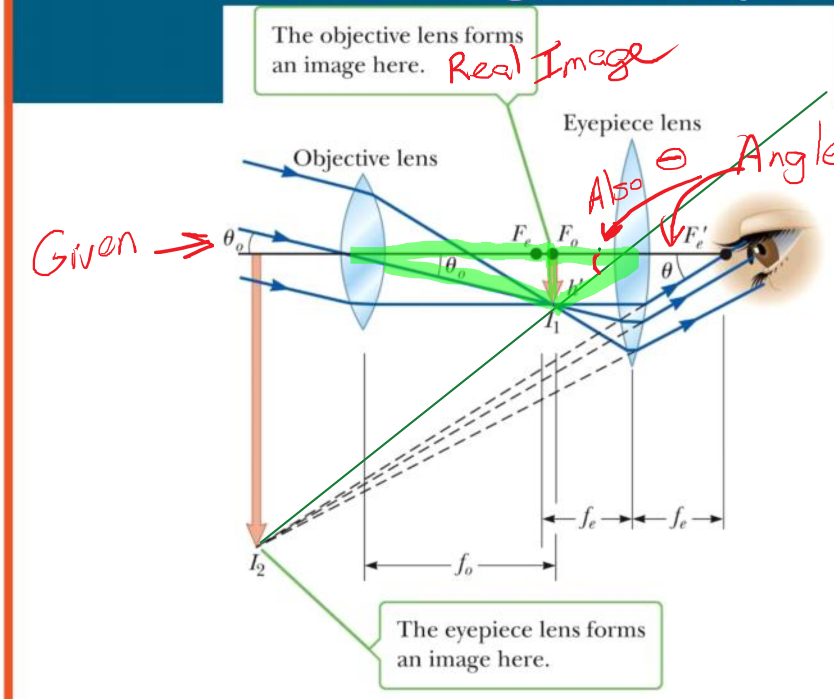
$$\frac{25\text{ cm}}{f_e}$$

Overall Mag: $\frac{-L(25\text{ cm})}{f_o f_e}$

What f's Lead to high mag?

What if we swap the lenses

Refracting Telescope



The objective lens forms an image here. Real Image

The eyepiece lens forms an image here.

Given →

Also θ Angle with telescope

$$\theta = \frac{h'}{f_e} \qquad \theta_o = \frac{-h'}{f_o}$$

$$\frac{\theta}{\theta_o} = \frac{-h'/f_e}{h'/f_o} = \frac{-f_o}{f_e}$$

Examples

Tuesday, August 6, 2019 3:27 PM

$f = 5\,cm$ simple Angular mag $= \frac{25\,cm}{f} = 5$

This was with "relaxed viewing".

max mag $= 5 + 1 = 6$

Max magnification is one notch higher.

What is the focal length of a 10 times magnifier?

$$9 = \frac{25\,cm}{f} \rightarrow f = \frac{25\,cm}{9} = 2.78\,cm$$

Ex: Telescope with $f_o = 1.5\,m$ $f_e = 25\,mm$

Mag = -1.5 / 0.025 = -60.0