Trying to power a resistor with a capacitor:

\[ C \frac{\Delta Q}{\Delta t} = I \]

\[ \text{Capacitor: } Q = CV \]

\[ \text{Resistor: } V = IR \]

This current drains the cap:

\[ I = -\frac{dQ}{dt} \]

\[ \frac{V}{R} = -\frac{C}{dt} \frac{dV}{dt} \]

\[ \frac{dV}{dt} = -\frac{1}{RC} V \]

\[ \text{Note: } \frac{d}{dt} e^t = e^t \quad \frac{d}{dt} e^{-t/2} = -\frac{1}{2} e^{-t/2} \]

\[ \frac{d}{dt} (V_0 e^{-t/2}) = V_0 \frac{1}{2} e^{-t/2} \]

\[ V = V_0 e^{-t/2} \text{ is the solution as long as } t = RC \]

Analogy: Drain a tank of water thru a straw.
What is \( e^{-t/\tau} \)?
Fraction of charge remaining.

What is \( t/\tau \)?
Amount of time since start, measured as a "number of time constants".

<table>
<thead>
<tr>
<th>( t/\tau )</th>
<th>( \exp(-t/\tau) )</th>
<th>( % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>0.5</td>
<td>0.61</td>
<td>61%</td>
</tr>
<tr>
<td>0.693</td>
<td>0.5</td>
<td>50%</td>
</tr>
<tr>
<td>1.0</td>
<td>0.37</td>
<td>37%</td>
</tr>
<tr>
<td>2.0</td>
<td>((0.37)^2=0.14)</td>
<td>14%</td>
</tr>
<tr>
<td>5.0</td>
<td>0.01</td>
<td>1%</td>
</tr>
</tbody>
</table>

After 50 time constants, basically done.

Note: \( C=2.718 \) vs. \( e=1.6\times10^{-19} \) C

\[ t = R \cdot C \]
RC Circuit Setup for Discharge

Initial Voltage = 8.0 V
Resistor = 500 Ω

After 2.0 s, current is measured:
I = 0.010 A

What is C?
How? Want to use \( \tau = RC \)

Need \( I = \frac{V}{R} \)

\[
V = V_0 \exp\left(\frac{-t}{\tau}\right)
\]

\[
(5.0 V) = (8.0 V) \exp\left(\frac{-(2.0 s)}{\tau}\right)
\]

\[
V = IR = (0.010 A)(500 \Omega) = 5.0 V
\]

\[
\frac{5.0 V}{8.0 V} = 0.625 = \exp\left(\frac{-t}{\tau}\right)
\]

\[
\ln(0.625) = -\frac{2.0 s}{\tau}
\]

\[-0.47 < -\frac{2.0 s}{\tau}
\]

\[0.47 \tau = (2.0 s)
\]

\[\tau = 4.26 s
\]

\[\tau = RC \rightarrow C = \frac{\tau}{R} = \frac{4.26 s}{500 \Omega} = 8.57 \text{ mF} = 8570 \mu F
\]
Charging:

\[ V_c + V_R = \varepsilon = \text{Battery voltage} \]

\[ V_c + \frac{\varepsilon e^{-t/\tau}}{C} = \varepsilon \]

\[ V_c = \varepsilon - \frac{\varepsilon e^{-t/\tau}}{1 - e^{-t/\tau}} \]

\[ I \text{ "looks like" } V_R \]

\[ Q \text{ "looks like" } V_c \]

\[ I = +\frac{dQ}{dt} \]

\[ V_f = \varepsilon \]
Temperature Coefficients

\[ \alpha \Delta T = \text{relative change} \]

For resistance, \( \frac{\Delta R}{R_0} = \alpha \Delta T \)

\[ \frac{R - R_0}{R_0} = \alpha (T - T_0) \]

\[ R = R_0 + R_0 \alpha (T - T_0) \]

\[ = R_0 (1 + \alpha (T - T_0)) \]

\[ \alpha = 0.003 \text{ means } 0.3 \% \text{ increase per degree} \]

\[ \alpha = + \text{ metals} \]

\[ \alpha = - \text{ semiconductor} \]
Gauss's Law

Metal shell with
\[ Q_{\text{net}} = 8 \text{ nC} \]

Small \( r \): Inside shell
Only 5 nC matters

\[ E = \frac{k(5 \text{ nC})}{r^2} \]

\[ \Phi = 4\pi k Q_{\text{enc}} = 4\pi \left( 9 \times 10^9 \text{ Nm}^2/\text{C}^2 \right) (5 \text{ nC}) \]
\[ = 180 \pi \left( \text{N/C} \right) \text{m}^2 \]

Large \( r \): Outside shell:

\[ \Phi = 4\pi k (5 + 8 \text{ nC}) = 468 \pi \left( \text{N/C} \right) \text{m}^2 \]

\[ E = \frac{k(13 \text{ nC})}{r^2} \]

Middle \( r \): Embedded in metal.

\[ E = 0 \]

\[ \Phi = EA = 0 \]

\[ Q = 0 = +5 \text{ nC} + Q_{\text{inner}} \]

\[ Q_{\text{inner}} = -5 \text{ nC} \]

\[ Q_{\text{net}} = Q_{\text{in}} + Q_{\text{out}} \]

\[ 8 \text{ nC} = -5 \text{ nC} + Q_{\text{out}} \]

\[ 13 \text{ nC} = Q_{\text{out}} \]