Capacitors in AC

What we know:
- Store Charge $Q = CV$
- Take time to charge or discharge $T = RC$
- Store Energy $Energy = \frac{1}{2} CV^2$

What happens in AC?

\[ Q = CV \]

Time derivative

\[ Q = CV \]

\[ I = C \frac{dV}{dt} \]

If $V = V_{\text{max}} \sin(2\pi ft)$

\[ V = V_{\text{max}} \cos(2\pi ft) \]

\[ I = V_{\text{max}} 2\pi f C \cos(2\pi ft) \]

\[ P = VI = V_{\text{max}} I_{\text{max}} \sin() \cos() \]

\[ = V_{\text{max}} I_{\text{max}} \frac{1}{2} \sin(2(2\pi ft)) \]

Average $P = 0$
\[ V = IR \]

AC Ohm's Law for Cap:

\[ I_{max} = V_{max} \frac{2\pi f}{C} \]

\[ V_{max} = I_{max} \left( \frac{1}{2\pi f C} \right) \]

\[ V_{max} = I_{max} X_C \]

Reactance \((X_C)\) is in ohms \((\Omega)\)

Capacitors have high reactance when:

- Low \(C\)
- Low \(f\) \((DC, f = 0)\)

Capacitors "block" DC and low \(f\).

Low \(f\) and High \(f\)

\(R_2\) gets more high \(f\)

\(R_1\) gets low \(f\)
Inductors - Devices that "feel" their own magnetic field.

\[
\text{Ex: Solenoid Corl} \quad B = \frac{\mu_0 N I}{l}
\]

Faraday's Law:
\[
\varepsilon = -\frac{d\Phi_B}{dt}
\]

Flux of coil's field:
\[
\Phi_B = B \cdot A \quad \text{(one loop)}
\]

\[
\Phi_B = NBA \quad \text{(whole coil)}
\]

\[
\Phi_B = N \left(\frac{\mu_0 NI}{l}\right) A
\]

\[
\Phi_B = \left(\frac{\mu_0 N^2 A}{l}\right) I = LI
\]

Inductance \((L)\) is measured in henries \((H)\)

Take derivative:
\[
\varepsilon = LI
\]

The EMF of \(L\) fight's \(I\)

As a passive device, we drop the \(\Theta\) sign.

\[
\text{Dv} = \varepsilon
\]
If \[ V = V_{\text{max}} \sin(2\pi ft) \]

Guess \[ I = -I_{\text{max}} \cos(2\pi ft) \]

\[ I = I_{\text{max}} \sin(2\pi ft) \cdot 2\pi f \]

\[ V = I_{\text{max}} 2\pi f L \sin(2\pi ft) \]

\[ V_{\text{max}} = I_{\text{max}} 2\pi f L \]

\[ V_{\text{max}} = I_{\text{max}} X_L \]

For the inductor:

Low \( f \) \( \Rightarrow \) Low \( X_L \) \( \Rightarrow \) Easy current flow.

Power of Inductor:

\[ P = VI = \sin() \cos() \]

\[ = \frac{1}{2} \sin(2\text{ }) \]

\[ P_{\text{avg}} = 0 \]

Energy of Inductor

\[ \text{Energy} = \frac{1}{2} LI^2 \]

Capacitor

\[ \text{Energy} = \frac{1}{2} CV^2 \]
Solenoid Relay:

Small Batt \[\rightarrow\] Small Switch \[\rightarrow\] Motor \[\rightarrow\] Large Batt

When you disconnect a coil, \( I \) is huge. This generates large EMF. This wrecks switches.

Protection options:
1. Diode
2. Capacitor

Series AC Circuit Intro

\[ V_{\text{max}} = V_{R} = I_{\text{max}} R \sin(2\pi ft) \]
\[ V_{L} = L \cdot I = I_{\text{max}} \frac{1}{2\pi f} L \cos(2\pi ft) \]
\[ V_{C} = \frac{1}{C} \int I = I_{\text{max}} \frac{1}{2\pi f} C (-\cos \frac{2\pi ft}{L} ) \]

Propose \( I = I_{\text{max}} \sin(2\pi ft) \)

\[ V_{\text{tot}} = V_{R} + V_{L} + V_{C} \]