Lec 1 - Intro, Charge, and E-Fields

Monday, August 26, 2019 1:17 PM

Why study Electricity and Magnetism (E&M)?

- Our society is electric
  - Energy
  - Information: Communication, Storage, Computation, Control, Measurement
- Basis of light and radio waves.
- Basis of Chemistry
- Practice with math and "learning new things".

Electrostatics

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Study of stationary charges. What are charges?

All matter is made of atoms, which are in turn made of protons, electrons, and neutrons. These are particles that interact electrically.

"Charge" is a measure of how electric something is.

A "charge" is an object or particle that has charge.

Proton: $q_p = 1.6 \times 10^{-19}\ C = +e$

Electron: $q_e = -1.6 \times 10^{-19}\ C = -e$

Neutron: $q_n = 0$

How much charge is in a 2 gram aluminum piece?

$1.66 \times 10^{-27}\ kg = 1\ u$

$m_{Al} = 27\ u = 27(1.66 \times 10^{-27}\ kg)$

$= 4.48 \times 10^{-26}\ kg$

$M = N m \qquad N = \frac{M}{m} = \frac{0.002\ kg}{4.48 \times 10^{-26}\ kg} = 4.46 \times 10^{22}$

Each aluminum atom has 13 protons and 13 electrons.

$Q_p = N_p q_p = (13)(4.46 \times 10^{22})(1.6 \times 10^{-19}\ C)$

$= 92800\ C$

$Q_e = -92800\ C$

$Q_T = 0$

We can "charge" the object by adding or removing charges (the particles).

- In solids, the protons are part of the nuclei, which stay in place.
- Some electrons are free to move around. Typically 1 per atom.
- In liquids, ions (nuclei without enough electrons to balance) can move around.

As it turns out, we can move 1 electron per atom, but we cannot steal one electron per atom. The amount we could steal is only around 1 nC worth.

Electrostatic Effects

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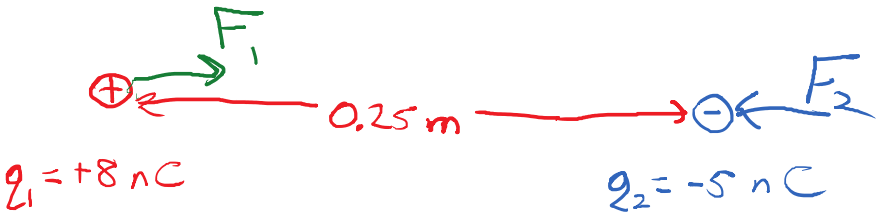
- Charges exert forces on each other.
- How? Charges generate "electric field". The electric field is what causes the force on other charges.



q1 is our "source" which generates electric field.

$$E_1 = \frac{k|q_1|}{r^2} \qquad F_2 = |q_2|E$$

Electric field (E) is measured in newtons per coulomb (N/C).

$$F_2 = k\frac{|q_1||q_2|}{r^2}$$

$$F_2 = \left(9\times10^9 \frac{Nm^2}{C^2}\right)\frac{(8\times10^{-9}\ C)(5\times10^{-9}\ C)}{(0.25\ m)^2} = 5.76\times10^{-6}\ N = 5.76\ \mu N$$

What is the direction of the force?

- Opposites attract
- Like charges repel

Is there a way to get the + and - to work out in the equation, automatically?

$$F_2 = \frac{k|q_1||q_2|}{r^2} \qquad \vec{F_2} = \frac{k q_1 q_2}{r^2}(+)$$

This comes out negative for the above calculation, which is exactly what we want. Does this work for F1?

$$\vec{F_1} = \frac{k q_1 q_2}{r^2}(-) \quad \text{should be } +.$$

We need something extra at the end of a generic formula, to set the direction of the force vector.

Extra Factor is (−)

Extra Factor is (+)

The extra factor is the relative direction, from the source, pointing toward the object we're studying.

$$\vec{F}_1 = \frac{k q_1 q_2}{r^2} \hat{r}$$

$\hat{r}$ for q_1 points toward q_1 from q_2

2. E-Fields

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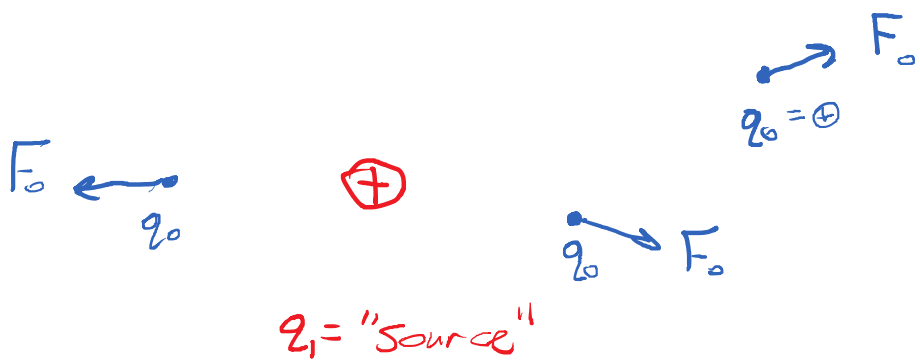
Electric field describes how electrified space is.
They are caused by charges.
(Also caused by fluctuating magnetic fields.)

Electric fields cause forces on charges.
Put a charge (q0) at a location in space that has an electric field (E) it will feel the force (F).

$$\vec{F} = q_0 \vec{E}$$

The direction of E is the same as the direction of F, if the charge (q0) is positive. Any q0 experiences the same E, but they experience different F.

- If q0 is negative, its force is in the opposite direction.



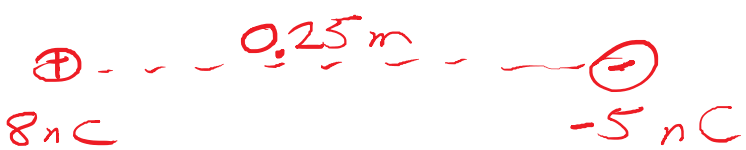
For a positive source, the electric field points away from the source.

How much electric field is present?

$$F_0 = \frac{k|q_0||q_1|}{r^2} = |q_0|E \qquad E = \frac{k|q_1|}{r^2}$$

This is the magnitude of E due to q1 (the source charge).

$$k = 9 \times 10^9 \frac{Nm^2}{C^2}$$



$$E = \frac{(9\times10^9)(5\times10^{-9})}{(0.25)^2} \qquad E = \frac{(9\times10^9)(8\times10^{-9})}{(0.25)^2}$$

$(0.25)^2$

$= 720\ N/C$

$\longrightarrow$ Toward $\ominus$

$(0.25)^2$

$= 1152\ N/C$

$\longrightarrow$ (away from $\oplus$)

These are two separate electric fields, in two different situations, where the other charge is thought of as the "source".

$F = (8 \times 10^{-9})(720)$

$= 5.76 \times 10^{-6}\ N$

$\longrightarrow$ (with E)

$F = (5 \times 10^{-9})(1152)$

$= 5.76 \times 10^{-6}\ N$

$\longleftarrow$ Against E

Charge distributions

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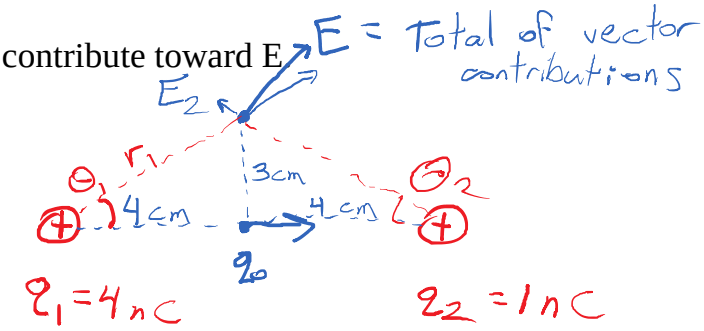
Multiple charges can contribute toward E



How would we calculate the electric field at the upper point?

$$E_1 = \frac{kq_1}{r_1^2} = \frac{(9\times10^9)(4\times10^{-9})}{(0.05)^2} = 14400\ N/C \qquad E_2 = 3600\ N/C$$

$$E_{1x} = E_1\cos\theta_1 = (14400)\left(\frac{4}{5}\right) = 11520\ N/C$$

$$E_{2x} = E_2\cos\theta_2 = -(3600)\left(\frac{4}{5}\right) = -2880\ N/C$$

$$E_x = 8640\ N/C$$

(coincidence)

$$E_{1y} = E_1\sin\theta_1 = 14400\left(\frac{3}{5}\right) = 8640\ N/C$$

$$E_{2y} = E_2\sin\theta_2 = 3600\left(\frac{3}{5}\right) = 2160\ N/C$$

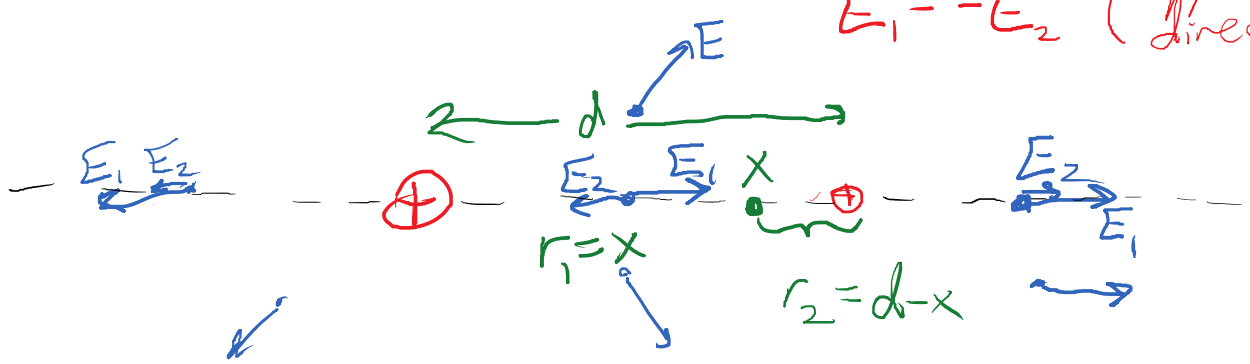
$$E_y = 10800\ N/C$$

Where would the total electric field be zero?

$$\vec{E}_1 + \vec{E}_2 = 0 \qquad \vec{E}_1 = -\vec{E}_2$$

$E_1 = E_2$ (same magnitude)

$\hat{E}_1 = -\hat{E}_2$ (opposite direction)



$$E_1 = E_2$$

$$\frac{k|q_1|}{r_1^2} = \frac{k|q_2|}{r_2^2}$$

Ex: d = 8 cm

$q_1 = 4\,nC$

$q_2 = 1\,nC$

$$\frac{\cancel{k}(4\,\cancel{nC})}{x^2} = \frac{\cancel{k}(1\,\cancel{nC})}{(d-x)^2}$$

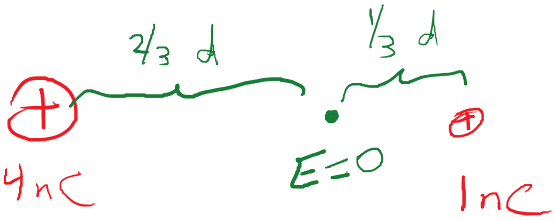
$$\frac{2}{x} = \frac{1}{d-x}$$

$$2d - 2x = x$$

$$2d = 3x$$

$$\frac{2}{3}d = x$$

$$E = \frac{kq}{r^2}$$



Continuous charge distributions

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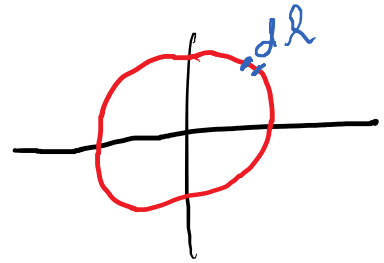
Line Charge: λ = charge per length
= linear charge density

$Q = \lambda L$ (if λ constant)

$Q = \int \lambda \, dx$ (IF along x-axis)

$Q = \int \lambda \, dl$ Arbitrary curve

$Q = \int dq$ Lots of little bits



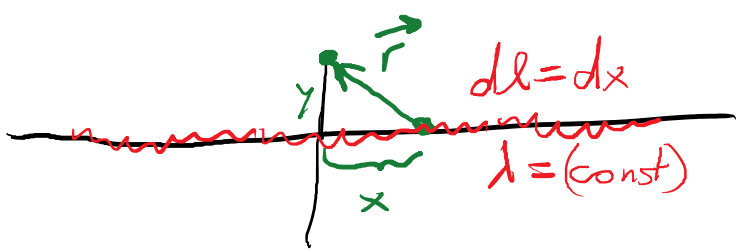
What is the electric field due to a continuous charge distribution?

$$\vec{E} = \int d\vec{E} = \int \frac{k\,dq}{r^2}\hat{r}$$

$dq = \lambda dl = \lambda dx$

$r^2 = x^2 + y^2$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{-x\hat{i} + y\hat{j}}{\sqrt{x^2+y^2}}$$



$$E_x = \int \frac{-k\lambda x}{(x^2+y^2)^{3/2}}\,dx = 0$$

$$E_y = \int \frac{k\lambda y}{(x^2+y^2)^{3/2}}\,dx = \frac{2k\lambda}{y}$$

3. Gauss's Law

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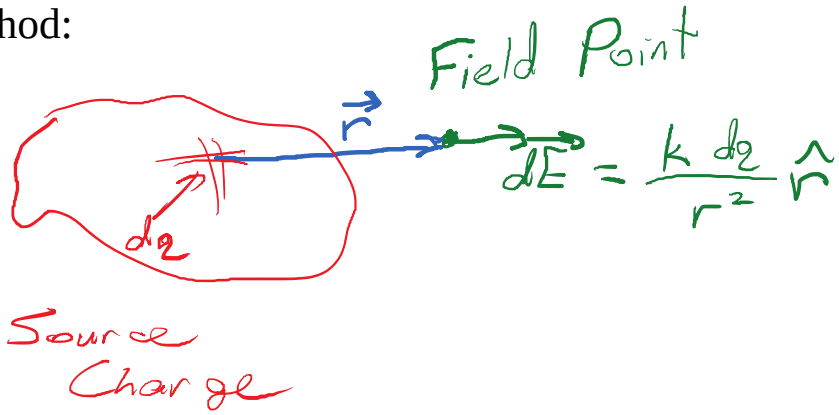
Alternative to integration method of finding E.

Integration method:



Gauss's Law is based off the idea of electric flux.

- In E&M, flux is like the "total amount of field" pointing through a surface. (Note: In fluids, flux is the amount of stuff per unit area.)

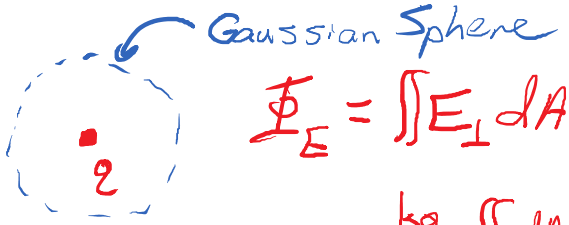
$$\Phi_E = \iint \vec{E}\cdot d\vec{A} = \iint \vec{E}\cdot\hat{n}\,dA = \iint E_\perp\,dA$$

$\hat{n}$ = normal vector to surface

$\bullet$ = dot product

$E_\perp$ = component of $\vec{E}$ $\perp$ to surface

Important fact: A point charge "emits" a certain amount of electric flux.



$$\Phi_E = \iint E_\perp\,dA = \iint \frac{kq}{r^2}\,dA$$

$$= \frac{kq}{r^2}\iint dA = \frac{kq}{r^2}A \qquad A = 4\pi r^2$$

$$\Phi_E = 4\pi kq$$

If you surround a point charge with a Gaussian surface (even if it's not a sphere), the same flux will be penetrate that surface.

Integration is a linear operation. $\int af(x) + bg(x)\,dx = a\int f(x)dx + b\int g(x)dx$

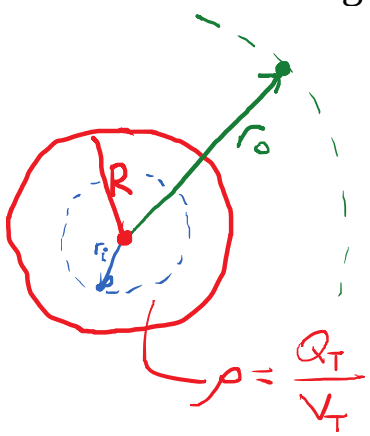
If multiple charges are inside a Gaussian surface, the flux is just the total flux "emitted" by the charges.

How is all of this useful? Can we get the value of E through Gauss's Law?

- If E_perp = constant, it factors out of the integral.
- If E = 0, the total flux must be zero.

In what situations is E_perp = constant? Symmetric geometries.

Ex: Spherical uniform charge distribution



For $r_o > R$:

All Q inside gaussian surface

$$\Phi_E = 4\pi k Q_{enc}$$

$$(E\, 4\pi r_o^2) = 4\pi k Q_T$$

$$E = kQ_T / r_o^2$$

The spherical ball of charge acts like a point charge, from the perspective of anything outside the sphere.

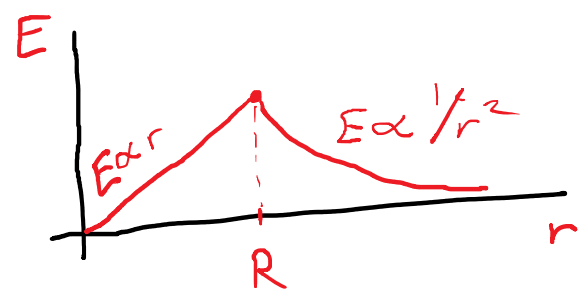
For $r_i < R$:

$$Q_{enc} = \rho V_{enc} = (Q_T/V_T) V_{enc} = Q_T \frac{\frac{4}{3}\pi r_i^3}{\frac{4}{3}\pi R^3} = Q_T \frac{r_i^3}{R^3}$$

$$\Phi_E = 4\pi k\, Q_{enc}$$

$$4\pi r_i^2 E = 4\pi k \left(Q_T \frac{r_i^3}{R^3}\right)$$

$$E = \frac{kQ_T r_i}{R^3}$$



All of this assumes spherical symmetry.

E of a line charge

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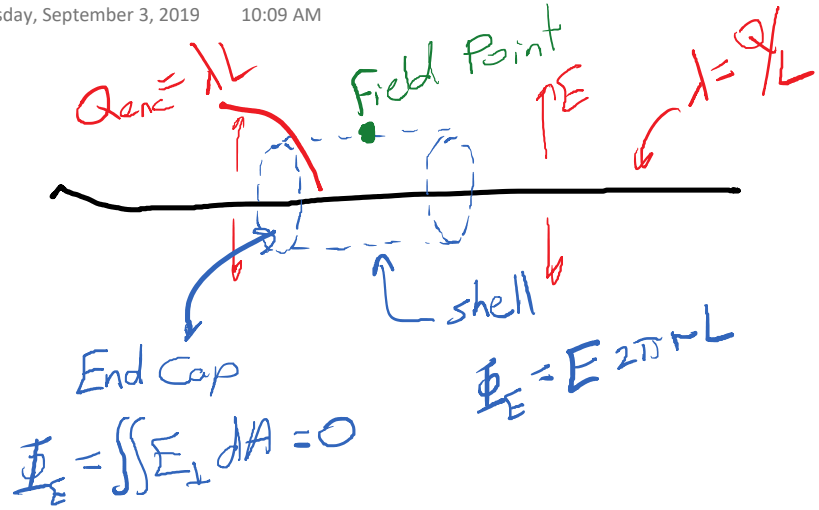

$$\Phi_E = 4\pi k Q_{enc}$$

$$E\, 2\pi r L = 4\pi k \lambda L$$

$$E = \frac{2k\lambda}{r}$$

Spherical shells with Gauss's Law

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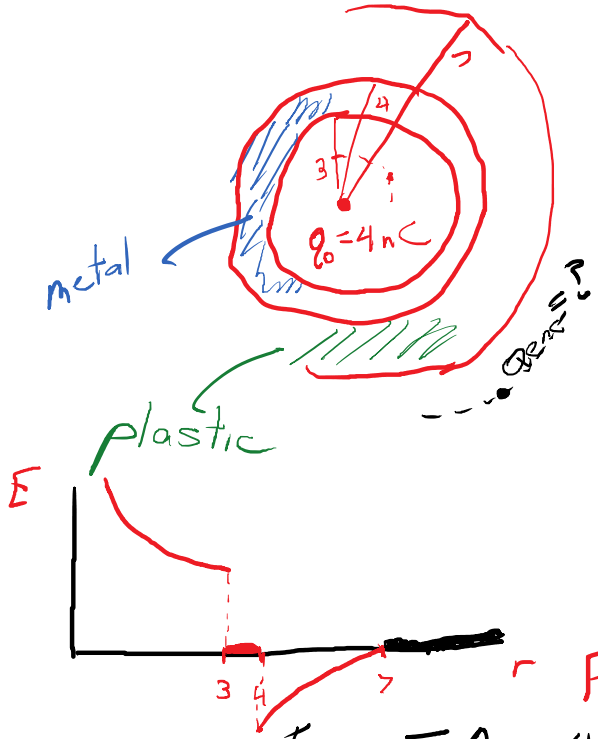


A 4 nC charge is in the center of a thick metal shell, Rinner = 3 cm, Router = 4 cm. The metal shell is coated with a 3 cm layer of insulating material. The metal has a total charge of -7 nC, and the plastic has a total charge of 3 nC. What is the electric field everywhere?

Important fact: E = 0 in electrostatics in the bulk of a conductor.

- If there was E != 0, it would exert force on any charges in the region.
- Conductors allow charges to move freely.
- That's not electrostatic.

For $r > 7$ cm: $Q_{enc} = 4nC - 7nC + 3nC = 0$

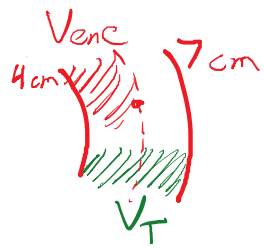
$$\Phi_E = EA = 4\pi k Q_{enc} \Rightarrow E = 0$$

For $r < 3$ cm: $Q_{enc} = 4nC$

$$EA = 4\pi k Q_{enc}$$

$$E 4\pi r^2 = 4\pi k (4nC) \Rightarrow E = \frac{k(4nC)}{r^2}$$

For $4 < r < 7$: $Q_{enc} = 4nC + -7nC + \rho V_{enc}$



$$\rho V_{enc} = \frac{Q V_{enc}}{V_T} = \frac{\frac{4}{3}\pi(r^3 - 0.04^3)(3nC)}{\frac{4}{3}\pi(0.07^3 - 0.04^3)}$$

$$EA = 4\pi k Q_{enc}$$

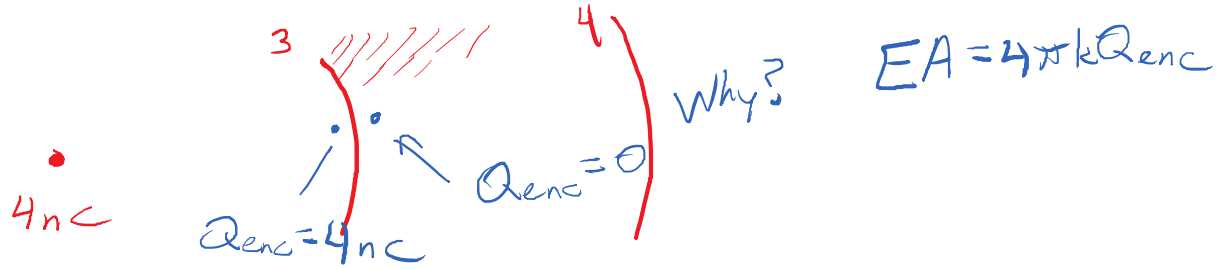
$$E 4\pi r^2 = 4\pi k\left(4nC - 7nC + (3nC)\left(\frac{r^3 - 0.04^3}{0.07^3 - 0.04^3}\right)\right)$$

Note: $r = 0.04$ m: $E = \frac{k(-3nC)}{r^2}$ (Discontinuous)

$r = 0.07$ m: $E = 0$ (Continuous)

It's surface charges that cause discontinuities in the electric field.

It's surface charges that cause discontinuities in the electric field.



The inner surface of the metal in this case must have a -4 nC charge.
The bulk of the metal has zero charge.
Any remaining charge must be on the outer surface.

$$Q_{metal} = Q_{inner} + Q_{bulk} + Q_{outer}$$

$$(-7nC) = (-4nC) + 0 + Q_{outer}$$

$$-3nC = Q_{outer}$$

4. Potential and Capacitors

Thursday, September 5, 2019 9:18 AM

Physics I review: Work $= W = \int \vec{F} \cdot d\vec{l}$

If you push something and it goes in the direction you push it, you give it energy.

Gravity is a good example to use to understand potential energy. Consider a dropped object:

$F_g = mg$

$W_g = \int -mg\, dy$

$= -mg\Delta y$

$U_g = mgy$

$U_i = mgh$

$\Delta U = -mgh$

$U_f = 0$

$K_f = \frac{1}{2}mv_f^2 = mgh$

(Only ⊖ variable is ΔU)

Δy is ⊖ so W_g is ⊕

$W_g = mgh = -\Delta U_g$

$\Delta U_g = -\int \vec{F} \cdot d\vec{l}$

$\vec{F} = -\nabla U_g$

Pure y force $\Delta U_g = -\int F_y\, dy$

$F_y = -\frac{dU_g}{dy}$

With electric forces, we can do something similar.

$$W_E = \int \vec{F}_E \cdot d\vec{l} = \int q_0 \vec{E} \cdot d\vec{l} = q_0 \int \vec{E} \cdot d\vec{l} = -q\Delta V$$

Usually, $\vec{E}$ doesn't depend on what q_0 we use and where we put it, so:

$W = -\Delta U$

$\vec{E}$ is the force per unit charge

V is the energy per unit charge

$-\Delta V$ is the work per unit charge

Many names: electric potential, potential difference, voltage, electromotive force (EMF)

What does electric potential do to our charge q_0?

$$E_x = -\frac{dV}{dx}$$

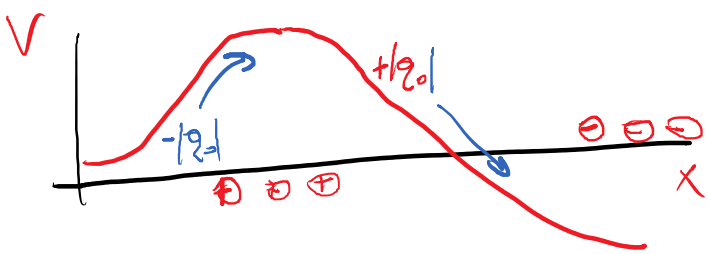
$\vec{E}$ points "downhill" toward lower (or more negative) V

$q_0 = \oplus$ $\quad \vec{F}_E = q_0\vec{E}$ same dir as $\vec{E}$

q_0 attracted to $V = \ominus$

$q_0 = \ominus$ $\quad \vec{F}_E = q_0\vec{E} = -|q_0|\vec{E}$ = opposite to $\vec{E}$

$-|q_0|$ attracted to $V = \oplus$



What causes V? Source charges

- Since +|q_0| is attracted to low V and attracted to negative sources: low (negative) V is caused by negative source charges.

Easiest case: point source charges

$$\vec{E} = \frac{kq_1}{r^2}\hat{r}$$

$$\frac{dV}{dx} = -E \qquad V = -\int E\,dx = -\int \frac{kq_1}{r^2}\,dx = -kq_1\left(\frac{-1}{r}\right) \quad (r = x)$$

$$V = \frac{kq_1}{r}$$

Note: V is a scalar (not a vector)

Just like with calculating "total E", we can calculate "total V" by adding

up the contributions of many sources.

$$V = \int dV = \int \frac{k\,dq}{r}$$

If we know E, we can certainly calculate V.

Using a sphere to store charge

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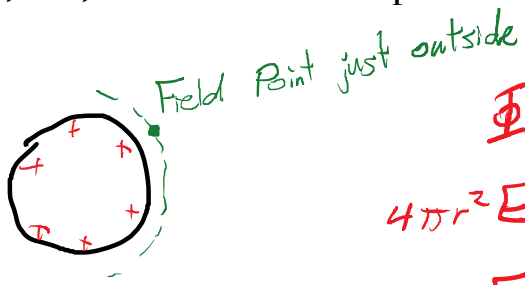
How much charge could we place on a BB?
Limit: E = 1,000,000 N/C causes sparks and discharge.



$\Phi_E = 4\pi k Q_{enc}$

$4\pi r^2 E = 4\pi k Q$

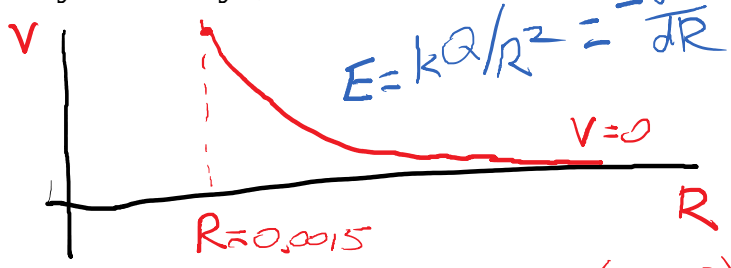
$E = \frac{kQ}{r^2}$

Dia = 3 mm
$R = 0.003/2$

$$Q = \frac{Er^2}{k} = \frac{(1\times10^6)(0.0015)^2}{(9\times10^9)} = 2.5\times10^{-10}\,C = 0.25\,nC$$

What is the electric potential of the BB with this charge?

- "Very far away", V=0



$E = kQ/R^2 = -\frac{dV}{dR}$

$V = \frac{kQ}{R}$ near our BB

At R=0.0015:

$$V = \frac{(9\times10^9)(2.5\times10^{-10})}{(0.0015)} = 1500\ V$$

Summary: A 3 mm diameter BB can hold up to 0.25 nC of charge, at which point it has a potential of 1500 V.

Electric potential is "like height" in the gravity analogy.
Electric potential is "like pressure" in a fluid analogy.

The ratio of charge gathered to voltage needed is called capacitance.

$Q = CV$

$(2.5\times10^{-10}\,C) = C\,(1500\ V)$

$C = \frac{2.5\times10^{-10}}{1500} = 1.67\times10^{-13}\,F$

capacitance

farads

$C = 0.167\ pF$

Parallel Plate Capacitor

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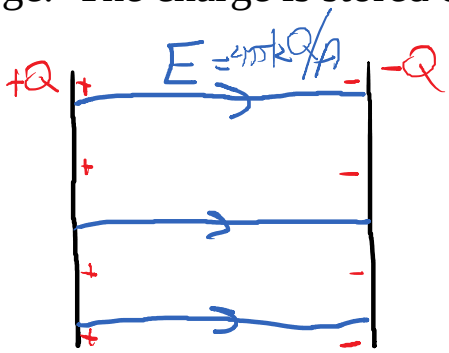
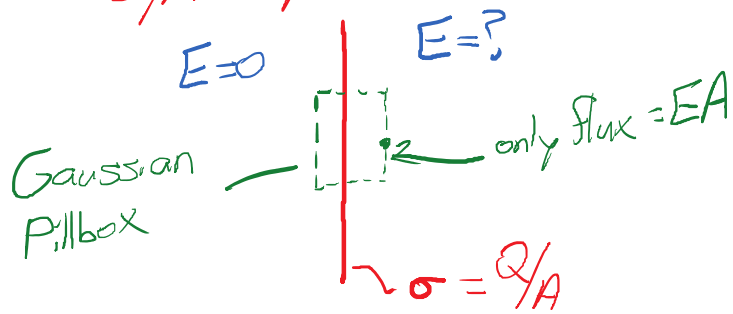
Why was it so difficult (high voltage) to gather even a little charge on the BB? The charges all repel each other.

To overcome this, a capacitor is built to store equal amounts of positive and negative charge. The charge is stored on flat metal surfaces called plates.



How strong is E?

Gauss's Law for Flat (Slab) symmetry:



$$\Phi_E = 4\pi k Q_{enc}$$

$$EA = 4\pi k \sigma A$$

$$E = 4\pi k \sigma$$

$$E = \sigma/\varepsilon_0$$

Note: $k = \frac{1}{4\pi\varepsilon_0}$ $\quad 4\pi k = \frac{1}{\varepsilon_0}$

This is the electric field everywhere inside the capacitor.

Just like $F_g = -mg \rightarrow U_g = mgy$

$$E_c = \frac{\sigma}{\varepsilon_0} \rightarrow V_c = \frac{\sigma d}{\varepsilon_0} = \frac{Qd}{\varepsilon_0 A}$$

$$CV = \left(\frac{\varepsilon_0 A}{d}\right)V = Q$$

So the capacitance is $C = \varepsilon_0 A/d$ $\quad E = \frac{V}{d}$

On the equation sheet: $C = \frac{K \varepsilon_0 A}{d}$ $\quad K$ = dielectric constant

Ex: $A = 1\,m^2$ $\quad K = 1$ $\quad C = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12})(1.0)}{\ldots} = 8.85\times10^{-8}\,F$

Ex: $A = 1\ m^2$ $\kappa = 1$

$d = 0.1\ mm$

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12})(1.0)}{1\times10^{-4}} = 8.85\times10^{-8}\ F$$

$$C = 85000\ pF$$

Capacitors store:

- Charge, Q
- Energy

Since voltage is energy per unit charge, you'd think that you could multiply charge * voltage to get energy.

$$\text{Energy} = q_0 \Delta V \quad \text{if } \Delta V \text{ not due to } q_0$$

$$\text{Self-Energy} = \tfrac{1}{2} Q V \quad \text{if } V \text{ is due to } Q$$

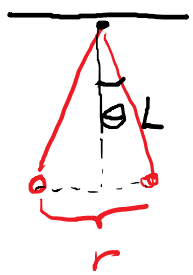
$$V = q/C$$

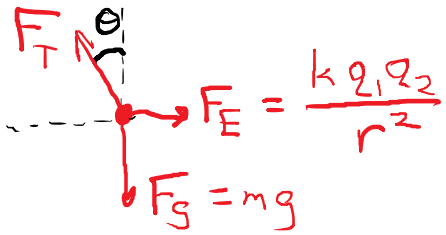
$$\text{Technically: Energy} = \int V\,dq = \int_0^Q \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C}$$

5. V, I, R, P

Tuesday, September 10, 2019 9:22 AM

Electrostatic charge detector





$$x: \frac{kq_1q_2}{r^2} - F_T \sin\theta = 0$$

$$y: F_T \cos\theta - mg = 0$$

$$F_T = \frac{mg}{\cos\theta}$$

$$\frac{1}{2}r = L\sin\theta$$

$$r = 2L\sin\theta$$

$$\frac{kq_1q_2}{r^2} - mg\tan\theta = 0$$

$$\frac{kq_1q_2}{(2L\sin\theta)^2} - mg\tan\theta = 0$$

What if we had to solve for theta? Messy algebra.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos\theta = \sqrt{1-\sin^2\theta}$$

Likely ends in quartic polynomial equation.
Ugh.

V, I, R, P and DC Circuits

Tuesday, September 10, 2019 9:46 AM

V = Voltage — Pressure to push current
I = Current — Energy per unit charge
R = Resistance — Charge flow per unit time
P = Power — Difficulty of pushing current
— Energy per unit time

All DC electrical circuits are formed from closed loops.
Why? Static electricity only allows a few nanocoulombs to flow.

Components are the pieces and parts we use to build circuits.
Wires: Allow current to flow easily.

- Inside a metal, the electric field is zero in electrostatics.
- In a good conductor, the electric field is close to zero.
- The potential difference between any two points of the metal is near zero.

$$\Delta V = -\int E_x \, dx$$

Battery: Source of voltage

- Ideally, generates a fixed amount of voltage (potential difference).

Alkaline: 1.5 V NiMH: 1.2 V Li-Ion: 3.8 V

- The voltage means that there are charges separated from each other.

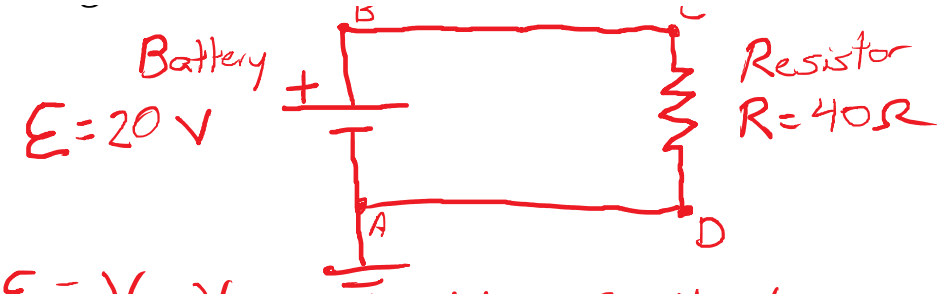
- The battery acts as a self-charging capacitor. As we draw current from the battery, the reactions inside the battery replenish the + and - charges to maintain the voltage. Ideally, we can draw many amperes (coulombs/second).

Resistor: Not-too-easy path for current.

- Resistance (R), measured in ohms (Ω) is the "difficulty".
- Ohm's Law relates a resistor's potential difference to the current.

$$V = IR$$

Basic circuit: Flashlight



$\Delta V_{Batt} = \mathcal{E} = V_B - V_A$

$V_A + \mathcal{E} = V_B$

$V_B = 0 + 20 = 20$

$\Delta V_{wire} = 0 = V_D - V_A$

$V_D = V_A = 0$

$V_C = 20$

$\Delta V_{wire} = V_C - V_B = 0$

Picking a point where V = 0 is called choosing a "ground".

Let A = Ground $\quad V_A = 0$

Connecting the resistor to the battery with two wires is called "applying a voltage to the resistor". It is important to stress that voltage does not flow in a circuit.

$\Delta V_R = \mathcal{E}$

$V = IR$

$(20\ V) = I\,(40\Omega)$

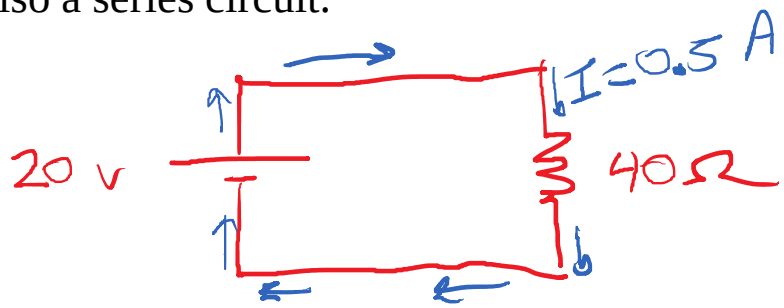
$0.5\ A = I$

amperes

Current

This geometry, where two components are connected at both ends, is called placing them "in parallel". Components in parallel always have the same voltage. If a circuit consists of only parallel components, it's a parallel circuit.

The above circuit is also a series circuit.



A series has the same current flowing through the components.

Power and the cost of energy.

Tuesday, September 10, 2019 10:28 AM

Resistor: $P_{used} = VI$

Battery: $P_{generated} = \mathcal{E} I$

Ex: $P_R = (20\ V)(0.5\ A) = 10\ W$ (used)

$P_B = (20\ V)(0.5\ A) = 10\ W$ (generated)

We pay for energy, not voltage, current, or power.

(cost) = (rate)(amount)

rate $\approx$ \$0.12/kWh

$1\ kWh = (1\ kW)(1\ hour)$
$= (1000\ W)(3600\ s)$
$= 3600000\ J$

rate $\approx \dfrac{\$0.12}{3.6 \times 10^6\ J}$

$\dfrac{1}{rate} = \dfrac{3.6 \times 10^6\ J / 0.12}{\$0.12 / 0.12}$

$= 30{,}000{,}000\ J/\$$

The main complicating factors in determining the cost of doing something are:

- Efficiency
- Duty cycle - how often are you doing this?

How does electricity compare to gasoline?

1 gallon $\Rightarrow$ 33.7 kWh of thermal energy

rate $= \dfrac{\$2.15}{33.7\ kWh} = \$0.064/kWh$

If your goal is to make heat, gasoline is much cheaper.

If your goal is to do work, there's an engine efficiency to consider.

car engine ~ 25% - 33%

power plant ~ 45%

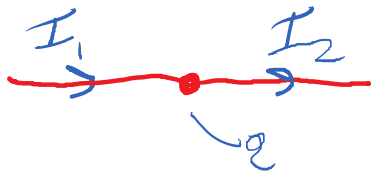
6. Series and Parallel

Thursday, September 12, 2019 9:19 AM

Two fundamental circuit principles:

- Charge conservation: Charge can't "build up" anywhere in a circuit.
- Energy conservation: As a charge moves around a loop, its energy gets back to the original value (otherwise energy would "build up" somewhere).

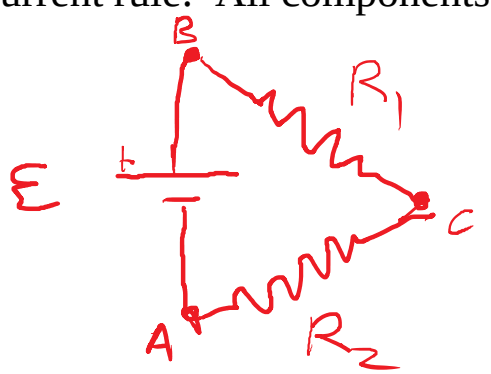
$$I_1 = \frac{dq}{dt} = 0$$



$$I_1 - I_2 = \frac{dq}{dt} = 0$$

$$I_1 = I_2$$

Series current rule: All components in a series have the same current.



Calculate $V_B - V_A$:

Thru Batt: $V_B - V_A = \mathcal{E}$

Thru R's: $V_B - V_C + V_C - V_A$

$$V_B - V_A = V_{R1} + V_{R2}$$

Result: $\mathcal{E} = V_{R1} + V_{R2}$

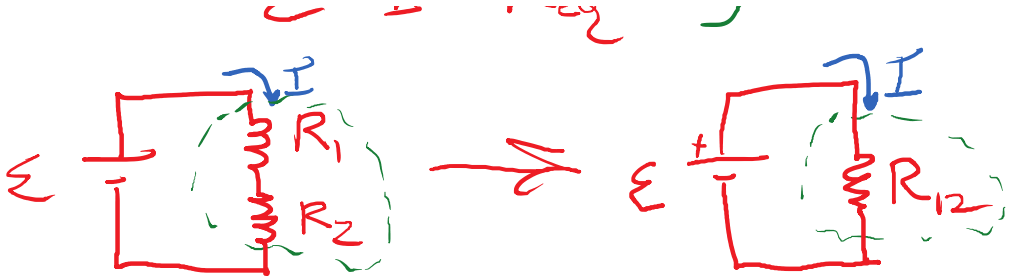
Series voltage rule: Voltages add in series.

Equivalent Resistance: From the battery's perspective, it just knows that something is connected to point A and B that allows current to flow.

$$\mathcal{E} = I_1 R_1 + I_2 R_2$$

$$= I R_1 + I R_2$$

$$\left.\begin{aligned} \mathcal{E} &= I(R_1 + R_2) \\ \mathcal{E} &= I \; R_{eq} \end{aligned}\right\} \quad R_{eq} = R_{12} = R_1 + R_2$$

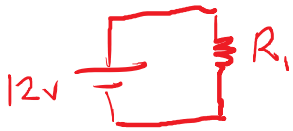


By analyzing the simpler equivalent, we can learn about the original circuit:

- Any value outside of the substitution doesn't change.
- The voltage of the equivalent resistor is the total voltage of the constituents.

$$V_{12} = V_1 + V_2$$

Ex: Two light bulbs are designed to be hooked to a 12 V battery, separately. One is a 12 W bulb, and the other is a 6 W bulb. If they are hooked up in series, which one is brighter?



$P_1 = 12\,W$

$P_1 = V_1 I_1$

$(12\,W) = (12\,v) I_1$

$1.0\,A = I_1$

$V_1 = I_1 R_1$

$12\,v = (1.0\,A) R_1$

$12\,\Omega = R_1$

$P_2 = V_2 I_2$

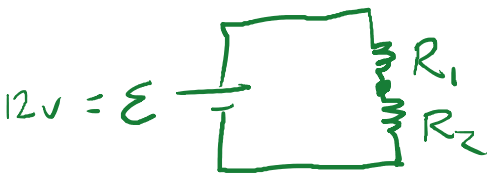
$6\,W = (12\,v) I_2$

$0.5\,A = I_2$

$V_2 = I_2 R_2$

$(12\,v) = (0.5\,A) R_2$

$24\,\Omega = R_2$



$\varepsilon = I(R_1 + R_2)$

$12\,v = I \; 36\,\Omega$

$0.333\,A = I$

$V_1 = IR_1 = (0.333)(12) = 4\,v$

$V_2 = (0.333)(24) = 8\,v$

$P_1 = IV_1$

$P_2 = IV_2$ ← more

Bulb 2 will be brighter, as it uses more power.
This was the 6 W bulb!

$P_2 = (0.333)(8\,v)$

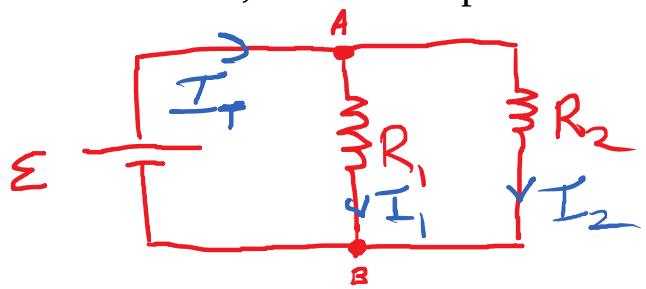
$= 2.66\,W$

The 6 W bulb will only use 2.66 W in the series circuit.

Parallel Circuits

Thursday, September 12, 2019 10:01 AM

In a parallel circuit, the current path branches and merges.



$$I_T = I_1 + I_2$$

$$V_{eq} = V_1 = V_2 = \dots$$

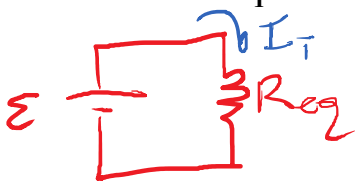
Calculate $V_A - V_B$:

Batt: $V_A - V_B = \mathcal{E}$

R_1: $V_A - V_B = V_1 = I_1 R_1$

R_2: $V_A - V_B = V_2 = I_2 R_2$

What about an equivalent resistance?



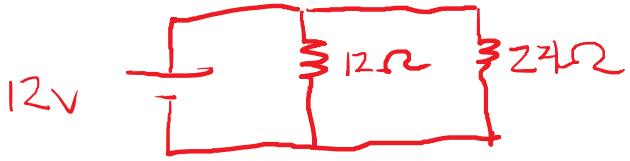
$$I_T = I_1 + I_2$$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2}$$

no subscript needed on V

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots \qquad R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots\right)^{-1}$$

What happens to those 12 W and 6 W bulbs when connected in parallel to a 12 V battery?



$$R_{eq} = \left(\frac{1}{12} + \frac{1}{24}\right)^{-1}$$

$$R_{eq} = \left(\frac{3}{24}\right)^{-1} = 8\Omega$$

(Parallel R_{eq} always less than lowest R_i)

$$I_T = \frac{12V}{8\Omega} = 1.5\,A$$

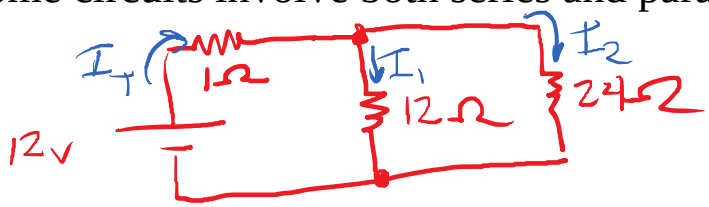
$$\left.\begin{aligned} I_1 &= \frac{12V}{12\Omega} = 1.0\,A \\ I_2 &= \frac{12V}{24\Omega} = 0.5\,A \end{aligned}\right\} I_T = 1.5\,A$$

The powers here would be the same as if they were connected individually. The bulbs in parallel don't "know" that the other bulb exists.

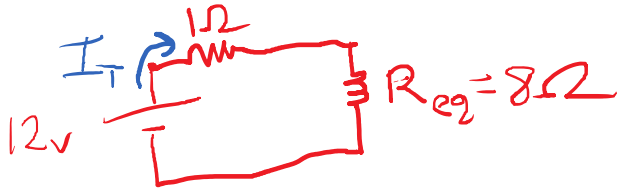
Combination Circuit

Thursday, September 12, 2019 10:18 AM

Some circuits involve both series and parallel parts.



The 1Ω models "internal resistance".



Substitute parallel resistors for their equivalent. (Calc on prev. page)

$$I_T = \frac{12v}{(1\Omega + 8\Omega)} = 1.33A$$

$V_{1\Omega} = 1.33$ V

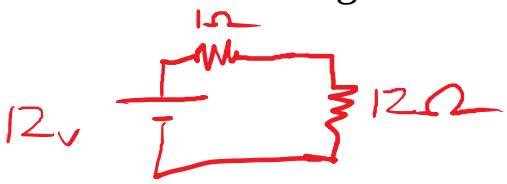
$V_{eq} = 12 - 1.33 = 10.67$ V

In orig circuit: $I_1 = \frac{10.67v}{12\Omega} = 0.89A$ $I_2 = \frac{10.67}{24} = 0.44A$

$I_T = 0.89 + 0.44 = 1.33$ ✓

$P_1 = (10.67)(0.89) = 9.5$ W

It wouldn't be fair to compare this to the 12 W design value.
Let's see what the bulb gets when connected to this battery.



$$I = \frac{12v}{13\Omega} = 0.923 \text{ A}$$

$$P_1 = V_1 I_1 = I_1^2 R_1 = (0.923)^2(12)$$

$$= 10.2 \text{ W}$$

12 ohm bulb connected to:

- 12 V ideal battery: P = 12 W
- 12 V real battery: P = 10.2 W
- 12 V real battery with other load: P = 9.5 W

Since the internal resistance is in series with the EMF, we often write this as:

$$\mathcal{E} = V_{int} + V_{ext}$$

$$\mathcal{E} = IR_{int} + V_{terminal}$$

$$(12\,V) = (1.33\,A)R_{int} + (10.67\,V)$$

$$1.33\,V = (1.33\,A)R_{int}$$

$$1.0\,\Omega = R_{int}$$

Keeping track of information:

Desc	V	I	R	P?
R_1	10.67V		12Ω	
R_2	10.67		22Ω	
R_{int}	1.33 V	1.33A	1Ω	
Total	12 V	1.33A	9Ω	
R_{12}	10.67V	1.33A	8Ω	

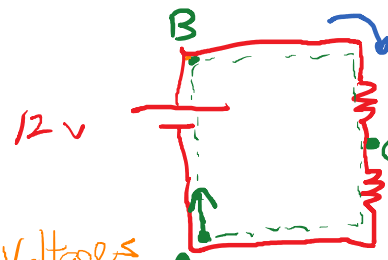
7. Kirchhoff's Laws

Tuesday, September 17, 2019 9:20 AM

(Useful Website: All About Circuits, Education section)

Kirchhoff's Current Law: At any node: $\sum I_{in} = \sum I_{out}$

Kirchhoff's Voltage Law: For any loop: $\sum \Delta V = 0$



$I = \frac{12v}{2\Omega + 4\Omega} = 2.0\,A$

$V_2 = IR = (2A)(2\Omega) = 4\,V$

$V_4 = (2A)(4\Omega) = 8\,V$

$+12v - 4v - 8v = 0$

$\sum \Delta V = 0$

$\mathcal{E} - IR_2 - IR_4 = 0$

$\mathcal{E} = IR_2 + IR_4$

Node Voltages

$V_B - V_A = 12$

$V_B - V_C = 2I$

$V_C - V_A = 4I$

At C: $\frac{V_B - V_C}{2} = \frac{V_C - V_A}{4}$

$V_{AB} + V_{BC} + V_{CA} = 0$

Alternate Form: $\sum \mathcal{E} = \sum IR$

"Voltage Drops"

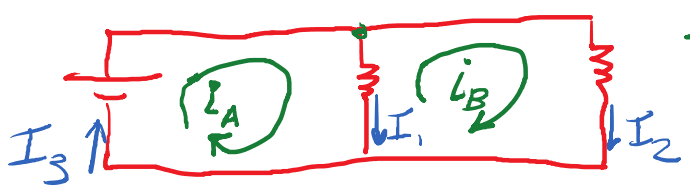
How do we apply these laws? Choose variables to track.

- Branch Current Analysis: Each branch gets a variable for the current.
- Mesh Current Analysis: Each inner loop gets a variable for the sub-current.

$I_3 = I_1 + I_2$

$\mathcal{E} = I_1 R_1$

$\mathcal{E} = I_2 R_2$



$\cancel{i_A + i_B = i_A + i_B \text{ (Nuts)}}$

$\mathcal{E} = (i_A - i_B) R_1$

$\mathcal{E} = i_B R_2$

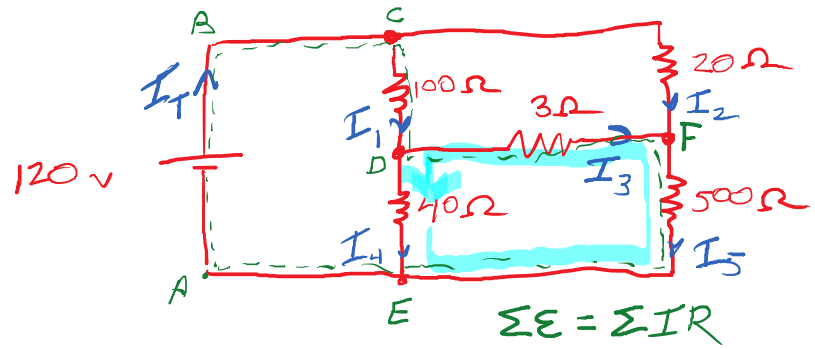
Mesh current analysis is popular, but tricky, so I won't use it.
Also, the mesh current variables aren't physical currents.

- Node Voltage Analysis: Each node has a voltage.
- Component Voltage Analysis? I made this up for completeness.

The above examples were series and parallel.
Are there cases when we NEED a more robust method?

Bridge Circuit

Tuesday, September 17, 2019 10:00 AM



$I_T = I_1 + I_2$

$I_1 = I_3 + I_4$

$I_2 + I_3 = I_5$

$\Sigma \mathcal{E} = \Sigma IR$

ABCDE: $120 = I_1 100 + I_4 40$ (1)

ABCFE: (outer) $120 = I_2 20 + I_5 500$

ABCDFE: $120 = I_1 100 + I_3 3 + I_5 500$ (2)

(1)−(2):

$0 = 40 I_4 - 3 I_3 - 500 I_5$

Loop DEF

Rephrase as linear algebraic equations:

$$0 = I_1 + I_2 - I_T$$
$$0 = -I_1 + I_3 + I_4$$
$$0 = -I_2 - I_3 + I_5$$
$$120 = 100 I_1 + 40 I_4$$
$$120 = 20 I_2 + 500 I_5$$
$$120 = 100 I_1 + 3 I_3 + 500 I_5$$

Matrix form for calculator:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 120 \\ 120 \\ 120 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & -1 \\ -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ 100 & 0 & 0 & 40 & 0 & 0 \\ 0 & 20 & 0 & 0 & 500 & 0 \\ 100 & 0 & 3 & 0 & 500 & 0 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_T \end{pmatrix}$$

Wolfram Alpha:

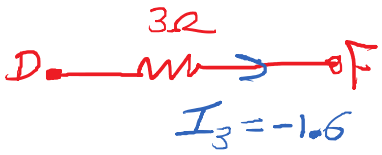
$a \approx 0.40104$, $b \approx 1.7657$, $c \approx -1.5964$, $d \approx 1.9974$, $f \approx 0.16937$, $x \approx 2.1668$

I_1 I_2 I_3 I_4 I_5 I_T

I_3 came out negative.

"The current in R_3 is 1.6 A flowing from right to left."

What is V_F - V_D?



$$\Delta V_{DF} = -I_3 R_3 = -(-1.6A)(3\Omega) = +4.8\ V$$

$$\Delta V_{CF} = -I_2 R_2 = -(1.77A)(20\Omega) = -35.4\ V$$

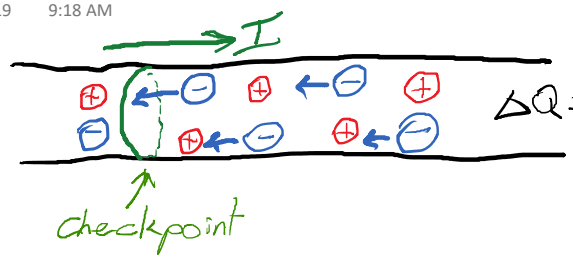
$$\Delta V_{DE} = -I_4 R_4 = -(2.00)(40\Omega) = -80\ V$$

$$\Delta V_{CFDE} = -35.4 - 4.8 - 80 = -120 = (-\mathcal{E})$$

8. Microscopic Current and RC Circuits

Thursday, September 19, 2019 9:18 AM

Reminder: Exam 1 Thursday.



$\Delta Q = \oplus$ means increasing positive imbalance.

$\Delta Q = \Delta N\, q$

$I = \frac{\Delta Q}{\Delta t}$

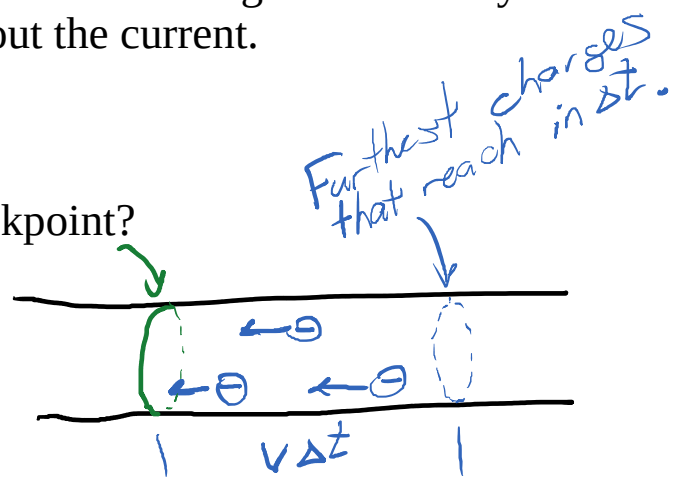
Change in the number of q charges.
In solids, $q = -e$.

To get $\Delta Q = \oplus$, need $\Delta N = \ominus$

The current pointing rightward makes Delta-Q positive.
To make Delta-Q positive, we could have electrons move leftward.
Most electrical currents are formed from electrons moving the other way.
Usually, we don't care and we just think about the current.

$$I\Delta t = \Delta Q = \Delta N q$$

How many individual charges pass the checkpoint?



All of the electrons in a section of wire of length $v\Delta t$ reach the checkpoint.

Each material has a certain density of conduction electrons. Often 1 per atom.

$$n = \frac{\#\text{ cond electrons}}{\text{Volume}}$$

$$\Delta N = n(A v \Delta t)$$

$$I\cancel{\Delta t} = nAv\cancel{\Delta t}\, q$$

$$I = nAv_d q$$

Drift Velocity

Example: Copper wire

Mass Density $\rho = 8.92\ g/cm^3 = 8920\ kg/m^3$

Atomic Mass $m = 63.5\ u = 1.05 \times 10^{-25}\ kg$

$1u = 1.66 \times 10^{-27}\ kg$

Density of Atoms $n = \frac{\rho}{m} = 8.41 \times 10^{28}\ atoms/m^3$

Density of Atoms $n = \frac{\rho}{m} = 8.41 \times 10^{28}$ atoms/m^3

Sample wire: $2mm = \phi = dia$ $A = 3 \times 10^{-6}\ m^2$

$I = 10.0\ A$ $q = -1.6 \times 10^{-19}\ C$

$I = nAvq$

$(10.0) = (8.41 \times 10^{28})(3 \times 10^{-6})\, v\, (-1.6 \times 10^{-19}\ C)$

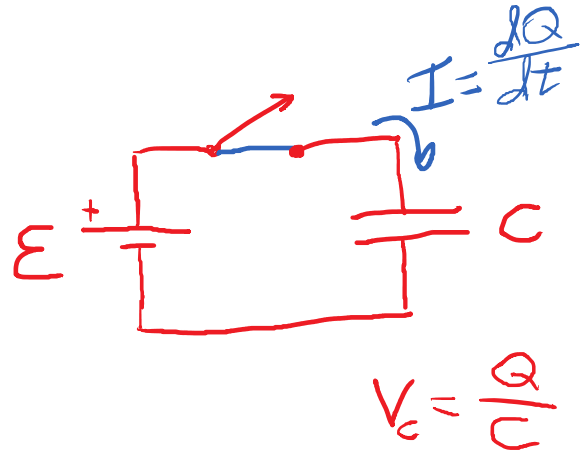
$[C/s] = [1/m^3]\ [m^2]\, v\ [C]$

$v = -2.5 \times 10^{-4}\ m/s = -0.25\ mm/s$

RC Circuits

Thursday, September 19, 2019 10:01 AM

RC Circuits are about HOW a capacitor gets charged and discharged.



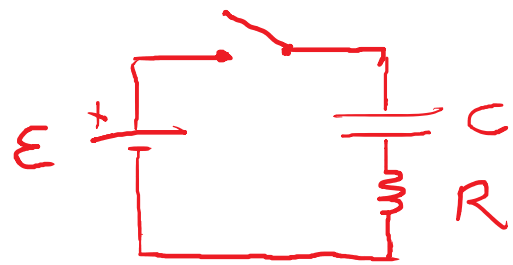
If we place a battery and empty capacitor together, and close the switch, the capacitor IMMEDIATELY wants to match the voltage of the battery.

That requires increasing Q very quickly.
That requires a HUGE current, I.

Even if we have "really good" wires, they will have some resistance.
This limits the amount of current.
Our circuit wasn't realistic, and must be updated.

$V_R = IR$

This is called an RC circuit.



Kirchhoff's Voltage Law

$$\mathcal{E} = V_R + V_C$$

$$\mathcal{E} = IR + Q/C$$

$$\mathcal{E} = \frac{dQ}{dt}R + \frac{1}{C}Q$$

Differential Equation

$t=0$: $Q=0$ $I = \mathcal{E}/R$

$t=1$: $Q=$(more) $I=$(less)

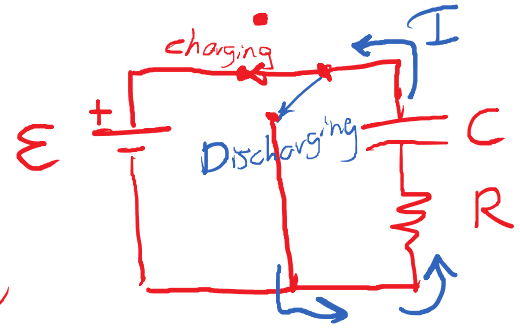
$t=\infty$: $Q=C\mathcal{E}$ $I=0$

The charge increases quickly at first, but flattens out to a stable final value.

Let's think about a discharging capacitor.

charging

Let's think about a discharging capacitor.

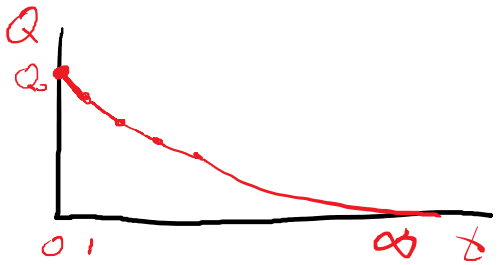


$$\mathcal{E} = V_C + V_R$$

$$0 = \frac{1}{C}Q + \left(+\frac{dQ}{dt}\right)R$$

One ⊖ : Kirchhoff's Law

One ⊖ ; Current Discharges



$t=0$ $\quad Q=Q_0$ $\quad I=-\frac{dQ}{dt}=\frac{Q_0}{RC}$

$t=1$ $\quad Q=(\text{less})$ $\quad I=(\text{less})$

$t=\infty$ $\quad Q=0$ $\quad I=0$

This looks like an exponential. Is it?

$$-\frac{dQ}{dt}R = \frac{1}{C}Q$$

$$\frac{dQ}{dt} = \frac{-1}{RC}Q$$

$$\frac{-1}{\tau}Q_0 e^{-t/\tau} = \frac{-1}{RC}Q_0 e^{-t/\tau}$$

Let $Q = Q_0 e^{-t/\tau}$

$$\frac{dQ}{dt} = Q_0 e^{-t/\tau}\left(\frac{-1}{\tau}\right)$$

Works if $\tau = RC$

The discharging capacitor discharges exponentially with the same time constant, no matter what initial charge it had.

The exponential function (exp(-t/tau)) tells us what fraction of the original charge remains.

t	$e^{-t/\tau}$	
0.0	1.0	100%
τ	0.37	37%
2τ	0.14	14%
3τ	0.05	5%
4τ	0.02	2%
5τ	0.006	<1%

By the time t = 5tau, the process is 99% finished.

At what point is the process half complete?

$$e^{-t/\tau} = 0.5$$

$-t/\tau = \ln(0.5) = -0.693$

$t = 0.693\,\tau$

For a discharging capacitor:

- Q_0 is the initial charge
- V_0 = Q_0 / C is the initial voltage
- I_0 = V_0 / R is the initial current

} All Decay Exponentially.

An unknown capacitor is charged up to 15 V and attached to a 2 k-ohm resistor. After 3.0 s, the voltage is measured to be 0.75 V. What is the capacitance?

$V = V_0 e^{-t/\tau}$

$(0.75\,V) = (15\,V)\, e^{-t/\tau}$

$\frac{0.75}{15} = e^{-t/\tau}$

$\ln(0.05) = \ln(0.75/15) = -t/\tau$

$-2.996 = -t/\tau$

$\tau = \frac{t}{2.996} = \frac{3.0}{2.996}$

$\tau = 1.0\ s$

Note: $\ln(15/0.75) = +2.996$

$\tau = RC$

$(1.0\ s) = (2000\ \Omega)\, C$

$0.0005\ F = C$

$500\ \mu F = C$

Charging capacitor is the same, except the charge "exponentially approaches a constant".

$Q_f - Q = Q_f e^{-t/\tau}$

$Q_f - Q_f e^{-t/\tau} = Q$

$Q_f(1 - e^{-t/\tau}) = Q$

$1 - e^{-t/\tau} = \frac{Q}{Q_f}$

Full Charge — % empty — % charged

full charge % empty % charged

9. Review

Tuesday, September 24, 2019 9:27 AM

Exam 1 Thursday. Bring:

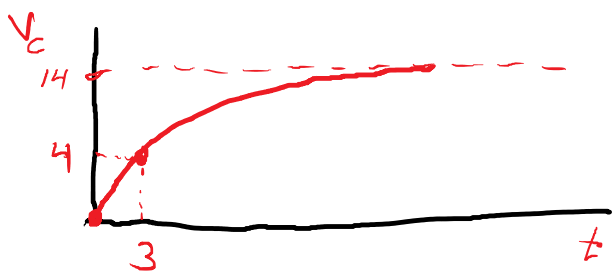
- Pencil
- Non-internet calculator

I will provide:

- Exam, equation sheet, scantron

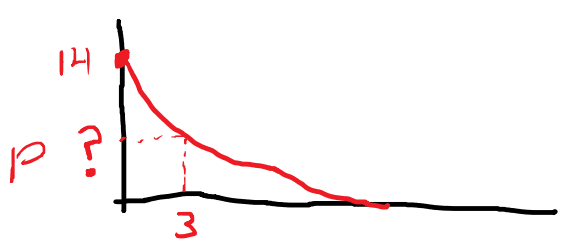
Topics: Electric Fields (Chap 22) thru DC Circuits (Chap 27)
(RC Circuits are in Chap 27)

RC Circuit Example (HW2-13)
A 7 micro-F capacitor charged by 14 V battery reaches 4 V in 3 s.
What is the resistance in the circuit?



$$V_c = V_f\left(1 - e^{-t/\tau}\right)$$
$$4 = 14\left(1 - e^{-t/\tau}\right)$$
$$4/14 = 1 - e^{-t/\tau}$$
$$e^{-t/\tau} = 1 - 4/14 = 10/14$$
$$-t/\tau = -0.336$$
$$RC = 8.92 = \frac{-3}{-0.336} = \tau$$

What if this (fully charged to 14 V) capacitor is discharged through the same resistor. What will the voltage be after 3.0 s?



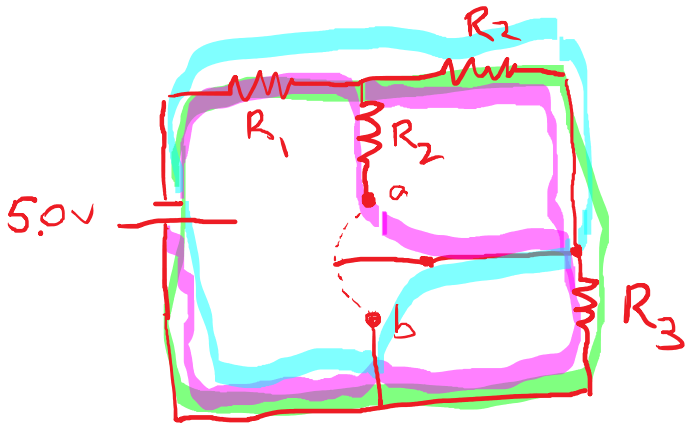
$$V = V_0 e^{-t/\tau} = V_0 e^{-3/8.92}$$
$$= V_0 e^{-0.336}$$
$$= V_0 \left(10/14\right)$$
$$V = 14\left(10/14\right)$$
$$V = 10$$

The charging capacitor gains 4.0 V out of 14 in the first 3 seconds,

and the discharging capacitor loses 4.0 V out of 14 in the same time.

Changing Resistor Circuit (HW2-14)

Tuesday, September 24, 2019 9:52 AM



open: $I_T = 1.08$ mA

a: $I_T = 1.24$ mA

b: $I_T = 2.10$ mA

open: $R_{eq} = \frac{5}{1.08} = 4.630$ kΩ

a: $R_{eq} = 4.032$ kΩ

b: $R_{eq} = 2.381$ kΩ

$\frac{1}{2}R_2 = 4.630 - 4.032$

$R_3 = 4.630 - 2.381$

Resistors in series: Add resistances to get equivalent.

Equal resistors in parallel:

$$R_{eq} = \left(\frac{1}{R_2} + \frac{1}{R_2} + \dots\right)^{-1} = \left(\frac{n}{R_2}\right)^{-1}$$

$$R_{eq} = R_2/n$$

Common but unnecessary simplification:

R_A and R_B in parallel

$$R_{AB} = \left(\frac{1}{R_A} + \frac{1}{R_B}\right)^{-1}$$

$$= \left(\frac{R_B + R_A}{R_A R_B}\right)^{-1}$$

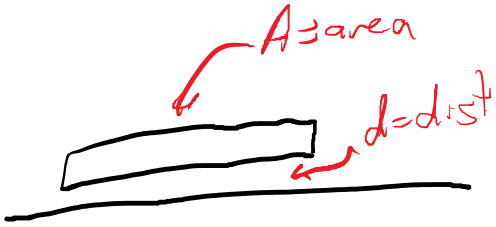
$$R_{eq} = \frac{R_A R_B}{R_A + R_B}$$

$\frac{R_A}{R_A R_B}$

Physical Capacitor

Tuesday, September 24, 2019 10:10 AM

$$C = \frac{K \varepsilon_0 A}{d}$$



K = "Kappa" = Dielectric Constant

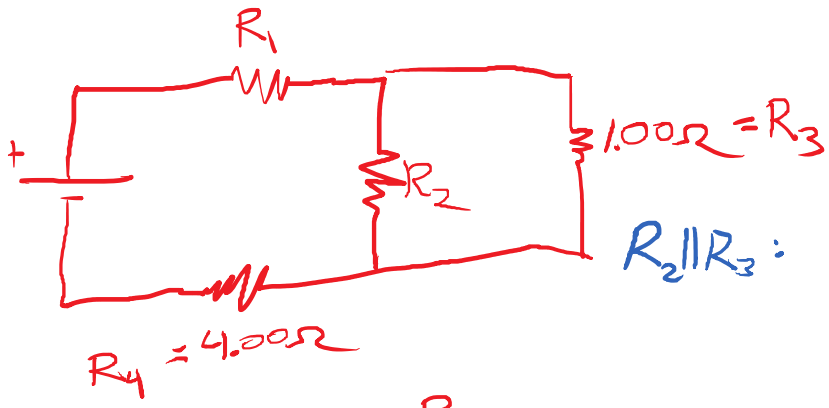
$K_{air} = 1.0$

$\varepsilon_0 = 8.85 \times 10^{-12}$

A = Overlap Area

d = distance between plates

$E = \frac{V}{d}$ in a capacitor



$R_2 \| R_3$: $R_{eq} = \left(\frac{1}{2} + \frac{1}{1}\right)^{-1}$

$= \left(\frac{3}{2}\right)^{-1}$

$R_{eq} = 0.667$

	V	I	R	P
R_1			3.0	
R_2			2.0	
R_3			1.0	
$R_2 \| R_3$			0.667	
R_4			4.0	
Batt	21.0		7.667	

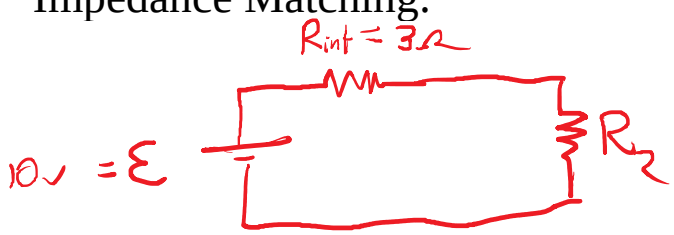
□ = Total Current

○ = R_2 or R_3 Voltage

Resistor Power:

$$P = IV = I^2R = \frac{V^2}{R}$$

Impedance Matching:



What is the Power used by R_2?

$$P_2 = I^2 R_2$$

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{10}{3 + R_2}$$

$$P_2 = \left(\frac{10}{3+R_2}\right)^2 R_2$$

R_2	P_2
0	0
0.5	4.08
1.0	6.25
2.0	8
3.0	8.33 ← Peak
4.0	8.16

(10 / 7)^2 * 4 = 8.1633

With a given internal resistance, the most power is

4.0 0.16

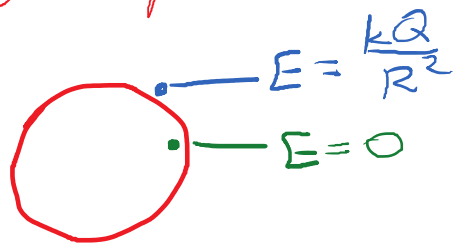
With a given internal resistance, the most power is transferred when the load equals the internal resistance.

$$P = \left(\frac{10}{3+x}\right)^2 x$$

With a given load resistance, the most power is transferred when the internal resistance is zero.

$$P = \left(\frac{10}{x+3}\right)^2 3$$

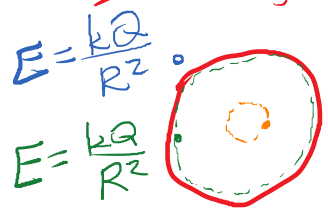
Metal Sphere



Q on sphere

Charge on Surface,
E jumps in value

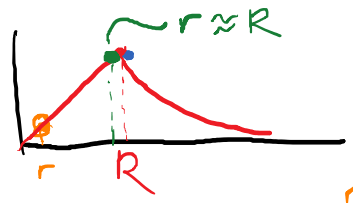
Insulating Sphere



Q = uniformly distributed

Just inside: $Q_{enc} \approx Q$

Charge spread in volume,
E continuous



$Q_{enc} = \frac{Q r^3}{R^3}$

$$E = \frac{k Q_{enc}}{r^2}$$

$$= \frac{k Q r}{R^3}$$