Why study Electricity and Magnetism (E&M)?

- Our society is electric
  - Energy
  - Information: Communication, Storage, Computation, Control, Measurement
- Basis of light and radio waves.
- Basis of Chemistry
- Practice with math and "learning new things".
Study of stationary charges. What are charges?

All matter is made of atoms, which are in turn made of protons, electrons, and neutrons. These are particles that interact electrically.

"Charge" is a measure of how electric something is.

A "charge" is an object or particle that has charge.

\[
\begin{align*}
\text{Proton:} & \quad q_p = 1.6 \times 10^{-19} \text{ C} = +e \\
\text{Electron:} & \quad q_e = -1.6 \times 10^{-19} \text{ C} = -e \\
\text{Neutron:} & \quad q_n = 0
\end{align*}
\]

How much charge is in a 2 gram aluminum piece?

\[
\begin{align*}
m_{\text{Al}} &= 2 \text{ g} = 2 \left( 1.66 \times 10^{-27} \text{ kg} \right) \\
&= 4.48 \times 10^{-26} \text{ kg} \\
N &= \frac{m}{m} = \frac{0.002 \text{ kg}}{4.48 \times 10^{-26} \text{ kg}} = 4.46 \times 10^{22}
\end{align*}
\]

Each aluminum atom has 13 protons and 13 electrons.

\[
\begin{align*}
Q_p &= N \cdot q_p = (13) \left( 4.46 \times 10^{22} \right) \left( 1.6 \times 10^{-19} \text{ C} \right) \\
&= 92800 \text{ C} \\
Q_e &= -92800 \text{ C} \\
Q_T &= 0
\end{align*}
\]

We can "charge" the object by adding or removing charges (the particles).
• In solids, the protons are part of the nuclei, which stay in place.
• Some electrons are free to move around. Typically 1 per atom.
• In liquids, ions (nuclei without enough electrons to balance) can move around.

As it turns out, we can move 1 electron per atom, but we cannot steal one electron per atom. The amount we could steal is only around 1 nC worth.
• Charges exert forces on each other.
• How? Charges generate "electric field". The electric field is what causes the force on other charges.

\[ q_1 = +8 \text{ nC} \quad q_2 = -5 \text{ nC} \]

\( q_1 \) is our "source" which generates electric field.

\[ E_1 = \frac{k|q_2|}{r^2} \quad F_2 = |q_2| E \]

Electric field (E) is measured in newtons per coulomb (N/C).

\[ F_2 = \left(9 \times 10^9 \frac{N \cdot m^2}{C^2}\right) \frac{(8 \times 10^{-9} C)(5 \times 10^{-9} C)}{(0.25 \text{ m})^2} = 5.76 \times 10^{-6} \text{ N} \]

What is the direction of the force?
• Opposites attract
• Like charges repel

Is there a way to get the + and - to work out in the equation, automatically?

\[ F_2 = \frac{k|q_2|}{r^2} \quad \Rightarrow F_2 = \frac{kq_1q_2}{r^2}(+) \]

This comes out negative for the above calculation, which is exactly what we want. Does this work for \( F_1 \)?

\[ F_1 = \frac{kq_1q_2}{r^2}(-) \text{ should be } +\]

We need something extra at the end of a generic formula, to set the direction of the force vector.
The extra factor is the relative direction, from the source, pointing toward the object we're studying.

\[ F = \frac{k q_1 q_2}{r^2} \hat{r} \]

\[ \hat{r} \text{ for } q_1 \text{ points toward } q_1 \text{ from } q_2 \]
Electric field describes how electrified space is. They are caused by charges. 
(Also caused by fluctuating magnetic fields.)

Electric fields cause forces on charges.
Put a charge \( q_0 \) at a location in space that has an electric field \( E \) it will feel the force \( F \).

The direction of \( E \) is the same as the direction of \( F \), if the charge \( q_0 \) is positive. Any \( q_0 \) experiences the same \( E \), but they experience different \( F \).
- If \( q_0 \) is negative, its force is in the opposite direction.

For a positive source, the electric field points away from the source.

How much electric field is present?

\[
E = \frac{k|q_1|}{r^2} = \frac{1}{2d} \quad E = \frac{k|q_1|}{r^2}
\]

This is the magnitude of \( E \) due to \( q_1 \) (the source charge).
These are two separate electric fields, in two different situations, where the other charge is thought of as the "source".

\[ F = (8 \times 10^{-9}) (720) \]
\[ = 5.76 \times 10^{-6} \text{ N} \]
\[ \rightarrow \text{(with E)} \]

\[ F = (5 \times 10^{-9}) (1152) \]
\[ = 5.76 \times 10^{-6} \text{ N} \]
\[ \rightarrow \text{(against E)} \]
Multiple charges can contribute toward $E$.

How would we calculate the electric field at the upper point?

$$E_1 = \frac{kq_1}{r_1^2} = \frac{(9 \times 10^9)(4 \times 10^{-9})}{(0.05)^2} = 14400 \text{ N/C}$$

$$E_2 = 3600 \text{ N/C}$$

$$E_{1x} = E_1 \cos \Theta_1 = (14400)(\frac{4}{5}) = 11520 \text{ N/C}$$

$$E_{2x} = E_2 \cos \Theta_2 = -(3600)(\frac{4}{5}) = -2880 \text{ N/C}$$

$$E_{1y} = E_1 \sin \Theta_1 = 14400(\frac{3}{5}) = 8640 \text{ N/C}$$

$$E_{2y} = E_2 \sin \Theta_2 = 3600(\frac{3}{5}) = 2160 \text{ N/C}$$

$$E_x = 8640 \text{ N/C}$$

$$E_y = 10800 \text{ N/C}$$

Where would the total electric field be zero?

$$E_1 + E_2 = 0$$

$$E_1 = -E_2$$

$$E_1 = E_2$$ (Same magnitude)

$$\hat{E}_1 = -\hat{E}_2$$ (Opposite direction)

$$E_1 = E_2$$

$$\frac{k|q_1|}{r_1^2} = \frac{k|q_2|}{r_2^2}$$
Ex: \( d = 8 \text{ cm} \)

\[ q_1 = 4 \text{ nC} \]
\[ q_2 = 1 \text{ nC} \]

\[ E = \frac{kq}{r^2} \]

\[ \frac{k(4 \text{ nC})}{x^2} = \frac{k(1 \text{ nC})}{(d-x)^2} \]

\[ \frac{2}{x} = \frac{1}{d-x} \]
\[ 2d - 2x = x \]
\[ 2d = 3x \]
\[ \frac{2}{3} d = x \]

\[ \frac{2}{3} \text{ d} \]
\[ \frac{1}{3} \text{ d} \]

\[ 4 \text{ nC} \]
\[ 1 \text{ nC} \]

\[ E = 0 \]
Continuous charge distributions

Line Charge: \( \lambda = \text{charge per length} \)
\( = \text{linear charge density} \)

\( Q = \lambda L \) (if \( \lambda \) constant)

\( Q = \int \lambda \, dx \) (IF along x-axis)

\( Q = \int \lambda \, dl \) Arbitrary curve

\( Q = \int dq \) Lots of little bits

What is the electric field due to a continuous charge distribution?

\[
\overrightarrow{E} = \int \overrightarrow{E} = \int \frac{k \, dq}{r^2} \hat{r}
\]

\( dq = \lambda \, dl = \lambda \, dx \)

\( r^2 = x^2 + y^2 \)

\( \hat{r} = \frac{\overrightarrow{r}}{r} = \frac{-x^\wedge + y^\wedge}{\sqrt{x^2 + y^2}} \)

\[
E_x = \int \frac{-k \lambda x}{(x^2 + y^2)^{3/2}} \, dx
\]

\( = 0 \)

\[
E_y = \int \frac{k \lambda y}{(x^2 + y^2)^{3/2}} \, dx
\]

\( = \frac{2k\lambda}{y} \)
Alternative to integration method of finding $E$.

Integration method:

\[
\begin{align*}
\Phi_E &= \iiint \vec{E} \cdot d\vec{A} = \iiint \vec{E} \cdot \hat{n} dA = \iint \vec{E}_\perp dA \\
\text{\hat{n}} &= \text{normal vector to surface} \\
\cdot &= \text{dot product} \\
\vec{E}_\perp &= \text{component of} \ \vec{E} \ \text{\perp to surface}
\end{align*}
\]

Important fact: A point charge "emits" a certain amount of electric flux.

- In E&M, flux is like the "total amount of field" pointing through a surface. (Note: In fluids, flux is the amount of stuff per unit area.)

\[
\begin{align*}
\Phi_E &= \iiint \vec{E}_\perp dA = \iint \frac{kq}{r^2} dA \\
&= \frac{kq}{r^2} \iint dA = \frac{kq}{r^2} A = 4\pi r^2 \\
\Phi_E &= 4\pi kq
\end{align*}
\]

If you surround a point charge with a Gaussian surface (even if it's not a sphere), the same flux will be penetrate that surface.

Integration is a linear operation.

\[
\int [af(x) + bg(x)] \, dx = a\int f(x) \, dx + b\int g(x) \, dx
\]
If multiple charges are inside a Gaussian surface, the flux is just the total flux "emitted" by the charges.

How is all of this useful? Can we get the value of E through Gauss's Law?
- If E<sub>perp</sub> = constant, it factors out of the integral.
- If E = 0, the total flux must be zero.

In what situations is E<sub>perp</sub> = constant? Symmetric geometries.

Ex: Spherical uniform charge distribution

The spherical ball of charge acts like a point charge, from the perspective of anything outside the sphere.

All of this assumes spherical symmetry.
E of a line charge

\begin{align*}
\Phi_E &= \lambda L \\
\Phi_E &= E_2 \pi r L \\
E &= \frac{2 k \lambda}{r} \\
E_{\text{Field Point}} &= \frac{4 \pi k Q_{\text{enc}}}{r^2} \\
E_{\text{2} \pi r L} &= \frac{4 \pi k \lambda L}{r^2}
\end{align*}
A 4 nC charge is in the center of a thick metal shell, $R_{inner} = 3$ cm, $R_{outer} = 4$ cm. The metal shell is coated with a 3 cm layer of insulating material. The metal has a total charge of -7 nC, and the plastic has a total charge of 3 nC. What is the electric field everywhere?

Important fact: $E = 0$ in electrostatics in the bulk of a conductor.
- If there was $E \neq 0$, it would exert force on any charges in the region.
- Conductors allow charges to move freely.
- That's not electrostatic.

It's surface charges that cause discontinuities in the electric field.
It's surface charges that cause discontinuities in the electric field.

\[ EA = 4\pi k Q_{enc} \]

The inner surface of the metal in this case must have a \(-4\) nC charge. The bulk of the metal has zero charge. Any remaining charge must be on the outer surface.

\[ Q_{metal} = Q_{inner} + Q_{bulk} + Q_{outer} \]
\[ (-7\,\text{nC}) = (-4\,\text{nC}) + 0 + Q_{outer} \]
\[ -3\,\text{nC} = Q_{outer} \]
Physics I review:

\[ \text{Work} = W = \int \vec{F} \cdot d\vec{x} \]

If you push something and it goes in the direction you push it, you give it energy.

Gravity is a good example to use to understand potential energy. Consider a dropped object:

\[
\begin{align*}
F_g &= mg \\
U_g &= mgy \\
W_g &= \int mg \, dy \\
\Delta y &= -m g \Delta y \\
\Delta U_g &= -\int \vec{F}_g \cdot d\vec{d} \\
\text{Pure } y \text{ force } \Delta U_g &= -\int F_y \, dy
\end{align*}
\]

\[ U_i = mgh \]

\[ \Delta U = -mgh \]

\[ m \]

\[ F_f = 0 \]

\[ \frac{1}{2} m v_f^2 = mgh \]

(Only \( \theta \) variable is \( \Delta U \))

\[ \vec{F} = -\nabla U_g \]

\[ F_y = -\frac{dU_g}{dy} \]

With electric forces, we can do something similar.

\[ W_E = \int \vec{F}_E \cdot d\vec{d} = \int q_0 \vec{E} \cdot d\vec{d} = q_0 \int \vec{E} \cdot d\vec{d} = -q \Delta V \]

Usually, \( E \) doesn't depend on what \( q_0 \) we use and where we put it, so:

\[ W = -\Delta U \]

\( \vec{E} \) is the force per unit charge

\( V \) is the energy per unit charge

\( -\Delta V \) is the work per unit charge

Many names: electric potential, potential difference, voltage, electromotive force (EMF)
What does electric potential do to our charge $q_0$?

$$E_x = -\frac{dV}{dx}$$

$E$ points "downhill" toward lower (or more negative) $V$

$q_0 = \Theta$

$F_E = \vec{q}_0 \vec{E}$ same dir as $\vec{E}$

$q_0$ attracted to $V = \Theta$

$q_0 = -\Theta$

$F_E = -\vec{q}_0 \vec{E} = -|q_0| \vec{E}$ = opposite to $\vec{E}$

$-|q_0|$ attracted to $V = \Theta$

What causes $V$? Source charges

- Since $+|q_0|$ is attracted to low $V$ and attracted to negative sources: low (negative) $V$ is caused by negative source charges.

Easiest case: point source charges

$$\vec{E} = \frac{kq_1}{r^2} \hat{r}$$

$$\frac{dV}{dx} = -E$$

$$V = -\int E \, dx = -\int \frac{kq_1}{r^2} \, dx = -kq_1 \left( \frac{1}{r} \right)$$

Note: $V$ is a scalar (not a vector)

Just like with calculating "total $E$", we can calculate "total $V$" by adding
up the contributions of many sources.

\[ V = \sum dV = \int \frac{k \cdot dQ}{r} \]

If we know E, we can certainly calculate V.
How much charge could we place on a BB?
Limit: $E = 1,000,000 \text{ N/C}$ causes sparks and discharge.

$$E = \frac{kQ}{r^2}$$

$Q = \frac{ER^2}{k} = \frac{(1 \times 10^6)(0.0015)^2}{(9 \times 10^9)} = 2.5 \times 10^{-10} \text{ C} = 0.25 \text{ nC}$

What is the electric potential of the BB with this charge?
- "Very far away", $V=0$

At $R=0.0015$:

$V = \frac{kQ}{R} = \frac{(9 \times 10^9)(2.5 \times 10^{-10})}{0.0015} = 1500 \text{ V}$

Summary: A 3 mm diameter BB can hold up to 0.25 nC of charge, at which point it has a potential of 1500 V.

Electric potential is "like height" in the gravity analogy.
Electric potential is "like pressure" in a fluid analogy.

The ratio of charge gathered to voltage needed is called capacitance.

$Q = CV$

$(2.5 \times 10^{-10} \text{ C}) = C(1500 \text{ V})$

$C = \frac{2.5 \times 10^{-10}}{1500} = 1.67 \times 10^{-13} \text{ F}$

$C = 0.167 \text{ pF}$
Why was it so difficult (high voltage) to gather even a little charge on the BB? The charges all repel each other.

To overcome this, a capacitor is built to store equal amounts of positive and negative charge. The charge is stored on flat metal surfaces called plates.

\[
\Phi_E = 4\pi k Q \quad \text{(on both sides of the slab)}
\]

\[
E = \frac{\Phi_E}{A} = \frac{4\pi k Q}{A}
\]

\[
E = \frac{Q}{\varepsilon_0}
\]

This is the electric field everywhere inside the capacitor. Just like

\[
F_g = -mg \quad \Rightarrow \quad U_g = mgy
\]

\[
E_c = \frac{\sigma}{\varepsilon_0} \quad \Rightarrow \quad V_c = \frac{\sigma d}{\varepsilon_0} = \frac{Qd}{\varepsilon_0 A}
\]

\[
CV = \left(\frac{\varepsilon_0 A}{d}\right) V = Q
\]

So the capacitance is

\[
C = \frac{\varepsilon_0 A}{d} \quad E = \frac{V}{d}
\]

On the equation sheet:

\[
C = k \frac{\varepsilon_0 A}{d} \quad k = \text{dielectric constant}
\]

\[
E_x: \quad A = 1 \, m^2 \quad k = 1 \quad \varepsilon = \frac{\varepsilon_0 A}{(8.85 \times 10^{-12}) (1.0)} = 8.85 \times 10^{-8}
\]
Capacitors store:
- Charge, Q
- Energy

Since voltage is energy per unit charge, you'd think that you could multiply charge * voltage to get energy.

\[
\text{Energy} = \varepsilon_0 \Delta V \quad \text{for } \Delta V \text{ not due to } \varepsilon_0
\]

\[
\text{Self-Energy} = \frac{1}{2} QV \quad \text{if } V \text{ is due to } Q
\]

\[
V = \frac{q}{\varepsilon}
\]

Technically:
\[
\text{Energy} = \int V \, dq = \int \frac{q}{\varepsilon} \, dq = \frac{1}{2} \frac{q^2}{\varepsilon}
\]