11. Exam1 Return, Magnets

Tuesday, October 1, 2019 9:24 AM

Exam1 Average: 59%
After grades are uploaded, use your Course Average to see where you stand.
(60% Exams, 15% HW, 25% Lab)

Equivalent units to the farad (F)?

This measures capacitance (C).

$C = \frac{\kappa \varepsilon_0 A}{d}$ $Q = CV$ $\tau = RC$

$[C] = [F][V]$ $[s] = [\Omega][F]$

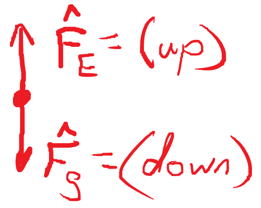
For the 25 mg styrofoam ball:

$25\ mg = 25 \times 10^{-3}\ g$
$= 25 \times 10^{-6}\ kg$

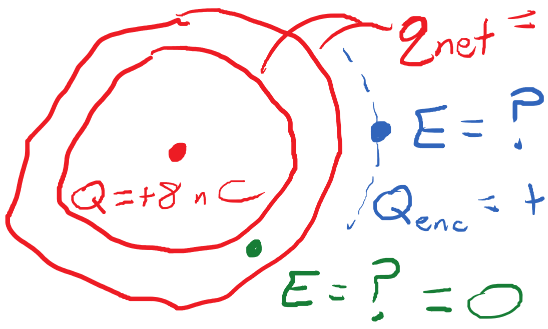
Mass is our only quantity where the fundamental unit is not the base unit.
Must convert to kg to use in F=ma, F=mg, etc.

$qE = mg$

$\overset{\uparrow}{\underset{\text{up}}{\vec{F}_E}} = \overset{\uparrow}{\underset{\text{negative}}{q}}\ \underset{\text{must be down}}{\vec{E}}$



Spherical Gauss's Law

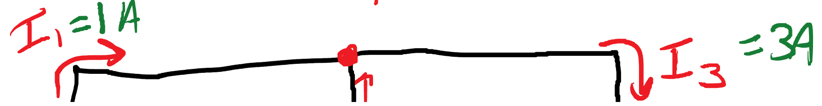


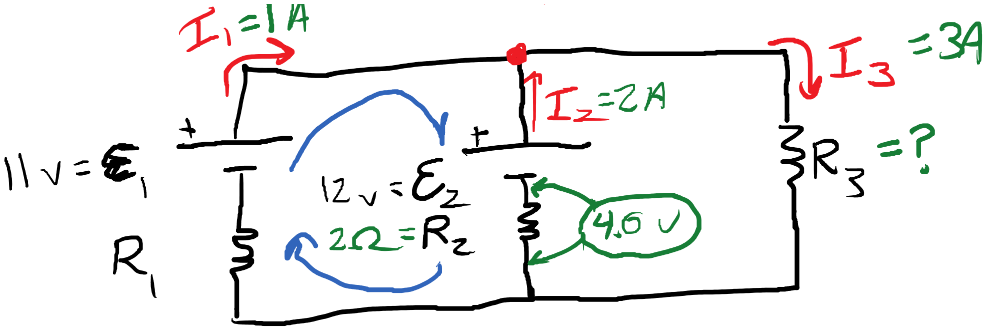
$Q_{enc} = +8\ nC - 5\ nC = +3\ nC$

$E = \frac{kQ_{enc}}{r^2} = \frac{(9)(3)}{(0.1)^2}$
$= 2700\ N/C$

Kirchhoff's Law Circuit

$I_1 + I_2 = I_3$





$$11 - 12 = -I_2R_2 + I_1R_1$$

$$11 - 12 + I_2R_2 - I_1R_1 = 0$$

$$\mathcal{E}_2 = V_3 + V_2$$

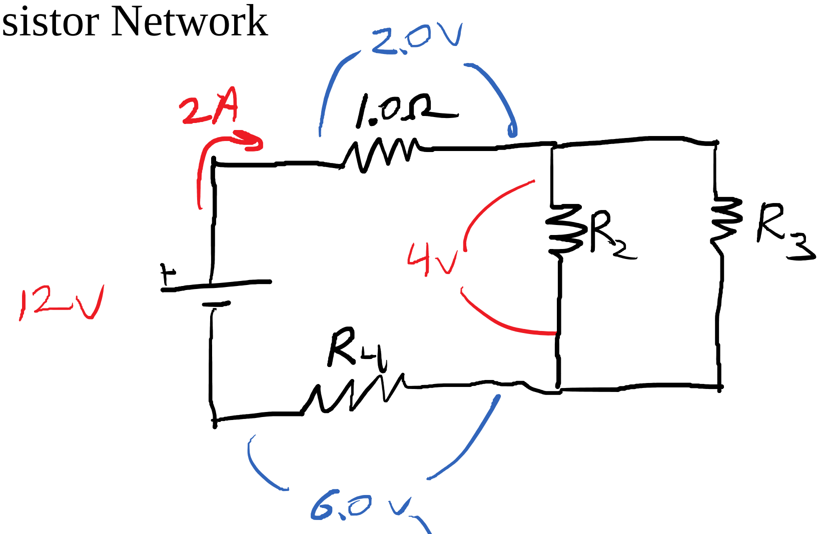
$$12 = V_3 + 4$$

$$8 = V_3$$

$$V_3 = I_3R_3$$

$$8 = 3R_3$$

Resistor Network



$$6.0v = (2.0A)R_4$$

$$I_2 + I_3 = 2A$$

$$I_2 = 2I_3$$

$$2I_3 + I_3 = (2A)$$

$$I_3 = 0.667A$$

$$V_3 = I_3R_3$$

$$(4v) = (0.667A)R_3$$

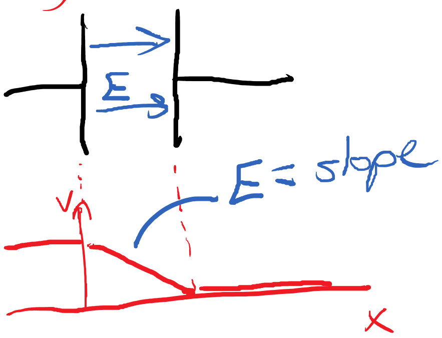
$$6\Omega = R_3$$

$$P_1 = V_1I_1 = (2v)(2A) = 4W$$

Capacitor in electrostatic and RC circuit

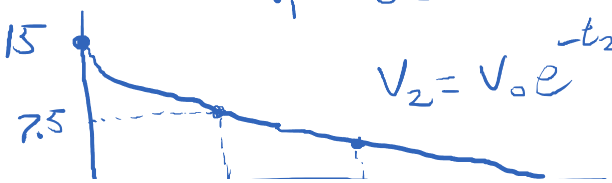
$$Q = CV = (400\mu F)(15v) = (6000\mu C)$$

$$E = \frac{V}{d}$$



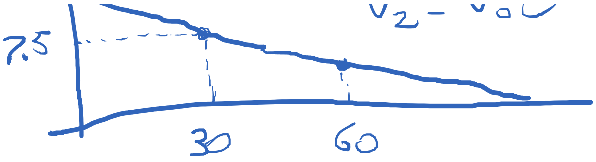
$$V_1 = V_0 e^{-t_1/\tau}$$

$$V_2 = V_0 e^{-t_2/\tau}$$



$$V_1 \quad e^{-t_1/\tau} = e^{-30/\tau}$$

$v_2 = v_0 e$



$$\frac{V_1}{V_0} = e^{-t_1/\tau} = e^{-30/\tau}$$

$$\frac{V_2}{V_0} = e^{-t_2/\tau}$$

$$\frac{2^4}{2^1} = 2^3$$

$$\frac{V_2}{V_1} = \frac{e^{-t_2/\tau}}{e^{-t_1/\tau}} = e^{-(t_2-t_1)/\tau} = e^{-30/\tau}$$

Every 30 second interval cuts the voltage in half.

$$0.5 = e^{-30/\tau}$$

$$\ln(0.5) = -30/\tau$$

$$\tau = \frac{-30}{\ln(0.5)} = 43\ s$$

hidden ⊖

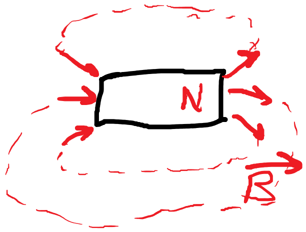
Magnetism Intro

Tuesday, October 1, 2019 10:18 AM

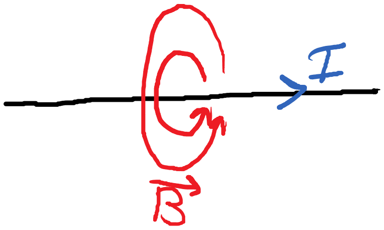
Magnetostatics: Magnetism without moving objects.
Any currents are DC currents.

Magnetism is a vector field, like the electric field.
The magnetic field ($\vec{B}$) is shaped differently.
Magnetic fields always form "loops".

Bar Magnet



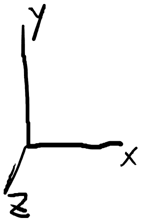
Straight Wire



What creates magnetic fields?
- Magnetic materials.
- Electric Currents
- Fluctuating electric fields

What do magnetic fields do?
- Force on moving charges and currents.
- Torque on magnetic dipoles.
- Attract or repel magnetic dipoles.
- Generate electric field and voltage.

We need 3-D directions and coordinate systems.

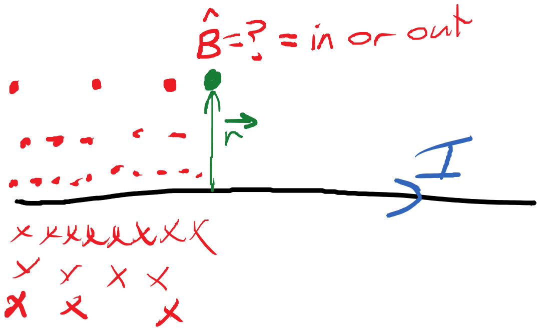


Paper	Map	Camera	Drawing
x = Right	x = East	x = Right	→ ←
y = Up = Top	y = North	y = Up	↑ ↓
z = Out	z = Up	z = Backward	⊙ ⊗

The word "Up" is ambiguous. Need clues to figure out what it means.

Always try to envision the 3-D situation separately from the diagram.

Always try to envision the 3-D situation separately from the diagram.



$\hat{I} = \hat{x}$ (Current in x-dir)

$\vec{r}$ points from source to field point

$\hat{r} = \hat{y}$

RHR for magnetic field of a current:

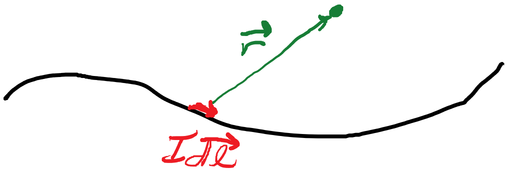
- Point thumb in direction of *I*.
- Curl fingers around current to find *B*.

12. Magnetic Sources

Thursday, October 3, 2019 9:24 AM

Generally, $\vec{B}$ is caused by currents.

Biot-Savart Law: $$\vec{B} = \int \frac{\mu_0 I\, d\vec{l} \otimes \vec{r}}{4\pi r^2}$$



$\mu_0 = 4\pi \times 10^{-7}\ N/A^2$

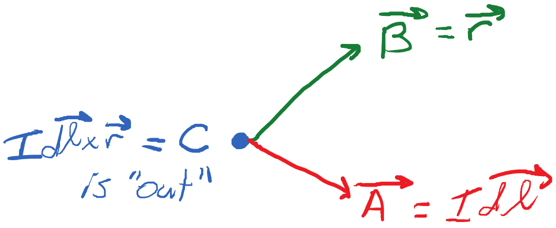
$\otimes$ = cross product

Vector cross product: $\vec{C} = \vec{A} \otimes \vec{B}$

- C is perpendicular to both A and B.
- C has a magnitude of AB sin(theta), where theta is the angle between A and B.
- Pick direction of C by RHR:

$\theta = 0 \rightarrow |\vec{A} \otimes \vec{B}| = 0$

$\theta = 90^\circ \rightarrow |\vec{A} \otimes \vec{B}| = AB$



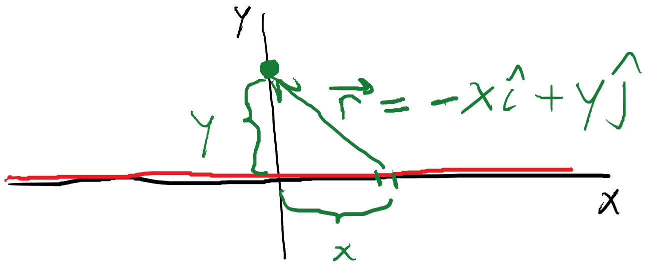
$\vec{C}$ = Thumb

$\vec{A}$ = Index

$\vec{B}$ = Middle

$$\vec{A} \otimes \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Example: Infinite, straight current



$r^2 = x^2 + y^2$

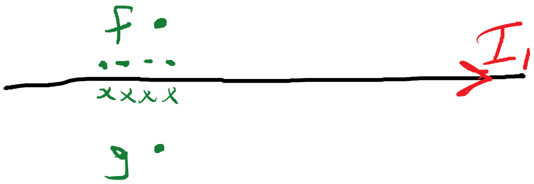
$d\vec{l} = dx\,\hat{i}$

$$d\vec{l} \otimes \vec{r} = dx\,\hat{i} \otimes (-x\hat{i} + y\hat{j}) = y\,dx\,\hat{k}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{y\,dx\,\hat{k}}{x^2 + y^2} = \frac{\mu_0 I}{2\pi y} \hat{k}$$

$$B_{wire} = \frac{\mu_0 I}{2\pi r}$$ r = distance from wire

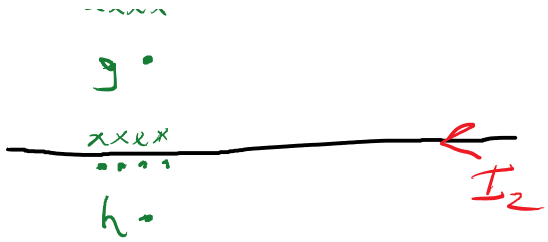
Common to think about magnetism of two wires.



$B = \frac{\mu_0 I}{2\pi r}$

Point	B_1	B_2	Net
f	out	in	$\lvert B_1 - B_2 \rvert$
g	in	in	$B_1 + B_2$

$B = \frac{\mu_0 I}{2\pi r}$



g	in	in	B_1+B_2
h	in	out	$\lvert B_1-B_2 \rvert$

Wires each carrying 1.5 A of current spaced 4 cm apart. B at midpoint?

$$B = B_1 + B_2 = \frac{\mu_0(1.5)}{2\pi(0.02)} + \frac{\mu_0(1.5)}{2\pi(0.02)} = 3\times10^{-5}\ T$$

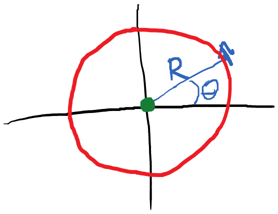
Hint: Type mu_0 as (pi*4e-7)

$\mu_0 = 4\pi\times10^{-7}\ \frac{T\cdot m}{A}$

At a point 1 cm above I1:

$$B = B_1 - B_2 = \frac{\mu_0(1.5)}{2\pi(0.01)} - \frac{\mu_0(1.5)}{2\pi(0.05)}$$

Circular Loop of Wire:



$\vec{dl} = R\,d\theta\,\hat{\theta}$

$\vec{r} = -R\,\hat{r}$

$\vec{dl}\otimes\vec{r} = R^2\,d\theta\,\hat{k}$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{\vec{dl}\otimes\vec{r}}{r^2} = \frac{\mu_0 I}{4\pi}\int_0^{2\pi}\frac{R^2\,d\theta\,\hat{k}}{R^2}$$

$$\vec{B} = \frac{\mu_0 I}{2R}\hat{k}$$

(Lost a factor of R?)

Along the axis, in front of and behind the loop,
B is weaker. It's kind of bell-shaped function.

Ampere's Law

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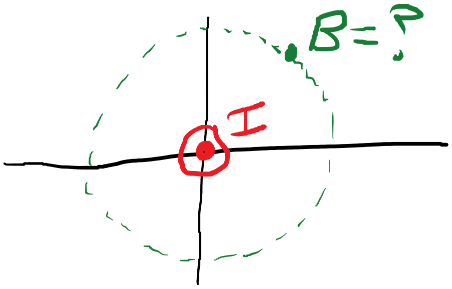
This is akin to Gauss's Law and summarizes the result of the Biot-Savart Law.

$$\int \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$



Strategy, make that integral easy.

- Make sure B = constant
- Keep dot product simple (theta=constant)

$$\int \vec{B}\cdot d\vec{l} = BL$$
$$= B2\pi R$$

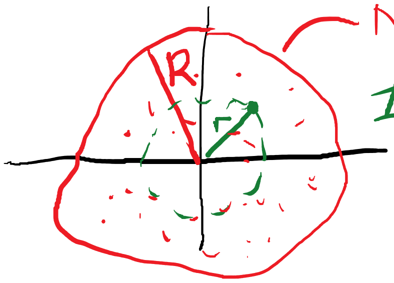
L= Length of loop

$$B_{\parallel} l = \mu_0 I_{enc}$$

Result:

$$2\pi R B = \mu_0 I$$
$$B = \frac{\mu_0 I}{2\pi R}$$

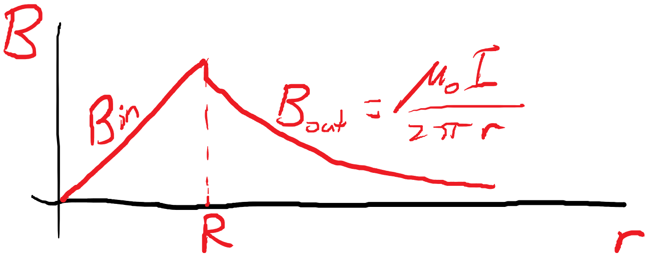
Magnetic field of a thick wire:



Net current = I

$$I_{enc} = I\frac{r^2}{R^2}$$

$$B_{\parallel} l = \mu_0 I_{enc}$$
$$2\pi r B = \mu_0 I\left(\frac{r^2}{R^2}\right)$$

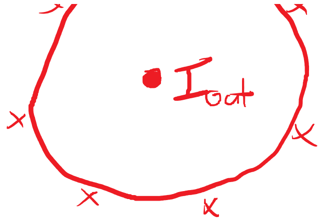


$$B_{in} = \frac{\mu_0 I r}{2\pi R^2}$$

Coaxial Cable:

If Iout = Iin, then I_enc = 0 for any point outside the coaxial cable.

outside the coaxial cable.



$B = 0$

Solenoid Coil

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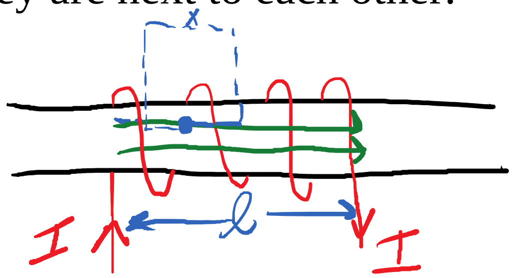
For a many-loop coil with all of the loops on top of each other, just multiply by N:

$$B = \frac{\mu_0 N I}{2R}$$

In a solenoid coil, the loops are not on top of each other, they are next to each other.



Inside: $\vec{B} = B_0 \hat{i}$

Outside: $\vec{B} \approx 0$

$$B_{\parallel} \ell = \mu_0 I_{enc}$$

$$B_0 x = \mu_0 I (N)\left(\frac{x}{\ell}\right) \rightarrow B_0 = \frac{\mu_0 N I}{\ell}$$

4 important Formulas:

$$B_{wire} = \frac{\mu_0 I}{2\pi r} \qquad B_{coil} = \frac{\mu_0 N I}{2R} \qquad B_{sol} = \frac{\mu_0 N I}{\ell}$$

$$B_{\parallel} \ell = \mu_0 I_{enc}$$

13. Magnetic Forces and Torques

Tuesday, October 8, 2019 9:23 AM

The most basic result of magnetism is forces on moving charges.

$$\vec{F}_B = q\vec{v} \otimes \vec{B}$$

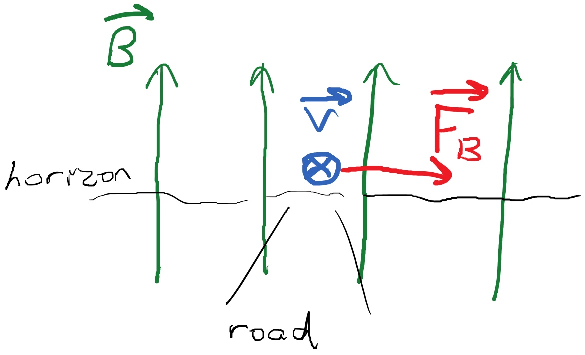
The cross product does a few things:

- The velocity must have a component perpendicular to B to generate force.
- The direction of the force is by the RHR for cross products.
  - F_B is perpendicular to velocity.
  - F_B is perpendicular to the magnetic field.

$$\vec{F}_B = q\vec{v} \otimes \vec{B}$$

$$\vec{B} = B_0 \hat{j}$$

$$\vec{v} = -v_0 \hat{k} \quad \text{(into page)}$$



$$\vec{F}_B = F_B \hat{i}$$

(Assuming ⊕ particle.)

Since the force is always perpendicular to the velocity:

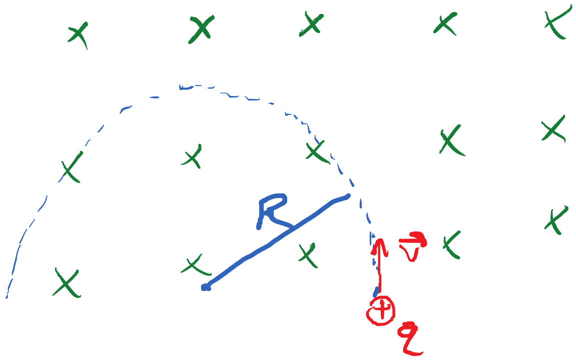
- F_B cannot change the speed of the particle's motion.
- F_B cannot transfer energy.

$$W = \int \vec{F} \cdot d\vec{l} = \int \vec{F} \cdot \vec{v}\, dt$$

- F_B does change the direction of motion of the particle. This makes the particle move in uniform circular motion.

$$\vec{F}_B = q\vec{v} \otimes \vec{B}$$

$$\vec{B} = -B_0 \hat{k}$$



$$\vec{F}_{net} = m\vec{a}$$

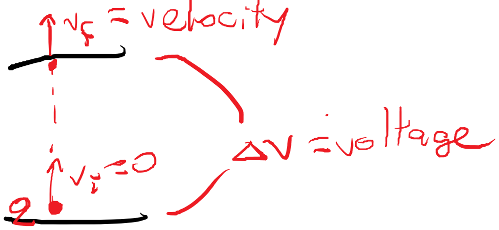
$$F_B = \frac{mv^2}{R}$$

$$qvB = mv^2/R$$

$$R = \frac{mv}{qB}$$

This radius of curvature is the basis of a mass spectrometer.

How do we generate a beam of particles with known velocity?
Method 1: Linear accelerator.



$$q\Delta V = \frac{1}{2}mv^2$$

Notation: ΔV = Accel voltage
v = velocity

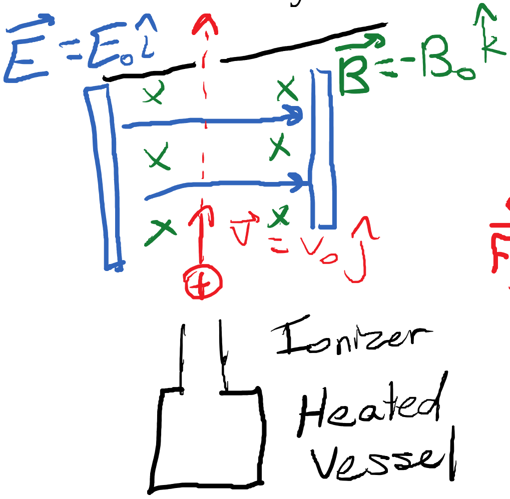
$$\text{velocity} = v = \sqrt{\frac{2q\Delta V}{m}}$$

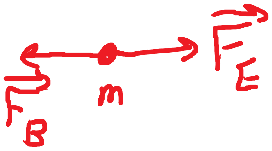
$$R = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \frac{1}{B}\sqrt{\frac{2m\Delta V}{q}}$$

If I have a bunch of particles with different masses and the same charge, and I accelerate them all with the same voltage, how does R depend on mass? (Here, R is proportional to the square root of mass.)

It would be nice to have a linear dependence between mass and radius. That would require all particles to have the same velocity (not voltage).

Method 2: Velocity Selector
- Start with particles of various velocities.
- Pick out the ones you want.





$$F_B = F_E$$

$$\cancel{q}v_0B_0 = \cancel{q}E_0$$

$$v = E/B$$

At this velocity, the net force is zero.
- Too fast: particle bends left.
- Too slow: particle bends right.

So now we have a beam of particles, all with the same velocity.

$R = \frac{mv}{2B}$ $R \propto m$

Magnetic force on currents

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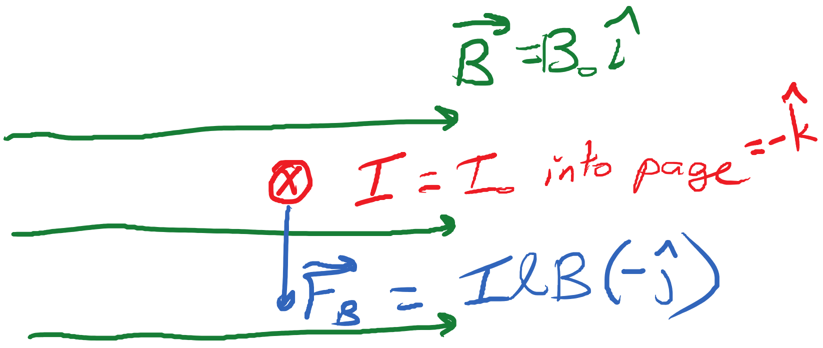


$$\vec{B} = B_0 \hat{i}$$

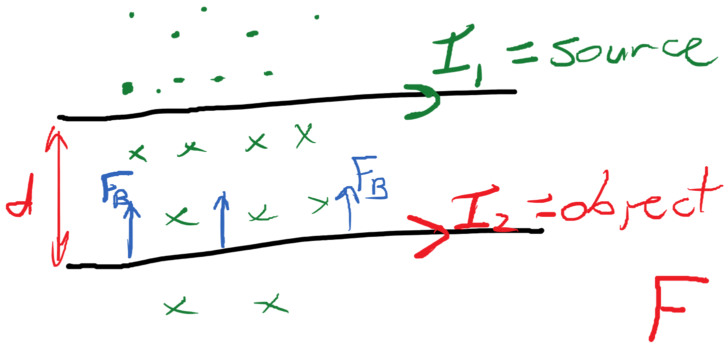
$$I = I_0 \text{ into page} = -\hat{k}$$

$$\vec{F}_B = I l B(-\hat{j})$$

$$\vec{F}_B = q\vec{v} \otimes \vec{B}$$

$$\vec{F}_B = I\vec{l} \otimes \vec{B}$$

The force on a current is boring, but it is the original definition of the ampere. Consider two parallel wires:



$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

Direction of F_B is "opposites repel".

$$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$$

$$F = I_2 l B_1$$

$$= \frac{\mu_0 I_1 I_2 l}{2\pi d}$$

$$= 2 \times 10^{-7} I_1 I_2 \frac{l}{d}$$

If the distances are "nice round numbers", then 1.0 A of current causes a "nice round number" value of force. This was the original definition of the ampere, and is why mu_0 is what it is.

Magnetic Torque

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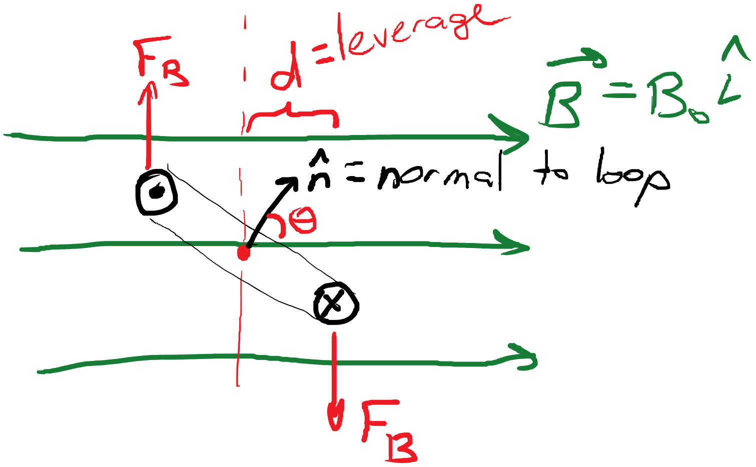
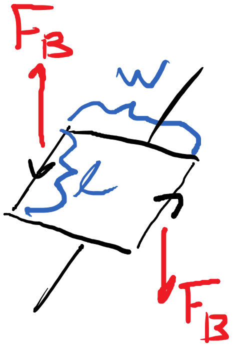




The two forces together generate a torque on the loop.
How much?

$$\tau_{net} = 2\tau_0 = 2(F_B d) = 2\left(I\ell B \frac{w}{2}\sin\theta\right)$$

$$\tau = BAI\sin\theta \qquad A = \ell w$$

Typically, the loop is a coil with many turns of wire.

$$\tau = NBAI\sin\theta$$

This torque tries to make n-hat point in the direction of B.

This is the torque produced by an electric motor.
Note: The motor doesn't need to be spinning to make torque.

Electric generators are also coils in magnetic fields. This means electric generators also exert torque. But, they only exert magnetic torque when current is flowing, i.e. when a load is attached.

14. Electromagnetic Induction

Thursday, October 10, 2019 9:24 AM

Vsauce video on Laws & Causes:
https://www.youtube.com/watch?v=_WHRWLnVm_M&t=308s

Electromagnetic Induction is covered by Faraday's Law:

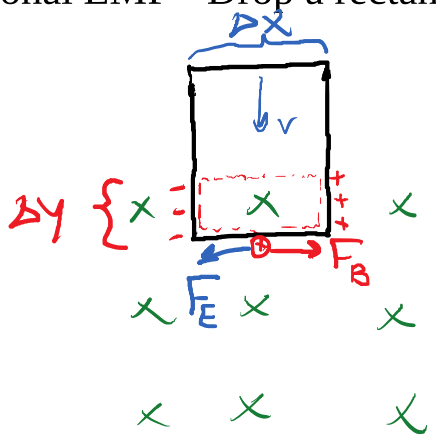
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = \iint \vec{B}\cdot d\vec{A} = NBA\cos\Theta$$

This law actually summarizes two physical effects:

- Moving a wire in a magnetic field - Motional EMF
- Fluctuating magnetic field - Need a new explanation

Motional EMF - Drop a rectangular loop "into a magnetic field".



$$\vec{B} = -B_0\hat{k}$$

$$E_0 = \frac{\Delta V}{\Delta x}$$

Charges in the wire that are falling in the magnetic field feel a magnetic force.

$$\vec{F_B} = qvB_0\hat{i}$$

$$\vec{F_E} = qE_0(-\hat{i})$$

$$\cancel{q}vB_0 = \cancel{q}\frac{\Delta V}{\Delta x}$$

$$\frac{\Delta y}{\Delta t}B_0 = \frac{\Delta V}{\Delta x}$$

Area of overlap

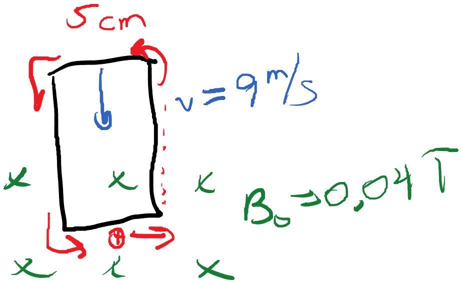
$$\frac{\Delta A\ B_0}{\Delta t} = \frac{\Delta x\,\Delta y\,B_0}{\Delta t} = \Delta V = |\mathcal{E}| \text{ Magnitudes}$$

$$\mathcal{E} = Bv\ell$$

$$\frac{\Delta(B_0 A)}{\Delta t} = \frac{\Delta\Phi_B}{\Delta t} = |\mathcal{E}|$$

This shows that Faraday's Law covers the case of a moving wire in a magnetic field.

Ex: A 25 gram wire that is 5 cm wide and 7 cm tall falls into a 0.04 T magnetic field. What is the resistance of the wire if it falls at 9 m/s?



$$\mathcal{E} = Bvl = (0.04\,T)(9\,m/s)(0.05\,m)$$

$$\mathcal{E} = 0.018 \quad V$$

Bottom Leg: $F_B = I l B$

RHR: + pushed right, so current is CCW.
For const-velocity, F_B = F_g

$$F_g = mg$$

$$I(0.05\,m)(0.04\,T) = (0.025\,kg)(9.8\,N/kg)$$

$$I = 122.5\ A$$

$$\mathcal{E} = IR \qquad R = \frac{\mathcal{E}}{I} = \frac{0.018\ V}{122.5\ A} = 1.47 \times 10^{-4}\ \Omega$$

$$= 0.147\ m\Omega$$

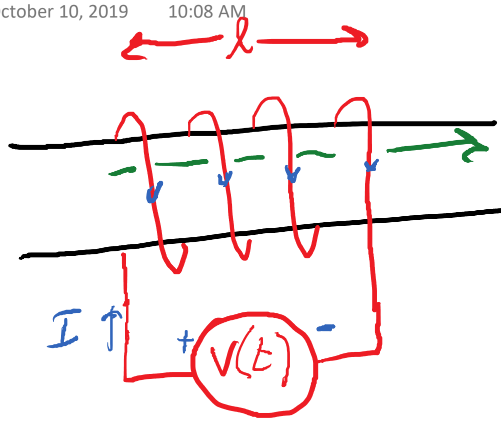
Stage 1: Charges in falling wire feel magnetic force along the wire. This is balanced by the generated electric force, which comes from the generated electric field, which is an EMF / length.
Stage 2: The EMF drives current around the loop. The current (along the wire) feels a magnetic force perpendicular to the wire. This is balanced by gravity, so the loop falls with zero acceleration.

We looked at the loop falling into a magnetic field. Only the bottom leg of the loop experienced motional EMF. Once the loop is "in" the magnetic field, the upper leg will also generate voltage, but the other way around the loop.

If the loop starts to fall "out of the magnetic field", then the bottom leg will stop generating voltage. The top leg will make current flow the other way.

EMF self-induced by a coil

Thursday, October 10, 2019 10:08 AM



$$\vec{B} = \frac{\mu_0 N I}{\ell}$$

$$\Phi_B = \iint \vec{B} \cdot \vec{dA}$$

$$= NBA \cancelto{1}{\cos}\Theta$$

$$\Phi_B = N\left(\frac{\mu_0 N I}{\ell}\right)A$$

$$\Phi_B = \frac{\mu_0 N^2 A}{\ell} I$$

Each loop forms a disc.
Each disc has a normal vector.
Theta is between the normal and B.

Faraday's Law says if the flux changes, there is an EMF.

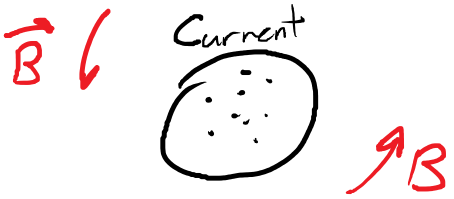
$$d\Phi_B/dt = \frac{\mu_0 N^2 A}{\ell} \frac{dI}{dt}$$

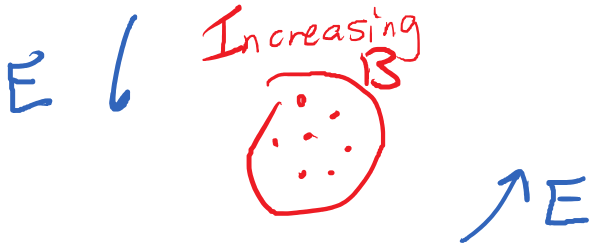
Changing the current in a coil generates EMF.

Ex: Unplug an electric motor. This makes the current stop VERY rapidly. The coil in the motor generates a HUGE EMF spike.

What's behind this effect?

- Fluctuating B "stirs up" electric field.



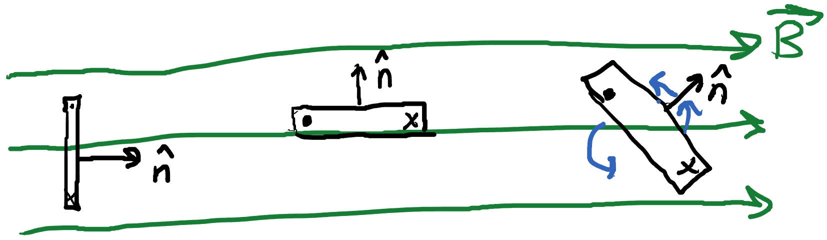


It's nice to not have to deal with the details.
Faraday's Law magically covers all cases.

Electric Generator

Thursday, October 10, 2019 10:22 AM

This is formed from a spinning coil in a magnetic field.



$$\Phi_B = NBA \cos\theta$$

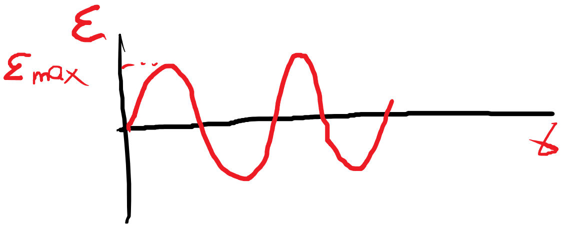
$$\frac{d\Phi_B}{dt} = NBA(-\sin\theta)\frac{d\theta}{dt}$$

$$\Phi_B = NBA \cos\theta$$
$$= NBA$$

$$\Phi_B = -n \cos(90^\circ)$$
$$= 0$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = NBA \sin\theta\, \omega$$

ω = angular velocity in rad/s



$$\mathcal{E}_{max} = NBA\,\omega$$

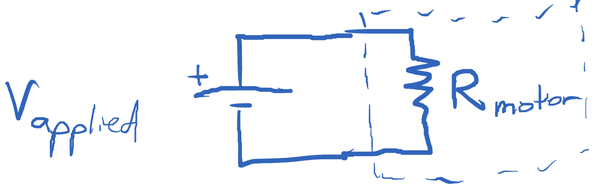
Motors vs. Generators

Both are coils in B fields.

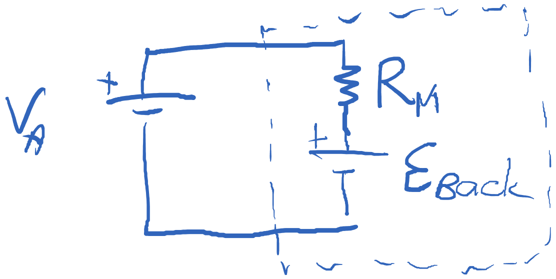
Generator	Motor
We apply torque to spin.	We apply voltage to push I
EMF pushes current.	I causes drive torque
Current causes drag torque.	Spinning causes Back EMF

Back EMF in a motor is actually good for us.

No Back EMF



With Back EMF



$$I = \frac{V_{applied}}{R}$$

$$V - \mathcal{E} = I R$$

$$I = \frac{V_{applied}}{R_{motor}}$$

$$V_A - \varepsilon_{Back} = I R_m$$

$$I = \frac{V_{app} - \varepsilon_{Back}}{R_m}$$

15. Inductance

Tuesday, October 15, 2019 9:25 AM

Self-Inductance (L) is the proportionality between a coil's magnetic flux (Φ_B) and its current (I).

Solenoid:

$$\Phi_B = NBA\,\overset{1}{\cancel{\cos\theta}}$$

$$B = \frac{\mu_0 N I}{\ell}$$

$$\Phi_B = \left(\frac{\mu_0 N^2 A}{\ell}\right) I$$

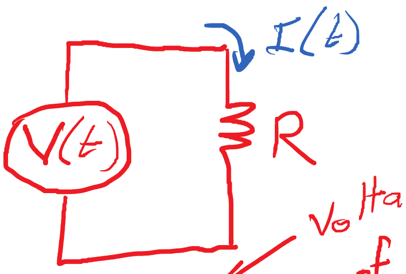
$$L = \frac{\mu_0 N^2 A}{\ell}$$

$$\Phi_B = L I$$

Magnetic flux is related to EMF:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

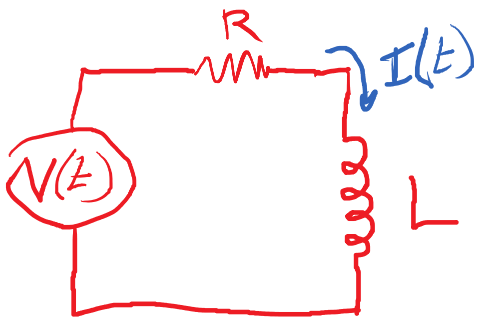
This is the basic relationship between current and voltage for an inductor. We need to understand how this affects our circuits.



Voltage Drop of Resistor

$$v(t) = V_R$$

$$v(t) = I(t)R$$



$$V(t) - L\frac{dI}{dt} = IR$$

$$V(t) = L\frac{dI}{dt} + IR$$

Now we are opposed by two things:

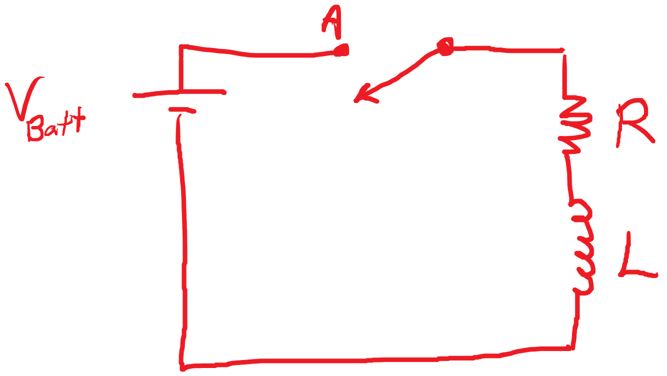
- The resistor opposes current.
- The inductor opposes changes in current. If the current is constant, the inductor doesn't do anything. It's only when the current tries to change that the inductor generates voltage.

I dI

the inductor doesn't do anything. It's only when the current tries to change that the inductor generates voltage.

$$V_R = IR \qquad V_L = L\frac{dI}{dt} \qquad \mathcal{E}_L = -L\frac{dI}{dt}$$

What happens when you first connect a battery to an electromagnet?



$$V_{Batt} = L\frac{dI}{dt} + IR$$

- Initial $\frac{dI}{dt} = \frac{V_{Batt}}{L}$
- Steady-state $I = \frac{V_{Batt}}{R}$

Shifted decaying exponential:

$$I = I_f + (I_i - I_f)e^{-t/\tau}$$

$$\frac{dI}{dt} = 0 + (I_i - I_f)e^{-t/\tau}\left(\frac{-1}{\tau}\right)$$

$I_i = 0 \qquad I_f = \frac{V_{Batt}}{R}$

Initial $$\frac{dI}{dt} = -I_f\left(\frac{-1}{\tau}\right) = -\left(\frac{V_{Batt}}{R}\right)\frac{-1}{\tau} = \frac{V_{Batt}}{L}$$

$$\frac{L}{R} = \tau \leftarrow \frac{1}{R\tau} = \frac{1}{L}$$

What does the time constant tell us?

- Bigger L = more time to build up current.
- Bigger R = Less time to build up current?
  We're asking the inductor for a more gentle current change, so we can do it faster.

$$I = I_f(1 - e^{-t/\tau}) \qquad \tau = L/R$$

$$I = I_f\left(1-e^{-t/\tau}\right) \qquad \tau = L/R$$

Like with the RC circuit, the exponential represents the "fraction of the process remaining to be completed".

One key time is when the process is 50% complete.

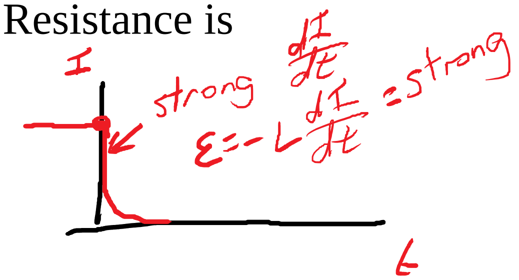
$$e^{-t/\tau} = 0.5$$

$$-t/\tau = \ln(0.5) \longrightarrow t = -\tau\ln(0.5)$$

$$\text{Half-Life:} \quad t_{1/2} = \tau\ln(2)$$

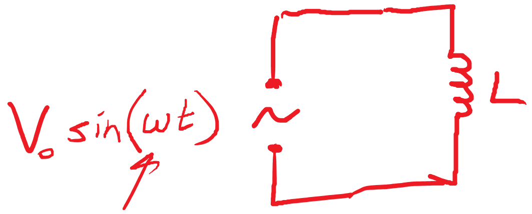
Inductors in DC Circuits:

- When current is steady: No voltage and hence no effect.
- When turning a circuit on or off: Inductor opposes that change.
  - Turning on: No big deal. Normal resistance makes for a normal time constant, and the inductor's voltage is reasonable (limited to battery voltage).
  - Turning off: Can produce EXTREME voltages. Resistance is very high (almost infinite), so tau is very small.
- A "Buck-Boost converter" generates voltage by turning an inductor's current on and off repeatedly.



Inductors in AC Circuits

Tuesday, October 15, 2019 10:14 AM



$\omega = \frac{d\theta}{dt}$ of generator

$$\mathcal{E}_{gen} = V_L$$

$$V_0 \sin(\omega t) = L \frac{dI}{dt}$$

$$\frac{V_0}{L} \sin(\omega t) = \frac{dI}{dt} \quad ①$$

Let $I(t) = -I_{max} \cos(\omega t)$

$$\frac{dI}{dt} = -I_{max}(-\sin(\omega t))\,\omega = I_{max}\, \omega \sin(\omega t)$$

$$① \rightarrow \quad \frac{V_0}{L} \cancel{\sin(\omega t)} = I_{max}\, \omega \cancel{\sin(\omega t)}$$

$$V_0 = I_{max}(\omega L)$$

Reactance

$X_L = \omega L$

This looks a lot like Ohm's Law.
Voltage is proportional to current.
Differences:

- These are voltage and current AMPLITUDES. The voltage was a sin() function, while the current is a cos() function.
- The proportionality factor isn't constant.
    - Slow oscillation: Low voltage for a given current.
    - Fast oscillation: High voltage for a given current.

The solenoid used in lab a few weeks ago:

$N = 1500$

$r = 0.03\text{ m}$

$\ell = 0.11\text{ m}$

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{(4\pi \times 10^{-7})(1500)^2(\pi(0.03)^2)}{0.11}$$

$$L = 0.0727\text{ H}$$

Inductance in henries

Wall outlet:

$V_{max} = 170\text{ V}$

$X = \omega L = 27.4\ \Omega$

wall outlet.

$V_{max} = 170$ V

$\omega = 377\ s^{-1}$

└ radians per sec

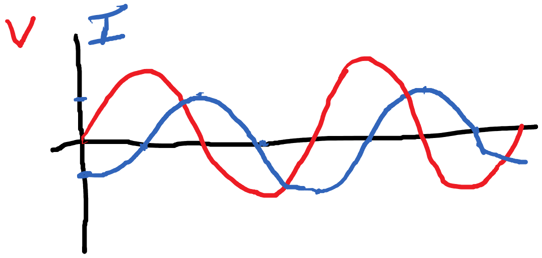
$X_L = \omega L = 27.4\ \Omega$

$V_0 = I_0 X_L$

$170 = I_0 (27.4)$

$6.2\ A = I_0$

This is the current amplitude that would (briefly) flow if we plugged one of those solenoid coils into a wall outlet.



$V = V_0 \sin(\omega t)$

$I = -I_0 \cos(\omega t)$

The current in an inductor "lags" behind the applied voltage.
How much? Exactly 1/4 of an oscillation.

16. AC Circuits

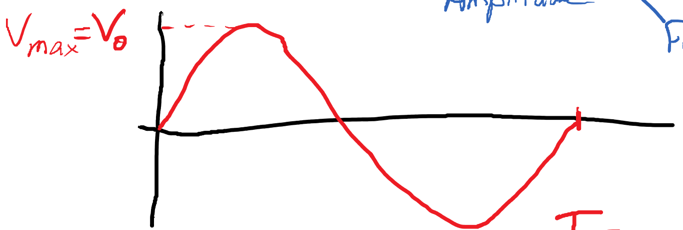
Thursday, October 17, 2019 9:22 AM

AC electricity involves sinusoidally-varying voltage and current.

Ex: $V = V_0 \sin(2\pi f t)$ or $V = V_0 \cos(2\pi f t)$

Amplitude

Frequency in Hz



T=period

$T = \frac{1}{f}$

$\sin\theta$ repeats every 2π

$$\sin(\theta) = \sin(\theta + 2\pi)$$

$$\sin(2\pi f t) = \sin(2\pi f t + 2\pi)$$

$$= \sin(2\pi f(t + T))$$

$\uparrow$

$2\pi f T = 2\pi$

$fT = 1$

Resistor in an AC Circuit:

$$V = I R$$

Sinewaves

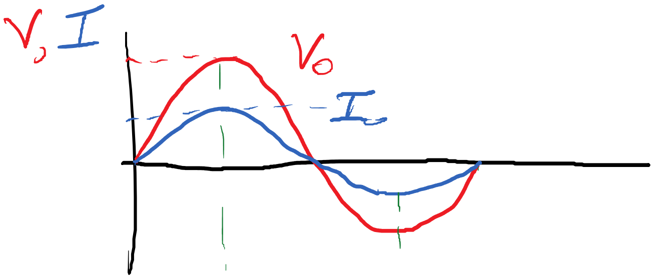
constant

$\omega t = 2\pi f t$

$\omega = 2\pi f$

$$V_0 \sin(\omega t) = I_0 \sin(\omega t) R$$

$$V_0 = I_0 R$$



Angular frequency (ω) is in radians per second. $= s^{-1}$

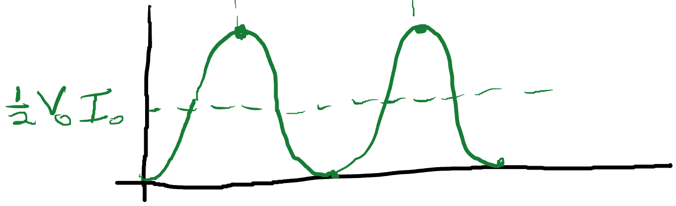
Frequency (f) is in cycles per second = Hz.

Power used by a resistor: $P = VI$

$$P = (V_0 \sin(\omega t))(I_0 \sin(\omega t))$$

$$= V_0 I_0 \sin^2(\omega t)$$

$$= V_0 I_0 \left(\frac{1}{2}(1 - \cos(2\omega t))\right)$$



The resistor receives energy in pulses instead of steadily.
The frequency of the pulses is twice the AC frequency.
On average,

$$P = \frac{d}{dt}(\text{Energy})$$

$$\Delta\text{Energy} = \int P\,dt = P_{avg}\,\Delta t$$

$$P_{avg} = \frac{1}{2} V_0 I_0 = \frac{1}{2} I_0^2 R$$

$$(\text{DC: } P = I^2 R)$$

Instead of using the too-large I_0 to describe current, let's use a smaller value,
so we don't need the 1/2 out front.

$$I_{RMS} = \frac{I_0}{\sqrt{2}} = \text{Average-ish current}$$

$$= \text{Effective current}$$

$$P_{avg} = I_{rms}^2 R$$

So for AC, we usually use RMS values to describe voltage and current.

Ex: Household Electricity

$V_{rms} = 120\ V$

$V_0 = \sqrt{2}(120) = 170\ V$

$f = 60\ Hz$

$\omega = 2\pi(60) = 377\ s^{-1}$

Inductor in AC

Thursday, October 17, 2019 10:02 AM

$$V_L = L\frac{dI}{dt}$$

$$V_o \sin(2\pi f t) = L\frac{d}{dt}\left(-I_o \cos(2\pi f t)\right)$$

$$V_o = I_o(2\pi f L) = I_o X_L$$

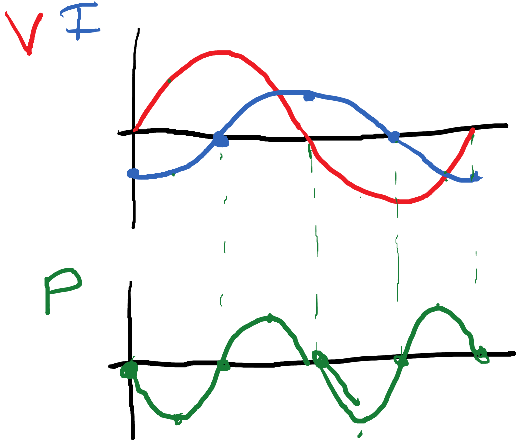
This works equally well for RMS voltage and current.

$$V_{rms} = I_{rms} X_L \qquad X_L = 2\pi f L$$

Power of an inductor:

$$P = VI$$

$$= V_o \sin(\omega t)\left(-I_o \cos(\omega t)\right)$$



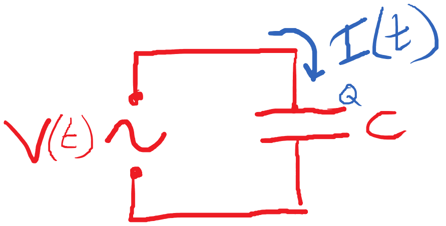
$$P_{avg,L} = 0$$

On average, inductors don't use power.

Capacitors in an AC Circuit

Thursday, October 17, 2019 10:14 AM

$Q = CV_c$



$V(t) = V_c$

$V_o \sin(2\pi f t) = Q/C$

$I = \frac{dQ}{dt}$

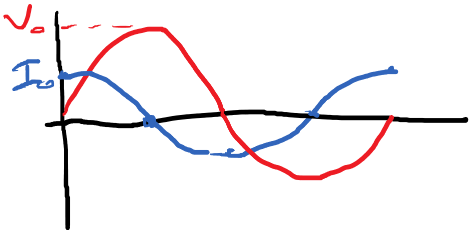
$\frac{d}{dt}\left(C V_o \sin(\omega t) = Q\right)$

$C V_o \cos(\omega t)\omega = I$

$I_o \cos(\omega t) = I$

What happens when you apply a sine function voltage to a capacitor?

- Current is a cosine function.



$I_o = V_o \omega C$

$V_o = \frac{I_o}{\omega C} = I_o X_c$

$\frac{1}{2\pi f C} = X_c = \frac{1}{\omega C}$

- The amplitudes obey AC Ohm's Law with capacitive reactance (X_C).

$V_R = IR$ $V_L = I(2\pi f L)$ $V_c = I\left(\frac{1}{2\pi f C}\right)$

- The capacitor's current "leads" the voltage.

What capacitor could be connected to a wall outlet and only allow 1 mA (RMS) to flow?

$f = 60\,\text{Hz}$ $V = 120\,\text{V}$

$I_c = 1\,\text{mA}$

$V_c = I X_c$

$X_c = \frac{1}{2\pi f C}$

$120 = 0.001\, X_c$

$120 = 0.001\,X_C$

$120000 = X_C$

$$C = \frac{1}{2\pi(60)(120000)}$$

$C = 2.21 \times 10^{-8}\,F$

$C = 22.1 \times 10^{-9}\,F$

$= 22.1\ nF$

Tuesday: Series AC Circuit

- Same current in each component. (cosine)
- Voltages add, but not the amplitudes.

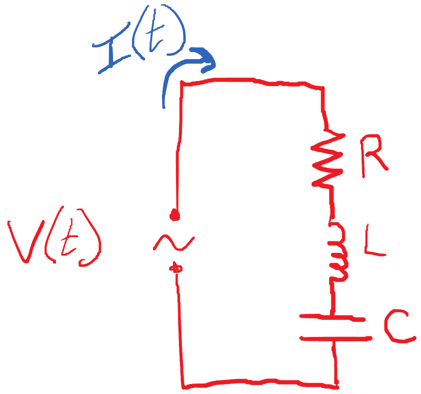
Exam 2 Thu 10/31

What do we know about the series circuit?

- Same current in each component.

$$I_R(t) = I_L(t) = I_C(t)$$

$$I_R = I_L = I_C \quad \text{(Amplitudes or RMS)}$$



- Voltages add up to total voltage.

$$V(t) = V_R(t) + V_L(t) + V_C(t)$$

But, Amplitudes don't add.

To get some intuition, we'll assume *I*(*t*) and see what the power supply voltage is.

Let $I(t) = I_o \cos(\omega t)$

Find $V_R(t) = I(t) R = I_o R \cos(\omega t)$

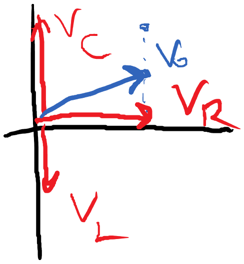
$$V_L(t) = L \frac{dI}{dt} = I_o \omega L\,(-\sin(\omega t))$$

$$V_C(t) = \frac{Q}{C} = I_o \frac{1}{\omega C}(\sin(\omega t))$$

$$V(t) = V_R \cos(\omega t) + V_L(-\sin(\omega t)) + V_C \sin(\omega t)$$

$$(V_C - V_L) \sin(\omega t)$$

To add these voltages, think of them as vectors. (Technically, they're phasors.)



$$V(t) = V_o \cos(\omega t + \phi)$$

Amplitude and angle of vector sum.

Total x: V_R

Total y: $V_C - V_L$

$$V_0 = \sqrt{V_R^2 + (V_C - V_L)^2}$$

Since each of these is proportional to the current amplitude, we can factor that out.

$$V_R = I_o R$$

$$V_L = I_o \omega L = I_o X_L$$

$$V_C = I_o \frac{1}{\omega C} = I_o X_C$$

$$\Big\} \; V_C - V_L = I_o \underbrace{(X_C - X_L)}$$

$$\left.\begin{aligned} V_L &= I_0\,\omega L = I_0 X_L \\ V_C &= I_0 \frac{1}{\omega C} = I_0 X_C \end{aligned}\right\} \quad V_C - V_L = I_0 \underbrace{(X_C - X_L)}_{X = X_C - X_L}$$

The circuit reactance is just the difference of the capacitive and inductive reactances.

$$V_0 = \sqrt{(I_0 R)^2 + (I_0 X)^2} = I_0 \underbrace{\sqrt{R^2 + X^2}}_{Z \;=\; \text{impedance}}$$

Impedance (*Z*) is like resistance, but for AC circuits.

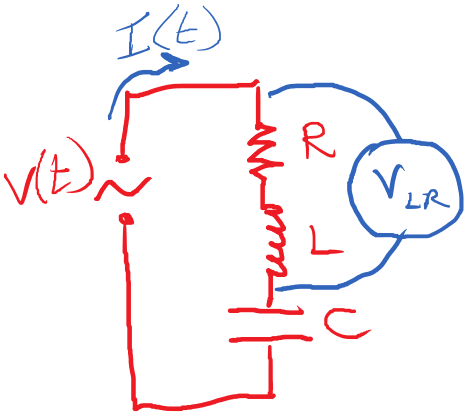
- For a resistor, *Z* = *R*
- For an inductor or capacitor, *Z* = *X*
- For a series circuit, use pythag: *Z* = sqrt(*R*^2 + *X*^2)
- Impedance reminds us that voltage and current aren't in phase.

$$V(t) = V_0 \cos(\omega t \underset{\llcorner\,\text{minus??}}{+\phi}) \qquad I(t) = I_0 \cos(\omega t)$$

$$V_0 = I_0 Z$$

RLC Example

Tuesday, October 22, 2019 9:56 AM



$V_{rms} = 120\text{ V}$ $f = 318.3\text{ Hz}$

$R = 300\,\Omega$ $L = 0.3\text{ H}$ $C = 2.5\,\mu\text{F}$

$P_{avg} = ?$ $V_{LR} = ?$

$$X_L = \underbrace{2\pi f}_{\omega = 2\pi f} L = 2\pi(318.3)(0.3) = 600\,\Omega$$

$$X_C = \frac{1}{2\pi f C} = 1/(2\pi(318.3)(2.5\times10^{-6})) = 200\,\Omega$$

$$X = X_C - X_L = 600 - 200 = -400\,\Omega$$

$$Z = \sqrt{R^2+X^2} = \sqrt{(300^2+400^2)} = 500\,\Omega$$

Overall: $V_{rms} = I_{rms} Z$

$$(120\text{ V}) = I_{rms}(500\,\Omega) \rightarrow I_{rms} = 0.24\text{ A}$$

Resistor Power: $P = I^2R = (0.24\text{ A})^2(300\,\Omega) = 17.3\text{ W}$

Individual component RMS Voltages:

- Resistor $V_{R\,rms} = (0.24)(300) = 72\text{ V}$
- Inductor $V_{L\,rms} = (0.24)(600) = 144\text{ V}$
- Capacitor $V_{C\,rms} = (0.24)(200) = 48\text{ V}$

$V_{rms} \neq 120\text{ V}$

$$V_{LC} = 48 - 144 = -96\text{ V}$$

gives phase

$$V_{LR} = \sqrt{(72^2+144^2)} = 161\text{ V}$$

The inductor and resistor have voltages that are 1/4 wave out of phase, so they add like perpendicular vectors.

Series RLC Behavior at different frequencies

Tuesday, October 22, 2019 10:10 AM

Very low frequencies:

$$X_C = \frac{1}{2\pi f C} = \text{huge} = Z$$

The capacitor blocks the current at low frequencies.

Very high frequencies:

$$X_L = 2\pi f L = \text{huge} = Z$$

The inductor blocks the current at high frequencies.

Special, resonance frequency:

$$X_L = X_C \qquad X = 0 \qquad Z = \sqrt{(R^2 + \cancelto{0}{X^2})} = R$$

$$\omega_R L = \frac{1}{\omega_R C}$$

$$\omega_R^2 LC = 1 \qquad \omega_R^2 = \frac{1}{LC} \qquad \omega_R = \frac{1}{\sqrt{LC}}$$

$$2\pi f_R = \frac{1}{\sqrt{LC}}$$

$$f_R = \frac{1}{2\pi\sqrt{LC}}$$

What is special about resonance?

$$Z = \sqrt{\underset{\uparrow \atop \text{fixed}}{R^2} + \underset{\uparrow \atop \text{variable, minimized}}{X^2}}$$

The overall series impedance is the lowest value.
The series circuit's current is the highest value.
The power used by the circuit is the highest value.

$$V_{rms} = 120\text{ V} \qquad R = 300\ \Omega \qquad L = 0.3H \qquad C = 2.5\mu F$$

$$f_R = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.3)(2.5\times10^{-6})}} = 183.8\text{ Hz}$$

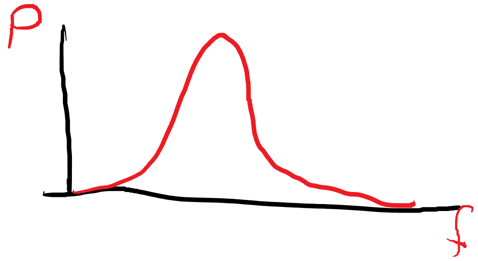
$V_{rms} = 120\text{ V} \qquad R = 300\ \Omega \qquad L = 0.3\text{ H} \qquad C = 2.5\ \mu\text{F}$

$$f_R = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.3)(2.5\times10^{-6})}} = 183.8\text{ Hz}$$

$$P_{Avg} = I_{rms}^2 R = \left(\frac{120}{300}\right)^2 (300) = 48\text{ W}$$

↑ $Z = R$ @ resonance

The maximum power is transferred at the resonant frequency.

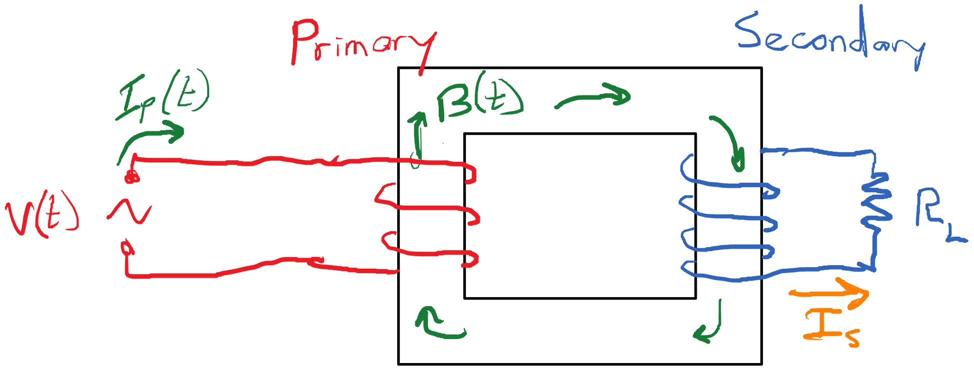


Width related to Q of circuit,
High Q: Narrow peak
Low Q: Wide peak

18. Transformers

Thursday, October 24, 2019 9:20 AM

A transformer is two coils that share the same magnetic flux (per loop).



- $I(t)$ increasing: $\mathcal{E}_p$ opposes $V(t)$
- Secondary generates $B(t)$ to oppose, by inducing $I_s(t)$
- I_s makes I_p larger.

The power supply drives current in the *Primary*, which makes a magnetic field and magnetic flux, and the changing magnetic flux generates voltage in both coils.

Primary Flux: $\Phi_p = N_p \underbrace{BA}_{\Phi_B \text{ per loop}}$

Secondary Flux $\Phi_s = N_s BA$

Induced voltage:

$$\mathcal{E}_p = -\frac{d\Phi_p}{dt} = -N_p \frac{dB}{dt} A$$

$$\mathcal{E}_s = \qquad = -N_s \frac{dB}{dt} A$$

$$\frac{\mathcal{E}_p}{\mathcal{E}_s} = \frac{N_p}{N_s}$$

The voltage ratio is equal to the ratio of the number of loops in each coil.

Most derivations: $\mathcal{E}_p = -N_p \frac{d\Phi_B}{dt}$ $\quad \Phi_B = BA =$ flux per loop.

By itself, the transformer generates two voltages (primary and secondary) that are strictly proportional.

- When current is flowing in the secondary, it generates magnetism to oppose the magnetism of the primary. This means it takes extra current in the primary to generate enough EMF to match the V(t) of the power supply.

- Summary: I_p is low when I_s is low, and I_p is high when I_s is high.

Ideal: $P_{in} = P_{out}$

$\mathcal{E}_p I_p = \mathcal{E}_s I_s$ $\mathcal{E}$ = caligraphy E
= EMF

$V_p I_p = V_s I_s$

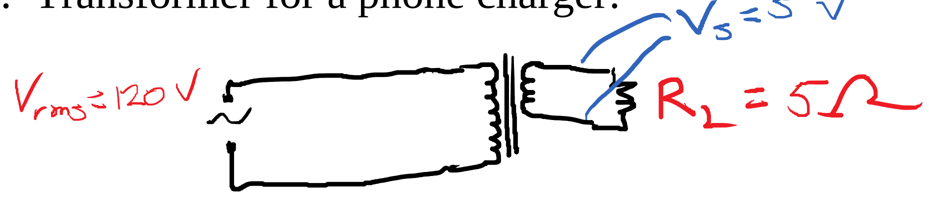
Practically, transformers do "lose" some energy along the way.

Efficiency = $\epsilon = \frac{P_{out}}{P_{in}}$ ϵ = epsilon

$P_{out} = \epsilon P_{in}$

$V_s I_s = \epsilon V_p I_p$

Ex: Transformer for a phone charger:



Turns ratio: $\frac{N_s}{N_p} = \frac{5V}{120V} = \frac{1}{24}$

If the secondary has 100 loops, the primary must have 2400 loops.

If ideal, what currents flow?

$V_s = I_s R_L$

$(5 V) = I_s (5\Omega)$

$1.0 A = I_s$

Ideal: $V_s I_s = \overset{1}{\cancel{\epsilon}} V_p I_p$

$(5V)(1.0A) = (120V) I_p$

$\frac{1}{24} A = 0.0625 = I_p$

Notice that the power supply doesn't "see" the same resistance as the load.

Notice that the power supply doesn't "see" the same resistance as the load.

$$V_p = I_p R_{eq}$$

$$(120\ v) = (0.0625\ A)\ R_{eq}$$

$$120 \cdot 24 = R_{eq} = 2880\ \Omega$$

How does this relate to the load resistance?

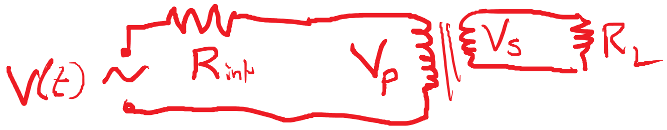
$$\frac{R_{eq}}{R_L} = \frac{2880}{5} = \frac{(120)(24)}{5} = (24)(24)$$

They are related by the turns ratio squared.

$$\frac{R_L}{R_{eq}} = \frac{1}{24 \cdot 24} = \left(\frac{N_s}{N_p}\right)^2$$

Why would we care about the equivalent resistance? Power efficiency is related to this.

Also, if the power supply has some internal resistance, reducing I_p reduces wasted power there.



The power used by R_int is:

$$P_{Rint} = V_{Rint}\ I_{Rint}$$

$$= (I_{Rint})^2 R_{int}$$

Wasted power $= P_{Rint} = I_p{}^2\ R_{int}$

Delivered power $= I_s{}^2 R_L$

Wasted % $= \dfrac{I_p{}^2 R_{int}}{I_s{}^2 R_L}$

Ideal: $V_p I_p = V_s I_s$

$$\frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p}$$

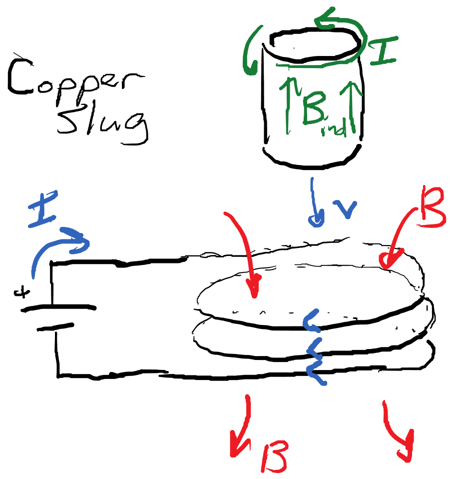
The current ratio is the inverse of the turns ratio (ideally).

How can we reduce the wasted energy?

- Have a low primary current = high primary voltage
- Have low internal resistance = thick wires
- Limit the load resistance to reasonably high values.

19. Review

Tuesday, October 29, 2019 9:23 AM

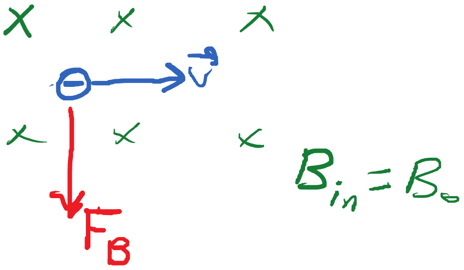


A copper slug is dropped into a coil as shown. What is the direction of the induced eddy current in the copper?

- velocity will be downward
- Current exits battery ⊕
- Coil's B points downward
- Slug's Flux magnitude is increasing
- EMF will oppose flux increase. (Lenz's Law)
  - Induced I makes B_{ind} upward
  - Induced I points right across front.

RHR for Magnetic Force

$$\vec{F}_B = q\vec{v} \otimes \vec{B}$$

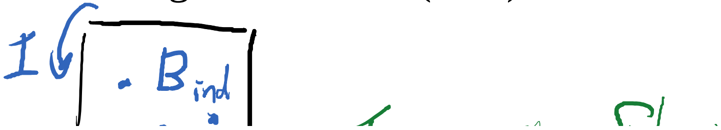


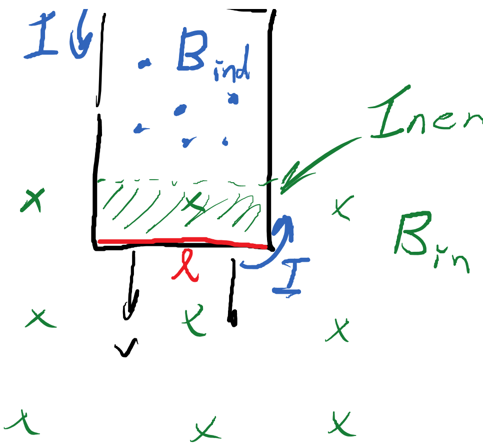
- Index finger = velocity = Rightward
- Middle finger = B = Into page
- Thumb = Top of page
- Negative q, so Force = Bottom of page

Quick Rules: $\vec{F} \perp$ to both $\vec{v}$ and $\vec{B}$; $\vec{v}$ can't be $\parallel$ $\vec{B}$

A proton is dropped into a downward-pointing magnetic field. What's the direction of the magnetic force? (n/a because there is no magnetic force)

A proton is dropped into a northward-pointing magnetic field. What's the direction of the magnetic force? (East)





Fall 2016 Practice 2 #14

$$F_g = F_B$$

$$mg = I\ell B$$

$$(6.6)(9.8) = I(2.0)(6.0)$$

$$I = \underline{\qquad}$$ ✓

$$\mathcal{E} = IR$$

$$vB\ell = IR$$

$$v(6.0)(2.0) = I(40)$$

(The 3.0 m height of the loop didn't matter.)

Resonance Frequency

Tuesday, October 29, 2019 10:06 AM

$$X_L = X_C \qquad f_R = \frac{1}{2\pi\sqrt{LC}}$$

$$2\pi f_R L = \frac{1}{2\pi f_R C}$$

$L = 4\ mH$

$f_R = 1200\ kHz$

$$2\pi(1.2\times10^6)(0.004) = \frac{1}{2\pi(1.2\times10^6)C}$$

To decrease the resonant frequency a
larger capacitor should be used.

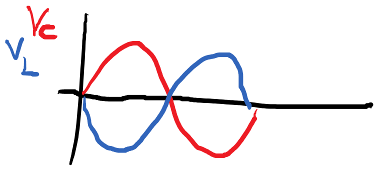
Small electronics use MUCH higher
frequencies, which allows smaller L and C.

If the current in the original question is 1 mA,
how much voltage is there across the inductor?

$$X_L = 2\pi(1.2\times10^6)(0.004)$$
$$= 30.2\ k\Omega$$

$$I X_L = V_L = (0.001)(30.2\ k) = 30.2\ V$$

The capacitor voltage is the same, because X_L = X_C.



The impedance of the LC series, at resonance, is zero.

Reading an AC voltage function:

$$V(t) = 140\ \sin(500\ t)$$

Amplitude $= 140 = V_0$

$\omega = 2\pi f = 500 =$ angular freq.

$$V_{Rms} = \frac{V_0}{\sqrt{2}} = \frac{140}{\sqrt{2}} \approx 100\ V \qquad f = \frac{\omega}{2\pi} = \frac{500}{(2\pi)} = 79.6\ Hz$$

$L = 1.0\ H \qquad X_L = 2\pi f L = (500)(1.0) = 500\ \Omega$

$C = 1.0\ \mu F \qquad = 2000\ \Omega$

$L = 1.0\ H$ $\quad X_L = 2\pi f L = (500)(1.0) = 500\ \Omega$

$C = 1.0\ \mu F$ $\quad X_C = \frac{1}{2\pi f C} = \frac{1}{(500)(10^{-6})} = 2000\ \Omega$

1/500e-6

1/(500 * 1e-6)

$$X = X_C - X_L = 1500\ \Omega$$

$R = 1000\ \Omega$ $\quad Z = \sqrt{R^2 + X^2}$

$$= \sqrt{1000^2 + 1500^2} = 1803\ \Omega$$

$V_{Crms} = I_{Rms} X_C$ $\quad V_{rms} = I_{rms} Z$

$V_{Crms} = V_{rms} \frac{X_C}{Z}$ $\quad \frac{V_{rms}}{Z} = I_{rms}$

$$= (100\ V)\left(\frac{2000}{1803}\right)$$

$$V_{Crms} = 110\ V$$

The power supply voltage is used with the total Z.
The capacitor voltage is used with the capacitor's Z = X_C.

DC Motor

Tuesday, October 29, 2019 10:29 AM

A 12-V DC motor draws 2.0 A when it is stopped, but only 0.5 A when it is spinning freely. What is the back-EMF?

spinning speed

Stopped, $\mathcal{E} = NBA\omega = 0$

$(12\,v) = (2.0\,A)\,R$

$6.0\,\Omega = R$

Spinning

$(12\,v) - \mathcal{E} = (0.5)(6)$

$12 - \mathcal{E} = 3$

$9 = \mathcal{E} =$ Back EMF

The back-EMF effectively subtracts from the power supply voltage.

Faraday's Law

Tuesday, October 29, 2019 10:34 AM

$$|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right|$$

Coil:

$$\Phi_B = NBA\cos\theta$$

$$\left|\frac{d\Phi_B}{dt}\right| = N\frac{dB}{dt}A$$

$$= (300)\frac{\Delta B}{\Delta t}(0.04)$$

$N = 300$

$A = (0.2)(0.2) = 0.04$

$\cos\theta = 1$

B "perpendicular to plane of coil" means B is "parallel to the normal vector" which is theta=0.

If the magnetic field was 0.5 T and is reversed over a period of 2.0 s, what is the average EMF?

$$\frac{dB}{dt} = \frac{1.0\ T}{2.0\ s}$$