Exam1 Average: 59%
After grades are uploaded, use your Course Average to see where you stand.
(60% Exams, 15% HW, 25% Lab)

Equivalent units to the farad (F)?

This measures capacitance (C).

\[ C = \frac{k \varepsilon_0 A}{d} \]

[ \[ \text{Q} = CV \] \]

[ \[ \text{S} = \text{AF} \] \]

For the 25 mg styrofoam ball:

\[ 25 \text{ mg} = 25 \times 10^{-3} \text{ g} \]
\[ = 25 \times 10^{-6} \text{ kg} \]

Mass is our only quantity where the fundamental unit is not the base unit. Must convert to kg to use in F=ma, F=mg, etc.

Spherical Gauss's Law

Kirchhoff's Law Circuit

\[ I_1 + I_2 = I_3 \]

\[ I_3 = 34 \]
Resistor Network

\[ 11 - 12 = -I_2 R_2 + I_1 R_1 \]
\[ 11 - 12 + I_2 R_2 - I_1 R_1 = 0 \]

\[ \epsilon_2 = V_3 + V_2 \]
\[ 12 = V_3 + 4 \]
\[ 8 = V_3 \]
\[ V_3 = I_3 R_3 \]
\[ 8 = 3 R_3 \]

I_2 + I_3 = 2A
\[ I_2 = 2I_3 \]
\[ 2I_3 + I_3 = (2A) \]
\[ I_3 = 0.667A \]
\[ V_3 = I_3 R_3 \]
\[ (4V) = (0.667A) R_3 \]
\[ 6.2 = R_3 \]

Capacitor in electrostatic and RC circuit

\[ Q = CV = (400 \mu F)(15V) = 6000 \mu C \]
\[ E = \frac{V}{d} \]

\[ V_1 = V_0 e^{-\frac{t}{1/2}} \]
\[ V_2 = V_0 e^{-\frac{t}{2/2}} \]
\[ V = 30/2 \]
\[ V = 0 \]
Every 30 second interval cuts the voltage in half.

\[
\frac{V_2}{V_1} = \frac{e^{-t_2/2}}{e^{-t_1/2}} = e^{-(t_2-t_1)/2} = e^{-30/2} = e^{-15}
\]

\[
\frac{V_1}{V_0} = e^{-t_1/2} = e^{-15}
\]

\[
\frac{V_2}{V_0} = e^{-t_2/2} = e^{-30/2} = e^{-15}
\]

\[
\frac{2}{2^1} = 2^{3}
\]

\[
0.5 = e^{-15}
\]

\[
\ln(0.5) = -15
\]

\[
T = \frac{30}{\ln(0.5)} = 43.5 \text{ s}
\]
Magnetostatics: Magnetism without moving objects. Any currents are DC currents.

Magnetism is a vector field, like the electric field. The magnetic field (\(\mathbf{B}\)) is shaped differently. Magnetic fields always form "loops".

What creates magnetic fields?
- Magnetic materials.
- Electric Currents
- Fluctuating electric fields

What do magnetic fields do?
- Force on moving charges and currents.
- Torque on magnetic dipoles.
- Attract or repel magnetic dipoles.
- Generate electric field and voltage.

We need 3-D directions and coordinate systems.

The word "Up" is ambiguous. Need clues to figure out what it means.

Always try to envision the 3-D situation separately from the diagram.
Always try to envision the 3-D situation separately from the diagram.

RHR for magnetic field of a current:
- Point thumb in direction of $I$.
- Curl fingers around current to find $B$. 
Generally, \( \mathbf{B} \) is caused by currents. 

\[
\mathbf{B} = \frac{\mu_0 I}{2\pi r^2}
\]

Vector cross product:
- \( \mathbf{C} \) is perpendicular to both \( \mathbf{A} \) and \( \mathbf{B} \).
- \( \mathbf{C} \) has a magnitude of \( AB \sin(\theta) \), where \( \theta \) is the angle between \( \mathbf{A} \) and \( \mathbf{B} \).
- Pick direction of \( \mathbf{C} \) by RHR:

Example: Infinite, straight current

\[
\mathbf{B} = \frac{\mu_0 I}{2\pi r} \int_{-\infty}^{\infty} \frac{y\,dx\,\hat{\mathbf{k}}}{x^2+y^2} = \frac{\mu_0 I}{2\pi y} \hat{\mathbf{k}}
\]

\[
B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} \quad r = \text{distance from wire}
\]

Common to think about magnetism of two wires.
Wires each carrying 1.5 A of current spaced 4 cm apart. B at midpoint?

\[ B = \frac{\mu_0 I}{2\pi r} \]

\begin{align*}
B_1 &= \frac{\mu_0 (1.5)}{2\pi (0.02)} = 3 \times 10^{-5} T \\
B_2 &= \frac{\mu_0 (1.5)}{2\pi (0.02)} = 3 \times 10^{-5} T
\end{align*}

Hint: Type \( \mu_0 \) as \( \pi \times 10^{-7} \frac{T \cdot m}{A} \)

At a point 1 cm above \( I_1 \):

\[ B = B_1 - B_2 = \frac{\mu_0 (1.5)}{2\pi (0.01)} - \frac{\mu_0 (1.5)}{2\pi (0.05)} \]

Circular Loop of Wire:

\[ \mathbf{B} = \frac{\mu_0 I}{4\pi R} \int_0^{2\pi} \hat{r} \frac{R^2}{r^2} d\theta = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{R^2}{R^2} d\theta = \frac{\mu_0 I}{2R} \hat{k} \]

Along the axis, in front of and behind the loop, B is weaker. It's kind of bell-shaped function.
This is akin to Gauss's Law and summarizes the result of the Biot-Savart Law.

\[ \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} \]

Strategy, make that integral easy.
- Make sure \( B = \text{constant} \)
- Keep dot product simple (\( \theta = \text{constant} \))

\[ \oint \mathbf{B} \cdot d\mathbf{l} = BL = B_z 2\pi R \]

Result:

\[ 2\pi RB = \frac{\mu_0 I}{2\pi R} \]

Magnetic field of a thick wire:

\[ B_{\text{in}} = \frac{\mu_0 I}{2\pi R} \]

If \( I_{\text{out}} = I_{\text{in}} \), then \( I_{\text{enc}} = 0 \) for any point outside the coaxial cable.
outside the coaxial cable.
For a many-loop coil with all of the loops on top of each other, just multiply by N:

\[ B = \frac{\mu_0 NI}{2R} \]

In a solenoid coil, the loops are not on top of each other, they are next to each other.

Inside: \( \vec{B} = B_0 \hat{i} \)

Outside: \( \vec{B} \approx 0 \)

\[ B_{\parallel} l = \mu_0 I_{\text{enc}} \]

\[ B_0 \times = \mu_0 I (N \frac{x}{l}) \Rightarrow B_0 = \frac{\mu_0 NI}{l} \]

4 important formulas:

\[ B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} \quad B_{\text{coil}} = \frac{\mu_0 NI}{2R} \quad B_{\text{sol}} = \frac{\mu_0 NI}{l} \]

\[ B_{\parallel} l = \mu_0 I_{\text{enc}} \]
The most basic result of magnetism is forces on moving charges.

\[ \mathbf{F}_B = q \mathbf{v} \times \mathbf{B} \]

The cross product does a few things:
• The velocity must have a component perpendicular to \( B \) to generate force.
• The direction of the force is by the RHR for cross products.
  ○ \( F_B \) is perpendicular to velocity.
  ○ \( F_B \) is perpendicular to the magnetic field.

Since the force is always perpendicular to the velocity:
• \( F_B \) cannot change the speed of the particle's motion.
• \( F_B \) cannot transfer energy.

• \( F_B \) does change the direction of motion of the particle. This makes the particle move in uniform circular motion.
This radius of curvature is the basis of a mass spectrometer.

How do we generate a beam of particles with known velocity?
Method 1: Linear accelerator.

\[
\begin{align*}
q \Delta V &= \frac{1}{2} mv^2 \\
\Delta V &= \text{accelerating voltage} \\
v &= \text{velocity}
\end{align*}
\]

\[
\text{Notation:} \quad v = \sqrt{\frac{2q \Delta V}{m}}
\]

\[
R = \frac{mv}{qB} = \frac{m}{qB} \sqrt{\frac{2q \Delta V}{m}} = \frac{1}{B} \sqrt{\frac{2m \Delta V}{q}}
\]

If I have a bunch of particles with different masses and the same charge, and I accelerate them all with the same voltage, how does R depend on mass? (Here, R is proportional to the square root of mass.)

It would be nice to have a linear dependence between mass and radius. That would require all particles to have the same velocity (not voltage).

Method 2: Velocity Selector
- Start with particles of various velocities.
- Pick out the ones you want.

\[
F_B = F_E \\
q\nu_B = qE_0 \\
v = \frac{E}{B}
\]

At this velocity, the net force is zero.
- Too fast: particle bends left.
- Too slow: particle bends right.

So now we have a beam of particles, all with the same velocity.
\[ R = \frac{mv}{2B} \quad R \propto m \]
The force on a current is boring, but it is the original definition of the ampere. Consider two parallel wires:

\[ \vec{B} = B_0 \hat{j} \]
\[ \vec{F}_B = \mu_0 \vec{I} \times \vec{B} \]
\[ \vec{F}_B = I I_2 B \cos \theta \]

Direction of \( \vec{F}_B \) is "opposites repel".

\[ \mu_0 = 4 \pi \times 10^{-7} \ \text{Tm/A} \]

If the distances are "nice round numbers", then 1.0 A of current causes a "nice round number" value of force. This was the original definition of the ampere, and is why \( \mu_0 \) is what it is.
The two forces together generate a torque on the loop. How much?

\[ T_{\text{net}} = 2 T_0 = 2(F_B d) = 2(IAI B \frac{W}{2} \sin \theta) \]

\[ T = BAI \sin \theta \]

\[ T = NBAI \sin \theta \]

Typically, the loop is a coil with many turns of wire.

This torque tries to make \( \mathbf{n} \)-hat point in the direction of \( \mathbf{B} \).

This is the torque produced by an electric motor. Note: The motor doesn't need to be spinning to make torque.

Electric generators are also coils in magnetic fields. This means electric generators also exert torque. But, they only exert magnetic torque when current is flowing, i.e. when a load is attached.
Electromagnetic Induction is covered by Faraday's Law:

\[ \mathcal{E} = -\frac{d\Phi_b}{dt} \]

\[ \Phi_b = \int \mathbf{B} \cdot d\mathbf{A} = NBA \cos \Theta \]

This law actually summarizes two physical effects:

- Moving a wire in a magnetic field - Motional EMF
- Fluctuating magnetic field - Need a new explanation

Motional EMF - Drop a rectangular loop "into a magnetic field".

Charges in the wire that are falling in the magnetic field feel a magnetic force.

\[ \mathbf{F}_B = q \mathbf{v} \times \mathbf{B} \]

\[ \mathbf{F}_E = q \mathbf{E} \times \mathbf{v} \]

This shows that Faraday's Law covers the case of a moving wire in a magnetic field.
Ex: A 25 gram wire that is 5 cm wide and 7 cm tall falls into a 0.04 T magnetic field. What is the resistance of the wire if it falls at 9 m/s?

\[ E = B v x = (0.04 T)(9 m/s)(0.05 m) \]
\[ E = 0.018 V \]

**Bottom Leg:**
\[ F_B = I L B \]

**For const-velocity,**
\[ F_B = F_g \]
\[ F_g = mg \]
\[ I (0.05 m)(0.04 T) = (0.025 kg)(9.8 \text{ N/kg}) \]
\[ I = 122.5 A \]
\[ E = I R \]
\[ R = \frac{E}{I} = \frac{0.018 V}{122.5 A} = 1.47 \times 10^{-4} \Omega \]

Stage 1: Charges in falling wire feel magnetic force along the wire. This is balanced by the generated electric force, which comes from the generated electric field, which is an EMF / length.

Stage 2: The EMF drives current around the loop. The current (along the wire) feels a magnetic force perpendicular to the wire. This is balanced by gravity, so the loop falls with zero acceleration.

We looked at the loop falling into a magnetic field. Only the bottom leg of the loop experienced motional EMF. Once the loop is "in" the magnetic field, the upper leg will also generate voltage, but the other way around the loop.

If the loop starts to fall "out of the magnetic field", then the bottom leg will stop generating voltage. The top leg will make current flow the other way.
Faraday's Law says if the flux changes, there is an EMF.

$$\frac{d\Phi_B}{dt} = \frac{\mu_0 N^2 A}{l} \frac{dI}{dt}$$

Changing the current in a coil generates EMF.

Ex: Unplug an electric motor. This makes the current stop VERY rapidly. The coil in the motor generates a HUGE EMF spike.

What's behind this effect?
- Fluctuating B "stirs up" electric field.

It's nice to not have to deal with the details. Faraday's Law magically covers all cases.
This is formed from a spinning coil in a magnetic field.

Motors vs. Generators

Both are coils in B fields.

Generator
We apply torque to spin.
EMF pushes current.
Current causes drag torque.

Motor
We apply voltage to push I.
I causes drive torque.
Spinning causes Back EMF.

Back EMF in a motor is actually good for us.
\[ I = \frac{V_{\text{applied}}}{R_{\text{motor}}} \]

\[ V_A - \varepsilon_{\text{back}} = IR_m \]

\[ I = \frac{V_{\text{applied}} - \varepsilon_{\text{back}}}{R_m} \]