What happened? The document version that was proofread was not the version that was split to Version A and Version B, and then printed.

- 5 directly affected questions were thrown out.
- Question #14 was counted as extra credit because it used the result of #13.

Lesson learned:

### #15-16

3 \( \mu \)F capacitor

\[
C = \frac{\varepsilon_0 A}{d}
\]

Freq of power supply halved

\( X_C \) doubles

\[
X_C = \frac{1}{2\pi fC}
\]

\[ V_0 = I_0 X_C \]

\[ \text{const} \quad \text{doubles} \]

\[ \text{must halve} \]

### #11

\[ \vec{F}_B = q \vec{v} \times \vec{B} \] on \( \oplus \) charges

drives current CW.

### #10:

\[ \vec{F}_B = IILB \] on current resists gravity

\[ F_g = F_B \]

\[ mg = IILB \]

upward.
#26-27:

\[ V_R = 30 \sin(4000\pi t) \]

\[ V_{R_{rms}} = 21.3 \text{ V} \]

\[ V_R = I R \]

\[ (21.3) = I (60) \]

\[ 0.35 \ldots = I_{rms} \]

\[ P_{avg} = I_{rms}^2 R = V_{rms} I_{rms} = (21.3)(0.35) = 7.5 \text{ W} \]

Whole circuit:

\[ V_{rms} = I_{rms} Z \]

\[ V_{rms} = (0.3534)(100\Omega) \]

Power Supply:

\[ V_{rms} = 35.3 \text{ V} \]
Simplest oscillator: Harmonic oscillator made from a mass and a spring.

\[ F_{\text{net}} = ma \]
\[ F_s = ma_s \quad (\text{Block's } x = x_s) \]
\[ -kx_s = m \frac{d^2x_s}{dt^2} \]

What is \( x_s \) as a function of time?
\[ x_s = x_0 \sin(\omega t) = x_0 \sin(2\pi ft) \]
\[ v_s = \frac{dx_s}{dt} = x_0 \cos(\omega t) \omega \]
\[ a_s = \frac{d^2x_s}{dt^2} = \omega x_0 (-\sin(\omega t)) \omega = -\omega^2 x_0 \sin(\omega t) \]

\[ -kx_s = m \left[ -\omega^2 x_s \right] \]
\[ k = m \omega^2 \]
\[ \omega = \sqrt{\frac{k}{m}} \]
\[ \omega = 2\pi f \]
\[ f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]

The mass-and-spring traces out a sinewave with an angular frequency as calculated above.

How is this related to the given situation?

\[ v_i = v_o = \text{given} \]
\[ v_s = \omega x_0 \cos(\omega t) \]
For an oscillation, the displacement amplitude ($x_0$) is proportional to the velocity amplitude ($v_0$).

How did we do this in Physics I? Conservation of energy.

\[ t=0 \Rightarrow v_0 = \omega x_0 \]

\[ v_f = \sqrt{\frac{k}{m}} x_f \]

\[ \max \ x_f = x_0 \sin(\omega t) \]

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The energy analysis and the oscillation analysis produce the same result.

If the mass is attached to the spring, it will continue to follow the sinewave function indefinitely. This is a basic oscillation.

\[ \text{Spring Energy:} \quad U_s = \frac{1}{2} k x_s^2 \]

\[ \text{Kinetic Energy:} \quad K = \frac{1}{2} m v^2 \]

\[ \frac{1}{2} m v_i^2 + 0 = 0 + \frac{1}{2} k x_f^2 \]

\[ v_f^2 = \frac{k}{m} x_f^2 \]

\[ v_f = \sqrt{\frac{k}{m}} x_f \]

\[ \text{Initial} = \text{Final} \]

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\[ \text{Generic:} \quad x = x_0 \sin(\omega t - \phi) \]

\[ \text{Frequency:} \quad \omega = \text{angular freq} \quad f = \text{frequency} \]

\[ \omega = 2 \pi f \quad \text{depend on which objects are involved!} \]

\[ \text{Amplitude:} \quad x_0 = \text{amplitude} \quad v_0 = \omega x_0 = \text{max speed} \]

\[ \text{depend on setup.} \]

\[ \text{Phase:} \quad \phi = \text{initial phase} \]

\[ (\omega t - \phi) = \omega (t - t_0) \]

\[ \sin(\omega (t - t_0)) \quad \text{starts at } t_0 \]

Note: It's common to adjust the phase by $\pi/2$
by switching from a sine to a cosine.
Another common way of dealing with phase is to split the function into sine and cosine components.

\[ x_0 \sin(wt) = A \sin(wt) + B \cos(wt) \]

\[ x_0 = A^2 + B^2 \]

\[ A = x_0 \sin \phi \]

\[ B = -\frac{B}{A} \]

This form is used when there is nonzero initial position *and* nonzero initial velocity.

Initial position = \( B \)

Initial velocity = \( \omega A \)

Mass-and-spring oscillator.

\[ m = 0.25 \text{ kg} \]

\[ k = 500 \text{ N/m} \]

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{500}{0.25}} = 2.25 \text{ Hz} \]

Given an initial displacement of 1 cm and an initial speed of 15 cm/s, what are the amplitude and max speed?

\[ x(t) = A \sin(wt) + B \cos(wt) \]

\[ B = 0.01 \text{ m} \]

\[ \omega = 2\pi f = \sqrt{\frac{500}{0.25}} = 14.14 \text{ rad/s} \]

\[ \omega A = v_i \]
\[ A = 0.15\, \text{m/s} = \text{initial speed} \]
\[ A = \frac{0.15}{14.14} = 0.0106\, \text{m} \]

Amplitude 
\[ x_0 = \sqrt{A^2 + B^2} = \sqrt{0.01^2 + 0.0106^2} \]
\[ = 0.0146\, \text{m} = 1.46\, \text{cm} \]

Max speed 
\[ v_0 = \omega x_0 = (14.14)(0.0146) \]
\[ = 0.206\, \text{m/s} = 20.6\, \text{cm/s} \]