What happened? The document version that was proofread was not the version that was split to Version A and Version B, and then printed.

- 5 directly affected questions were thrown out.
- Question #14 was counted as extra credit because it used the result of #13.

Lesson learned:

Lesson Learned:

#15-16

3 μF capacitor

\[ C = \frac{\kappa \varepsilon_0 A}{d} \]

Freq of power supply halved

\[ X_C \text{ doubles} \]

\[ V_0 = I_0 \times X_C \]

\( \text{const} \uparrow \text{doubles} \)

must halve

#11

\[ \mathbf{F}_B = q \mathbf{v} \times \mathbf{B} \] on \( q \) charges

drives current \( \text{CW} \)

#10:

\[ \mathbf{F}_B = IILB \] on current resists gravity

\[ F_g = F_B \]

\[ m \mathbf{g} = IILB \]
#26-27:

Given:

\[ V_R = 30 \sin(4000 \pi t) \]

\[ V_{R_{rms}} = 21.3 \text{ V} \]

\[ V_R = I_R R \]

\[ (21.3) = I (60) \]

\[ 0.35 \ldots = I_{rms} \]

\[ P_{avg} = I_{rms}^2 R = V_{rms} I_{rms} = (21.3)(0.35) = 7.6 \text{ W} \]

Whole circuit:

\[ V_{rms} = I_{rms} Z \]

\[ V_{rms} = (0.3534)(100 \Omega) \]

Power supply: \[ V_{rms} = 35.3 \text{ V} \]
Simplest oscillator: Harmonic oscillator made from a mass and a spring.

\[ F_{\text{net}} = m a \]
\[ F_S = m a_S \quad \text{(Block's } x = x_S) \]
\[ -k x_S = m \frac{d^2 x_S}{dt^2} \]

What is \( x_S \) as a function of time?
\[ x_S = x_0 \sin(\omega t) = x_0 \sin(2\pi f t) \]
\[ v_S = \frac{dx_S}{dt} = x_0 \cos(\omega t) \omega \]
\[ a_S = \frac{d^2 x_S}{dt^2} = \omega x_0 (-\sin(\omega t)) \omega = -\omega^2 x_0 \sin(\omega t) \]

\[ -k x_S = m \left[-\omega^2 x_S \right] \]
\[ k = m \omega^2 \]
\[ \omega = \sqrt{\frac{k}{m}} \]

The mass-and-spring traces out a sine wave with an angular frequency as calculated above.

\[ \omega = 2\pi f \]
\[ f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]

How is this related to the given situation?

\[ v_i = v_o = \text{given} \quad v_S = \omega x_0 \cos(\omega t) \]
For an oscillation, the displacement amplitude ($x_0$) is proportional to the velocity amplitude ($v_0$).

How did we do this in Physics I? Conservation of energy.

\[ t = 0 \Rightarrow v_i = \omega x_0 \]

\[ v_i = \sqrt{\frac{k}{m}} x_f \]

\[ \text{Max } x_f = x_0 \sin(\omega t) \]

\[ \text{Spring Energy: } U_s = \frac{1}{2} k x_s^2 \]

\[ \text{Kinetic Energy: } K = \frac{1}{2} m v^2 \]

\[ \frac{1}{2} m v_i^2 + 0 = 0 + \frac{1}{2} k x_f^2 \]

\[ v_f^2 = \frac{k}{m} x_f^2 \]

\[ v_i = \sqrt{\frac{k}{m}} x_f \]

The energy analysis and the oscillation analysis produce the same result.

If the mass is attached to the spring, it will continue to follow the sinewave function indefinitely. This is a basic oscillation.

\[ \text{Generic: } x = x_0 \sin(\omega t - \phi) \]

**Frequency:**
- $\omega = \text{angular freq}$
- $f = \text{frequency}$
- $w = 2\pi f = \text{depend on which objects are involved}$

**Amplitude:**
- $x_0 = \text{amplitude}$
- $v_0 = \omega x_0 = \text{max speed}$
- depend on setup.

**Phase:**
- $\phi = \text{initial phase}$
- $t_0 = \text{start time}$

\[ (\omega t - \phi) = \omega (t - t_0) \]

\[ \sin(\omega (t - t_0)) \text{ starts @ } t_0 \]

Note: It's common to adjust the phase by $\pi/2$ by switching from a sine to a cosine.
Another common way of dealing with phase is to split the function into sine and cosine components.

\[ x_0 \sin(\omega t - \phi) = A \sin(\omega t) + B \cos(\omega t) \]

\[ x_0^2 = A^2 + B^2 \]

\[ A = x_0 \sin \phi \]

\[ B = -x_0 \cos \phi \]

This form is used when there is nonzero initial position \*and\* nonzero initial velocity.

Initial position = \( B \)

Initial velocity = \( \omega A \)

Mass-and-spring oscillator.

\[ m = 0.25 \text{ kg} \]

\[ k = 50 \text{ N/m} \]

\[ f = \frac{1}{2\pi \sqrt{\frac{k}{m}}} = \frac{1}{2\pi} \sqrt{\frac{50}{0.25}} = 2.25 \text{ Hz} \]

Given an initial displacement of 1 cm and an initial speed of 15 cm/s, what are the amplitude and max speed?

\[ x(t) = A \sin(\omega t) + B \cos(\omega t) \]

\[ B = 0.01 \text{ m} = \text{ initial displacement}, \]

\[ \omega = 2\pi f = \sqrt{\frac{50}{0.25}} = 14.14 \text{ rad/s} \]

\[ \omega A = V_i \]
14.14 \[ A = 0.15 \text{ m} = \text{ initial speed} \]

\[ A = \frac{0.15}{14.14} = 0.0106 \text{ m} \]

Amplitude \[ x_0 = \sqrt{A^2 + B^2} = \sqrt{0.01^2 + 0.0106^2} \]

\[ = 0.0146 \text{ m} = 1.46 \text{ cm} \]

Max speed \[ v_0 = \omega x_0 = (14.14)(0.0146) \]

\[ = 0.206 \text{ m/s} = 20.6 \text{ cm/s} \]
Waves are not stuff.

A wave is an organized disturbance in a set of coupled oscillators.
- Coupled: Displacement of one oscillator exerts a force on its neighbors.
- Disturbance: Can be a pulse, but often an oscillation

General wave solution:

\[ f(x - vt) \]

Oscillating wave solutions:

\[ \sin \left( \frac{2\pi x}{\lambda} - 2\pi f t \right) \]

Repeats when:
- \( \Delta \left( \frac{2\pi x}{\lambda} \right) = 2\pi \)
- \( \Delta x = \lambda = \text{wavelength} \)
- \( \Delta (2\pi ft) = 2\pi \)
- \( \Delta t = \frac{1}{f} = T = \text{period} \)

Wave speed:
- \( \left( \frac{2\pi x}{\lambda} - 2\pi f t \right) = \frac{2\pi}{\lambda} (x - f t) \)
- \( \mathbf{v} = \mathbf{f} \lambda \)
- \( \mathbf{v} = \frac{1}{T} \)

The wave speed is the speed of propagation of the disturbance.
No actual material is moving at that speed.

Sample wave speeds:

\[
\begin{align*}
\text{Sound} & : \quad v \approx 340 \, \text{m/s} \\
\text{Light} & : \quad v = 3 \times 10^8 \, \text{m/s} \\
\text{String} & : \quad v = \sqrt{\frac{F}{\mu}} \\
\text{Water} & : \quad v \approx 1 - 10 \, \text{m/s}
\end{align*}
\]

All waves involve oscillations of at least 2 variables.
At least one of these variables is a vector. (no pure scalar waves)

The direction of the vector is either parallel or perpendicular to the
direction of the wave propagation velocity.
- Longitudinal: oscillations parallel to propagation.
- Transverse: oscillations perpendicular to propagation.

\[ v = f \lambda \]

Range of audible sound waves: 20 Hz to 20 kHz

\[ f_1 = 20 \text{ Hz} \quad v = 340 \text{ m/s} \quad \lambda_1 = \frac{340 \text{ m/s}}{20 \text{ Hz}} = 17 \text{ m} \]

Typical musical instrument is 1/4 wavelength.

\[ f_2 = 20 \text{ kHz} \quad \lambda_2 = \frac{340 \text{ m/s}}{20 \text{ kHz}} = 17 \text{ mm} = 0.017 \text{ m} \]
When oscillators are coupled, the frequency of the oscillations is maintained as the disturbance moves from one to the other.  

\[ f = 440 \text{ Hz} \]

\[ f = 440 \text{ sound} \]

\[ f = 440 \text{ in ear} \]

Exception: When the source and/or observer is moving toward or away from the wave.

When the source is moving away from the wave, each successive peak has to travel a further distance. This stretches out the wavelength of the wave.  

By the time an observer hears the sound, the peaks occur at more separated times.

Source moving away -> observed frequency is slower (lower freq)

The moving observer detects successive peaks at different times than a stationary observer, because each peak must travel further.

Observer moving away -> observed frequency is slower (lower freq)

\[ f_0 = f_\text{s} \frac{v + v_0}{v - v_\text{s}} \]

+ \( v_0 \) = toward wave

+ \( v_\text{s} \) = toward wave

For light waves, the formula is more complicated.
\[ f_0 = f_s \sqrt{\frac{V + V_{\text{rel}}}{V - V_{\text{rel}}}} \]

Simple version (use this whenever possible):

\[ \frac{\Delta f}{f} = \frac{V_{\text{rel}}}{V_{\text{wave}}} \]

Good when \( V_{\text{rel}} < 15\% V_{\text{wave}} \)
Waves in general:

\[ f(x, t) = A \sin \left( \frac{2\pi}{\lambda} x - 2\pi f t \right) \]

Wavelength: \( \lambda = \text{repeat distance} \)
Frequency: \( f = \frac{1}{T} \) \( T = \text{period = repeat time} \)

Wave interference: When two waves meet, their functions add. (superposition principle)

When two sinewaves meet, and they have the same frequency, they can add or subtract depending on their relative phase.

\[ \begin{align*}
\text{wave} + \text{wave} &= \text{wave} \\
\text{wave} + \text{wave} &= \text{null}
\end{align*} \]

The relative phase between the two waves determines how they add.

\[ \Delta \phi = 0, \frac{2\pi}{\lambda}, \frac{4\pi}{\lambda}, \ldots \text{ constructive} \]
\[ f_1 = \sin \left( \frac{2\pi}{\lambda} x - 2\pi f_1 t + \phi_1 \right) \]
\[ f_2 = \sin \left( \ldots x - \ldots \frac{2\pi}{\lambda} t + \phi_2 \right) \]
\[ \Delta \phi = \pi, 3\pi, 5\pi, \ldots \text{ destructive} \]

Reflection: A wave's energy can "bounce" off a surface or a discontinuity in the medium. During reflection, a wave basically generates a reverse-direction copy of itself. At the point of reflection, either the wave or its conjugate must be zero.

Ex: Reflection of string waves.
- Fixed end must have zero displacement. The reflected wave must combine with the original wave to cancel out and achieve a zero value at the point of reflection.
• Free end must have peak displacement. The reflected wave is "right-side-up" as compared to the original wave.

An upside-down reflection is like a phase addition of pi.

Traveling waves can add to form standing waves.

\[ y(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t) = 2y_m \sin(kx) \cos(\omega t) \]

Standing wave resonance is caused when a wave reflects back and forth in a **cavity**. (A region that carries the wave and has reflective ends.) The wave can interfere with itself.

This only happens if the copy of the wave "lines up" with the original wave. The travel distance is \(2L\), twice the cavity length.

\[ 2L = n\lambda \]

Often we're concerned with frequencies.

\[ f = \frac{v}{2L} \]

\[ f = \frac{\nu}{L} = \frac{\nu}{2L} \]

\[ f = f_1n \]

This can happen with sound waves as well. The reflection is a little different, but the same math applies.
If the two ends of the cavity produce *different* kinds of reflections (one reversing and one not), then we're looking for a time when the sinewave is *opposite* of what it is at the origin.

\[ 2L = (n+\frac{1}{2})\lambda \]  
\[ 4L = (2n+1)\lambda \]

With *different* ends, the longest wavelength is 4L.

\[ f = \frac{v}{\lambda} = \frac{v}{4L} n' \quad (n' = 1, 3, 5, \ldots) \]

Ex: Air cavity with open ends, length = 0.8 m.

\[ v = 340 \text{ m/s} \quad \lambda_1 = 2L = 1.6 \text{ m} \quad f_1 = \frac{340 \text{ m/s}}{1.6 \text{ m}} = 212.5 \text{ Hz} \]

\[ f_2 = 2f_1 = 425 \text{ Hz} \]

\[ f_3 = 3f_1 = 617.5 \text{ Hz} \]

\[ \Delta f = 212.5 \text{ Hz} \]

What if one end is blocked?

\[ \lambda_1 = 4L = 3.2 \text{ m} \quad f_1 = \frac{340 \text{ m/s}}{3.2 \text{ m}} = 106.25 \text{ Hz} \]

\[ f_2 \text{ not allowed} \quad f_3 = 3f_1 = 318.75 \text{ Hz} \]

\[ \Delta f = 318.75 - 106.25 = 212.5 \text{ Hz} \]
Example from RF theory: 1/4 wave antenna:

According to "standard" antenna calculations, the speed of RF waves on the wire is 2.85e8 m/s.

\[ c = 3 \times 10^8 \text{ m/s} \]
Energy (J)
\[ P = \frac{\Delta \text{Energy}}{\Delta t} \]
\[ \text{Spread thru Time} \]
\[ \text{Accumulate over Time} \]
\[ \Delta \text{Energy} = \int P \, dt = P \, dt \]

Power (W = J/s)
\[ I = \frac{P}{\text{Area}} \]
\[ \text{Spread over Area} \]
\[ \text{Accumulate on Surface} \]
\[ P = \iint I \, dA \cos \theta \]
\[ P = IA \cos \theta \]

Intensity (W/m\(^2\))
(Flux density)

Two basic "shapes" of waves:

- **Plane waves**: Uniform intensity and propagation direction.
  Wave fronts form planes.
- **Spherical waves**: Generated by a point source and spread from there.
  Propagation direction is radially away from source.
  Intensity is strong near the source and gets weaker with distance.
  Wave fronts form spheres.

For travelling waves, the power of each wave front is consistent.

\[ P = IA \]

\[ P = I \frac{4\pi r^2}{4\pi r^2} = I \]

\[ (\cos \theta = 1 \text{ b/c propagation \ perp to wave front}) \]

Ex: A light source has an intensity of 100 W/m\(^2\) at 10 m, how bright is it at 20 m?

\[ I \propto \frac{1}{R^2} \]

\[ \frac{I_2}{I_1} = \left( \frac{1/R_2}{1/R_1} \right)^2 \]
Energy, power, and intensity span huge ranges of values.

\[
\frac{I_2}{I_1} = \left(\frac{R_1}{R_2}\right)^2
\]

\[
I_2 = \frac{1}{4} I_1 = 25 \text{ W/m}^2
\]

\[
\frac{I_2}{I_1} = \left(\frac{10}{20}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}
\]

\[
\frac{I_{\text{max}}}{I_{\text{min}}} \sim 10^{10}
\]
"Level" is a way of describing energy-like quantities on a logarithmic scale. This compresses large values and stretches small values. Ratios between related values always have the same size. There is no way to represent zero intensity.

Levels *always* represent ratios. The value (in decibels) represents some factor between two intensities.

\[
\frac{I}{I_0} = 10^{\text{Level in } dB/10}
\]

\[
\log_{10} \left( \frac{I}{I_0} \right) = \text{Level in } dB/10 \implies \text{Level in } dB = 10 \log_{10} \left( \frac{I}{I_0} \right)
\]

Because beta is an exponent, addition of levels (in dB) corresponds to multiplying the intensity several times.

<table>
<thead>
<tr>
<th>( \beta ) (dB)</th>
<th>( \frac{I}{I_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
</tr>
<tr>
<td>1</td>
<td>1.26</td>
</tr>
<tr>
<td>3</td>
<td>2.0</td>
</tr>
<tr>
<td>5</td>
<td>( \sim 3 )</td>
</tr>
<tr>
<td>7</td>
<td>5.0</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>-37</td>
<td>( \frac{1}{5000} )</td>
</tr>
</tbody>
</table>

Zero decibels means no change. One decibel is a 26% increase.

3 dB is a doubling of intensity.

5 dB is approx. tripling intensity.

7 dB is a factor of 5.

10 dB is a tenfold increase.

\[ 10^{-3.7} = 0.0002 \]
\[ \frac{1}{5000} = 0.0002 \]

A speaker produces a 75 dB sound when it is 3.0 m away. How loud is the sound from 5.0 m?
How loud is the sound from 5.0 m?

\[ \frac{I}{I_o} = \left( \frac{3}{5} \right)^2 = 0.36 \quad \log_{10}(0.36) = -0.44 \]

\[ \Delta \beta = -4.4 \text{ dB} \]

\[ \beta = \beta_o + \Delta \beta = 75 \text{ dB} - 4.4 \text{ dB} = 70.6 \text{ dB} \]

One quirk: Sometimes EEEN's talk about decibel levels for voltages. Power is proportional to voltage squared.

\[ P = IV = I^2R = \frac{V^2}{R} \]

A tenfold increase in V actually increases P by 100 times. So this is +20 dB increase.

\[ \beta = 10 \log_{10}\left( \frac{P}{P_o} \right) \quad \beta = 20 \log_{10}\left( \frac{V}{V_o} \right) \]
Transverse waves (like light) involve oscillations perpendicular to the propagation direction. This is not unique. Polarization is the direction of oscillations in a transverse wave.

The polarization of light is the direction of the electric field oscillations.

Light can be of one polarization or it can be a mixture.
- Most light is non-polarized. The E oscillations fluctuate.
- Polarized light has E oscillations that are organized.
  - Sky glare off of the surface of a body of water tends to be horizontally polarized. (Brewster's Angle)
  - Polarizing filters will filter out a particular polarization, and pass light of the opposite polarization. Vertical polarizers filter out horizontally polarized light.

How much light gets through a polarizer?

\[
\text{Incoming Unpolarized: } I = \frac{1}{2} I_0 \quad \text{Incoming Polarized: } I = I_0 \cos^2 \theta
\]

Stack of polarizers:

\[
\begin{align*}
I_0 &= 100 \text{ W/m}^2 \\
0^\circ \text{ Polarizer: } I_1 &= 50 \text{ W/m}^2 \\
30^\circ \text{ Polarizer: } I_2 &= 37.5 \text{ W/m}^2 \\
90^\circ \text{ Polarizer: } I_3 &= 9.4 \text{ W/m}^2
\end{align*}
\]

\[
I_2 = I_1 \cos^2 \theta = (50) \cos^2 (30^\circ) = (50) \left(\frac{3}{4}\right) = 37.5 \text{ W/m}^2
\]
What would happen if the second polarizer was removed?
(No light would get through the third polarizer.)

How many decibels of change did the intensity have?

\[
\frac{I}{I_0} = \frac{9.4}{100} = 0.094 \\
\beta = 10 \log \left( \frac{I}{I_0} \right) = 10 \log (0.094) \\
\quad = -10.3 \text{ dB}
\]