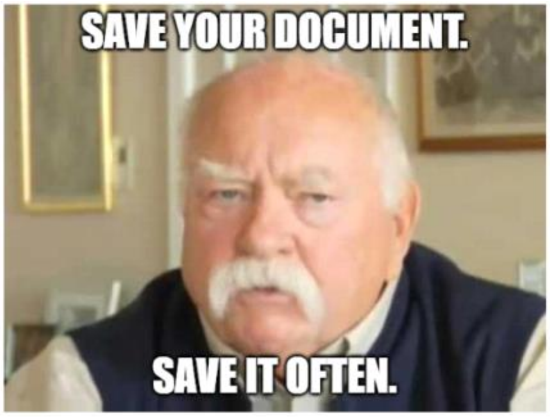
Exam2 Aftermath

Tuesday, November 5, 2019 8:55 AM

What happened? The document version that was proofread was not the version that was split to Version A and Version B, and then printed.

- 5 directly affected questions were thrown out.
- Question #14 was counted as extra credit because it used the result of #13.

Lesson learned:



Avg: 67%

"Lost" question Avg: 63%
(From other students)

#15-16

3 μF capacitor $\qquad C = \frac{\kappa \varepsilon_0 A}{d} \qquad X_C = \frac{1}{2\pi f C}$

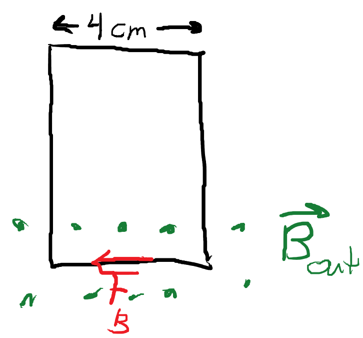
Freq of power supply halved

X_C doubles

$$V_0 = I_0 X_C$$

const $\qquad$ must halve $\qquad$ doubles

#11



$\vec{F}_B = q\vec{v} \otimes \vec{B}$ on ⊕ charges
drives current CW.

#10: $F_B = I\ell B$ on current resists gravity

$$F_g = F_B$$

$$mg = I\ell B$$

$$mg = I \ell B$$

$0.001\ kg \to m$, $4\ cm \to \ell$, $0.25\ T \to B$

#26-27:

given

$$V_R = 30 \sin(4000t)$$

$$V_{R_{rms}} = 21.3\ V$$

$$V_R = IR$$

$$(21.3) = I(60)$$

$$0.35\ldots = I_{rms}$$

$$P_{avg} = I_{rms}^2 R = V_{R_{rms}} I_{rms} = (21.3)(0.35) = 7.5\ W$$

Whole circuit: $V_{rms} = I_{rms} Z$

$$V_{rms} = (0.353A)(100\Omega)$$

Power Supply: $V_{rms} = 35.3\ V$

Oscillations

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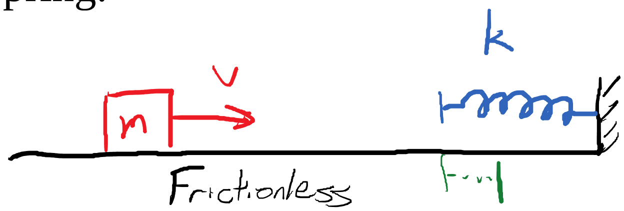
Textbook: Chap 15, especially 15.1-15.4.

Simplest oscillator: Harmonic oscillator made from a mass and a spring.

Spring Force:

$$F_s = -k x_s$$

Phys I:



Δx_f = max compression

Using spring force to find the max compression is tough.

$$F_{net} = ma$$

$$F_s = m a_s \quad (\text{Block's } x = x_s)$$

$$-k x_s = m \frac{d^2 x_s}{dt}$$

What is x_s as a function of time?

$$X_s = X_0 \sin(\omega t) = X_0 \sin(2\pi f t)$$

$$V_s = \frac{dx_s}{dt} = X_0 \cos(\omega t)\,\omega$$

$$a_s = \frac{d^2 x_s}{dt^2} = \omega X_0 (-\sin(\omega t))\,\omega = -\omega^2 \overbrace{X_0 \sin(\omega t)}^{x_s}$$

$$-k x_s = m\left[-\omega^2 x_s\right]$$

$$k = m\omega^2 \qquad \omega^2 = \frac{k}{m} \qquad \omega = \sqrt{k/m}$$

The mass-and-spring traces out a sinewave with an angular frequency as calculated above.

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{k/m}$$

How is this related to the given situation?

$$V_i = V_0 = \text{given} \qquad V_s = \omega X_0 \cos(\omega t)$$

$t=0 \Rightarrow v_s = \omega x_0$

$v_i = \sqrt{\frac{k}{m}}\, x_f$

max=1

Max $x_s = x_0 \sin(\omega t)$

For an oscillation, the displacement amplitude (x0) is proportional to the velocity amplitude (v0).

How did we do this in Physics I? Conservation of energy.

Spring Energy: $U_s = \frac{1}{2} k x_s^2$

Kinetic Energy: $K = \frac{1}{2} m v^2$

Initial = Final

spring

$\frac{1}{2} m v_i^2 + 0 = 0 + \frac{1}{2} k x_f^2$

$v_i^2 = \frac{k}{m} x_f^2$

$v_i = \sqrt{\frac{k}{m}}\, x_f$

The energy analysis and the oscillation analysis produce the same result.

If the mass is attached to the spring, it will continue to follow the sinewave function indefinitely. This is a basic oscillation.

Generic: $x = x_0 \sin(\omega t - \phi)$

Frequency: ω = angular freq $\quad f$ = frequency

$\omega = 2\pi f$ = depend on which objects are involved

Amplitude: x_0 = amplitude $\quad v_0 = \omega x_0$ = max speed

depend on setup.

Phase: ϕ = initial phase $\quad t_0$ = start time

$(\omega t - \phi) = \omega(t - t_0)$

$\sin(\omega(t-t_0))$ starts @ t_0

Note: It's common to adjust the phase by pi/2 by switching from a sine to a cosine.

$$x_0 \sin(\omega t) = x_0 \cos(\omega t - \pi/2)$$

Cofunctions of Complimentary Angles are equal

$$\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right)$$

Cosine is even

$$\cos\left(\frac{\pi}{2} - \theta\right) = \cos\left(\theta - \frac{\pi}{2}\right)$$

Another common way of dealing with phase is to split the function into sine and cosine components.

$$x_0 \sin(\omega t - \phi) = A \sin(\omega t) + B \cos(\omega t)$$

$$x_0^2 = A^2 + B^2 \qquad A = x_0 \sin\phi$$

$$\tan\phi = -B/A \; (?) \qquad B = -x_0 \cos\phi$$

This form is used when there is nonzero initial position *and* nonzero initial velocity.

Initial position = B

Initial velocity = ωA

Mass-and-spring oscillator. $m = 0.25\ \text{kg} \qquad k = 50\ \text{N/m}$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{50}{0.25}} = 2.25\ \text{Hz}$$

Given an initial displacement of 1 cm and an initial speed of 15 cm/s, what are the amplitude and max speed?

$$x(t) = A \sin(\omega t) + B \cos(\omega t)$$

$$B = 0.01\ \text{m} = \text{initial displacement.}$$

$$\omega = 2\pi f = \sqrt{\frac{50}{0.25}} = 14.14\ \text{rad/s}$$

$$\omega A = v_i$$

$14.14 A = 0.15 \ m/s =$ initial speed

$$A = \frac{0.15}{14.14} = 0.0106 \ m$$

Amplitude $x_0 = \sqrt{A^2 + B^2} = \sqrt{0.01^2 + 0.0106^2}$

$= 0.0146 \ m = 1.46 \ cm$

Max speed $v_0 = \omega x_0 = (14.14)(0.0146)$

$= 0.206 \ m/s = 20.6 \ cm/s$

Waves

Thursday, November 7, 2019 9:23 AM

Wave visuals: https://www.acs.psu.edu/drussell/demos.html
(or Google: Dan Russell waves)

Waves are not stuff.

A wave is an organized disturbance in a set of coupled oscillators.

- Coupled: Displacement of one oscillator exerts a force on its neighbors.
- Disturbance: Can be a pulse, but often an oscillation

General wave solution: $f(x - vt)$

Oscillating wave solutions: $\sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right)$

Repeats when: $\Delta\left(\frac{2\pi x}{\lambda}\right) = 2\pi$ $\quad \Delta x = \lambda =$ wavelength

$\Delta(2\pi f t) = 2\pi$ $\quad \Delta t = \frac{1}{f} = T =$ period

Wave speed: $\left(\frac{2\pi x}{\lambda} - 2\pi f t\right) = \frac{2\pi}{\lambda}(x - f\lambda t)$

$$v = f\lambda \qquad v = \frac{\lambda}{T}$$

The wave speed is the speed of propagation of the disturbance.
No actual material is moving at that speed.

Sample wave speeds:

Sound	Light	String	Water
$v \approx 340\ m/s$	$v = 3\times10^{8}\ m/s$	$v = \sqrt{F_T/\mu}$	$v \approx 1-10\ m/s$

All waves involve oscillations of at least 2 variables.
At least one of these variables is a vector. (no pure scalar waves)

The direction of the vector is either parallel or perpendicular to the direction of the wave propagation velocity.

- Longitudinal: oscillations parallel to propagation.
- Transverse: oscillations perpendicular to propagation.

$v = f\lambda$

Range of audible sound waves: 20 Hz to 20 kHz

$f_1 = 20\ Hz$ $\quad v = 340\ m/s$ $\quad \lambda_1 = \frac{340\ m/s}{20\ Hz} = 17\ m$

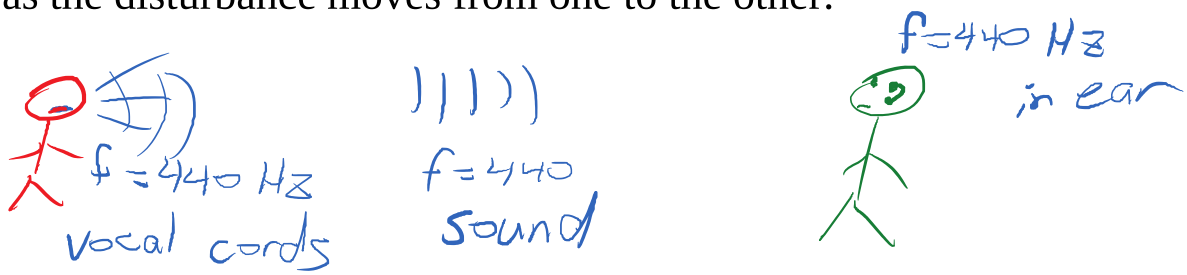
Typical musical instrument is 1/4 wavelength.

$f_2 = 20\ kHz$ $\quad \lambda_2 = \frac{340\ m/s}{20\ kHz} = 17\ mm = 0.017\ m$

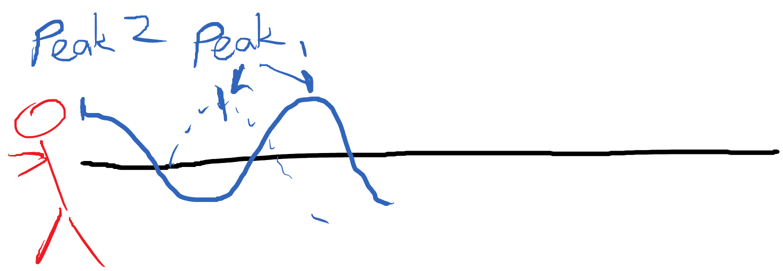
Doppler Effect

Thursday, November 7, 2019 9:23 AM

When oscillators are coupled, the frequency of the oscillations is maintained as the disturbance moves from one to the other.



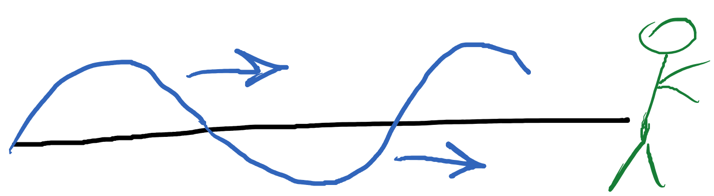
Exception: When the source and/or observer is moving toward or away from the wave.



When the source is moving away from the wave, each successive peak has to travel a further distance. This stretches out the wavelength of the wave.

By the time an observer hears the sound, the peaks occur at more separated times.

Source moving away -> observed frequency is slower (lower freq)



The moving observer detects successive peaks at different times than a stationary observer, because each peak must travel further.

Observer moving away -> observed frequency is slower (lower freq)

$$f_o = f_s \frac{v + v_o}{v - v_s}$$

$+V_o$ = toward wave

$+V_s$ = toward wave

For light waves, the formula is more complicated.

$$f_o = f_s \sqrt{\frac{v + v_{rel}}{v - v_{rel}}}$$ $+v_{rel}$ = toward each other

Simple version (use this whenever possible):

$$\frac{\Delta f}{f} = \frac{v_{rel}}{v_{wave}}$$ Good when $v_{rel} < 15\%\ v_{wave}$

23. Standing Waves

Tuesday, November 12, 2019 9:31 AM

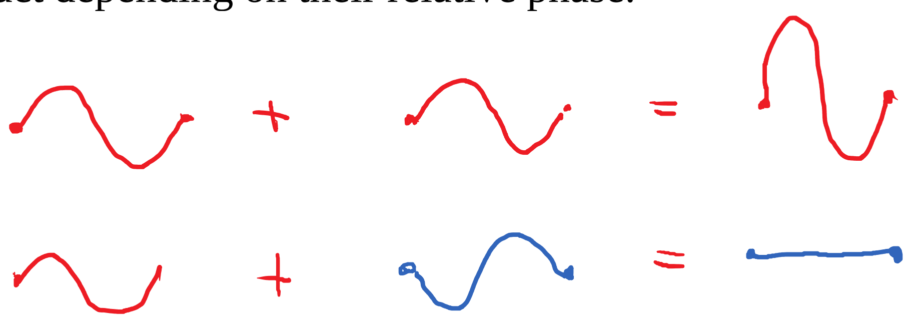
Waves in general: $f(x,t) = A \sin\left(\frac{2\pi}{\lambda} x - 2\pi f t\right)$

Wavelength: λ = repeat distance

Frequency: $f = 1/T$ T = period = repeat time

Wave interference: When two waves meet, their functions add.
(superposition principle)

When two sinewaves meet, and they have the same frequency, they can add or subtract depending on their relative phase.



The relative phase between the two waves determines how they add.

$\Delta\phi = 0, 2\pi, 4\pi, \ldots$ constructive

$f_1 = \sin\left(\frac{2\pi}{\lambda} x - 2\pi f t + \phi_1\right)$

$f_2 = \sin(\ldots x - \ldots t + \phi_2)$ $\Delta\phi = \phi_2 - \phi_1$

$\Delta\phi = \pi, 3\pi, 5\pi, \ldots$ destructive

Reflection: A wave's energy can "bounce" off a surface or a discontinuity in the medium. During reflection, a wave basically generates a reverse-direction copy of itself.
At the point of reflection, either the wave or its conjugate must be zero.

Ex: Reflection of string waves.

- Fixed end must have zero displacement. The reflected wave must combine with the original wave to cancel out and achieve a zero value at the point of reflection.

- Free end must have peak displacement. The reflected wave is "right-side-up" as compared to the original wave.

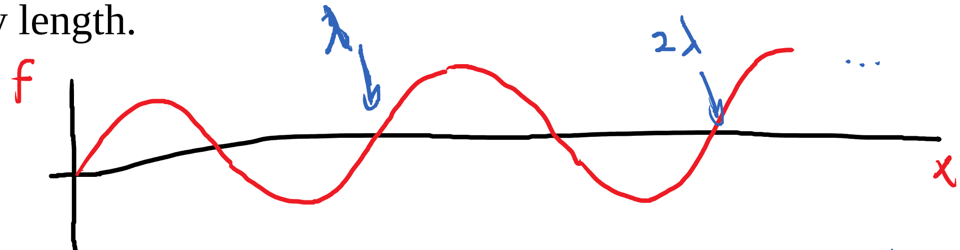
An upside-down reflection is like a phase addition of pi.

Traveling waves can add to form standing waves.

$$y(x,t) = y_m \sin(\mathrm{kx} - \omega \mathrm{t}) + y_m \sin(\mathrm{kx} + \omega \mathrm{t}) = 2y_m \sin(\mathrm{kx}) \cos(\omega \mathrm{t})$$

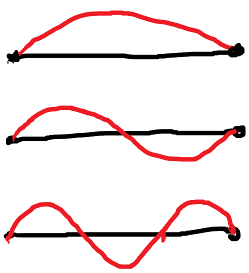
Standing wave resonance is caused when a wave reflects back and forth in a **cavity**. (A region that carries the wave and has reflective ends.) The wave can interfere with itself.

This only happens if the copy of the wave "lines up" with the original wave. The travel distance is 2*L*, twice the cavity length.



$n\lambda$ = point where wave looks like $x=0$

$2L = n\lambda$ wave constructively interferes with its own reflection.



$n=1$ $\lambda = 2L$

$n=2$ $\lambda = L$

$n=3$ $\lambda = \frac{2L}{3}$

$\lambda = \frac{2L}{n}$

$f_1 = \frac{v}{2L}$ = fundamental

Often we're concerned with frequencies.

$v = f\lambda$ v = speed of wave on the string

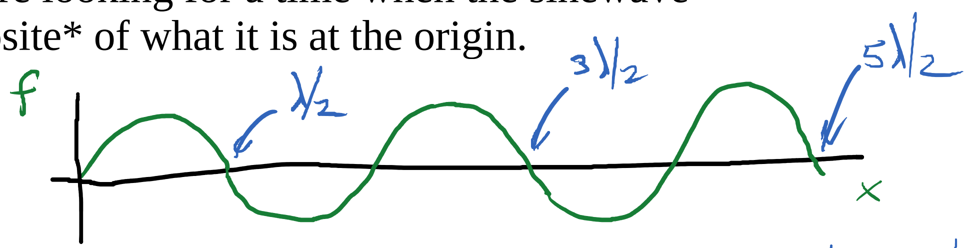
$v = \sqrt{F_T/\mu}$

$f = \frac{v}{\lambda} = \left(\frac{v}{2L}\right) n$

$f = f_1 n$

This can happen with sound waves as well. The reflection is a little different, but the same math applies.

If the two ends of the cavity produce *different* kinds of reflections (one reversing and one not), then we're looking for a time when the sinewave is *opposite* of what it is at the origin.



$(n+\frac{1}{2})\lambda$

$2L = (n+\frac{1}{2})\lambda$ constructive (diff ends)

$4L = (2n+1)\lambda$ $\quad 2n+1 = 1, 3, 5, \ldots$

$4L = n'\lambda$ $\quad n' = 1, 3, 5, \ldots$

With *different* ends, the longest wavelength is 4L. $\lambda = \frac{4L}{n'}$

$$f = \frac{v}{\lambda} = \left(\frac{v}{4L}\right)n' \quad (n' = 1, 3, 5, \ldots)$$

$$f_1 = \frac{v}{4L}$$

$$f = f_1 n' \quad \text{(only odd harmonics)}$$

Ex: Air cavity with open ends, length = 0.8 m.

$v = 340\,m/s$ $\quad \lambda_1 = 2L = 1.6\,m$ $\quad f_1 = \frac{340\,m/s}{1.6\,m} = 212.5\,Hz$

$f_2 = 2f_1 = 425\,Hz$ $\quad f_3 = 3f_1 = 637.5\,Hz$

$\Delta f = 212.5\,Hz$

What if one end is blocked?

$\lambda_1 = 4L = 3.2\,m$ $\quad f_1 = \frac{340\,m/s}{3.2\,m} = 106.25\,Hz$

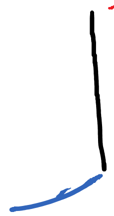
f_2 not allowed $\quad f_3 = 3f_1 = 318.75\,Hz$

$\Delta f = 318.75 - 106.25 = 212.5\,Hz$

Same Ends	Diff. Ends
$\lambda_1 = 2L$	$\lambda_1 = 4L$
$f = n f_1 \quad n = 1, 2, 3, \ldots$	$f = n' f_1 \quad n' = 1, 3, 5, \ldots$

Example from RF theory: 1/4 wave antenna:

— Free end, $I = 0$



Different Ends $\quad \lambda_1 = 4L$

Feed point, $I = \text{peak}$

$f_1 = 4 \text{ MHz}$

$L = 58.5 \text{ ft} = 17.83 \text{ m} \qquad \lambda_1 = 4L = 71.32 \text{ m}$

$v = f\lambda = 285.28 \times 10^6 \text{ m/s}$

According to "standard" antenna calculations, the speed of RF waves on the wire is 2.85e8 m/s.

$c = 3 \times 10^8 \text{ m/s}$

Energy, Power, and Intensity

Thursday, November 14, 2019 9:21 AM

Energy (J)

$P = \frac{\Delta Energy}{\Delta t}$

Spread thru Time ↓ ↑ Accumulate over Time

$\Delta Energy = \int P\,dt$
$= P \Delta t$

Power (W = J/s)

$I = \frac{P}{Area}$

Intensity

Spread over Area ↓ ↑ Accumulate on Surface

$P = \iint I\,dA \cos\theta$

$P = IA\cos\theta$

Intensity (W/m^2)
(Flux density)

Two basic "shapes" of waves:

- Plane waves: Uniform intensity and propagation direction.
  Wave fronts form planes.
- Spherical waves: Generated by a point source and spread from there.
  Propagation direction is radially away from source.
  Intensity is strong near the source and gets weaker with distance.
  Wave fronts form spheres.

For travelling waves, the power of each wave front is consistent.

Spherical: $P = IA$

($\cos\theta = 1$ b/c propagation perp to wave front)

$P = I\,4\pi r^2$

$\frac{P}{4\pi r^2} = I$

Ex: A light source has an intensity of 100 W/m^2 at 10 m, how bright is it at 20 m?

$I \propto \frac{1}{R^2}$

$\frac{I_2}{I_1} = \left(\frac{1/R_2}{1/R_1}\right)^2$

$$\frac{I_2}{I_1} = \left(\frac{R_1}{R_2}\right)^2$$

$$I_2 = \frac{1}{4} I_1 = 25\,W/m^2 \longleftarrow \frac{I_2}{I_1} = \left(\frac{10}{20}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Energy, power, and intensity span huge ranges of values.

$$\frac{I_{max}}{I_{min}} \sim 10^{10}$$

decibels and intensity levels

Thursday, November 14, 2019 9:49 AM

"Level" is a way of describing energy-like quantities on a logarithmic scale.
This compresses large values and stretches small values.
Ratios between related values always have the same size.
There is no way to represent zero intensity.

Levels *always* represent ratios. The value (in decibels) represents some factor between two intensities.

Level in dB

$$\frac{I}{I_0} = 10^{\beta/10}$$

$$\log_{10}\left(\frac{I}{I_0}\right) = \beta/10 \Rightarrow \beta = 10\log_{10}\left(\frac{I}{I_0}\right)$$

Because beta is an exponent, addition of levels (in dB) corresponds to multiplying the intensity several times.

β (dB)	I/I_0
0	1.0
1	1.26
3	2.0
5	~3
7	5.0
10	10
20	100
-37	$\frac{1}{5000}$

Zero decibels means no change.
One decibel is a 26% increase.

3 dB is a doubling of intensity.

5 dB is approx. tripling intensity.
7 dB is a factor of 5

10 dB is a tenfold increase

10^-3.7 = 0.0002
1/5000 = 0.0002

A speaker produces a 75 dB sound when it is 3.0 m away.
How loud is the sound from 5.0 m?

How loud is the sound from 5.0 m?

$$\frac{I}{I_0} = \left(\frac{3}{5}\right)^2 = 0.36$$

$$\log_{10}(0.36) = -0.44$$

$$\Delta\beta = -4.4 \text{ dB}$$

$$\beta = \beta_0 + \Delta\beta = 75\text{dB} - 4.4\text{dB} = 70.6 \text{ dB}$$

One quirk: Sometimes EEEN's talk about decibel levels for voltages.
Power is proportional to voltage squared.

$$P = IV = I^2R = V^2/R$$

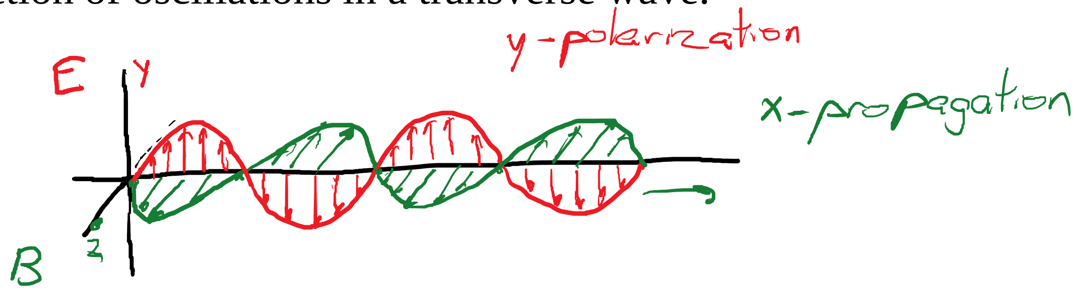
A tenfold increase in V actually increases P by 100 times.
So this is +20 dB increase.

$$\beta = 10 \log_{10}\left(\frac{P}{P_0}\right) \qquad \beta = 20 \log_{10}\left(\frac{V}{V_0}\right)$$

Polarization

Thursday, November 14, 2019 10:15 AM

Transverse waves (like light) involve oscillations perpendicular to the propagation direction. This is not unique. Polarization is the direction of oscillations in a transverse wave.



The polarization of light is the direction of the electric field oscillations.

Light can be of one polarization or it can be a mixture.

- Most light is non-polarized. The E oscillations fluctuate.
- Polarized light has E oscillations that are organized.
  - Sky glare off of the surface of a body of water tends to be horizontally polarized. (Brewster's Angle)
  - Polarizing filters will filter out a particular polarization, and pass light of the opposite polarization. Vertical polarizers filter out horizontally polarized light.

How much light gets through a polarizer?

Incoming Unpolarized $I = \frac{1}{2} I_0$

Incoming Polarized $I = I_0 \cos^2\theta$

Twist angle

Stack of polarizers:

Unpolarized $I_0 = 100\ W/m^2$ | 0° Polarizer | 0° Polarized $I_1 = 50\ W/m^2$ | 30° Polarizer | 30° Polarized $I_2 = 37.5\ W/m^2$ | 90° Polarizer | 90° Polarized $I_3 = 9.4\ W/m^2$

$$I_2 = I_1 \cos^2\theta_{12} = (50)\cos^2(30°) = (50)\left(\frac{3}{4}\right) = 37.5\ W/m^2$$

$I_2 = I_1 \cos^2\theta_{12} = (50)\cos^2(30^\circ) = (50)\left(\frac{3}{4}\right) = 37.5\ W/m^2$

$I_3 = I_2 \cos^2\theta_{23} = (37.5)\cos^2(60^\circ) = (37.5)\left(\frac{1}{4}\right) = 9.4\ W/m^2$

What would happen if the second polarizer was removed?
(No light would get through the third polarizer.)

How many decibels of change did the intensity have?

$\frac{I}{I_0} = \frac{9.4}{100} = 0.094$ $\beta = 10\log\left(\frac{I}{I_0}\right) = 10\log(0.094)$

$= -10.3\ dB$

Ray Optics

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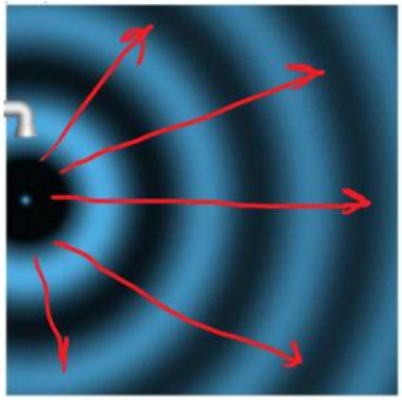
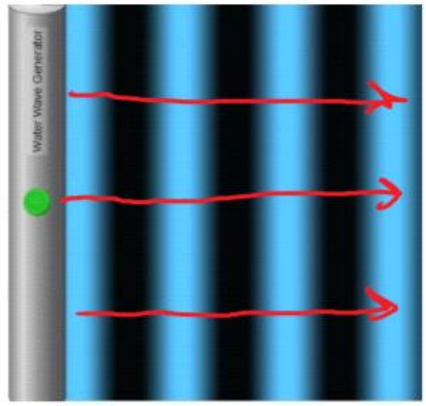


Spherical Waves
Diverging Rays



Plane Waves
Parallel Rays

When you see something clearly focused in front of you, you're seeing a field of points, each of which emits rays like above.

- Nearby objects produce strongly diverging rays.
- Far objects produce weakly diverging rays.
- Infinitely far objects produce parallel rays.

What are rays? Paths that trace the flow of energy.

- Velocity: For light: $v = c = 3 \times 10^8 \, m/s$
    - In a material: $v = \frac{c}{n}$ $n =$ index of refraction

Ex: $n_w = 1.33$ $v = \frac{3 \times 10^8 \, m/s}{4/3} = 2.25 \times 10^8 \, m/s$

Normally, light travels in straight lines. How can we change this?

- Doppler shift. (Frequency changes due to motion.
- Diffraction: Light passing a barrier spreads out to fill the shadow region.
- Absorption: Energy being absorbed by a material.
- Scattering: Energy deflected by small objects.
- Reflection: Organized deflection of energy by a surface.
- Refraction: Bending of light by a surface of a new material.

Any of these can combine to simulate the rays we are used to seeing.

We start by looking at refraction of a single ray.

Refraction

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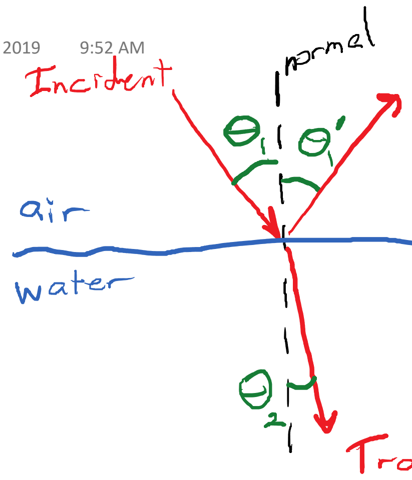


$\theta_1 = 40°$
$n_1 = 1$
$n_2 = 1.33$

$\theta_1' = \theta_1$

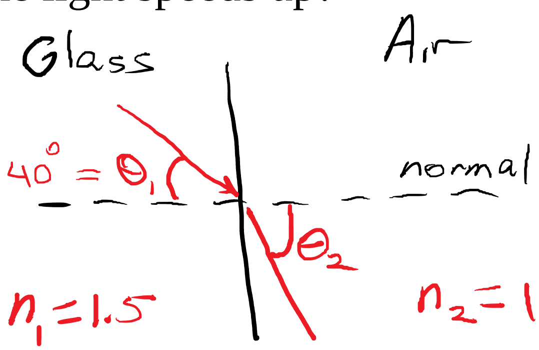
$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$(1)\sin(40°) = (1.33)\sin\theta_2$

$0.4833 = \sin\theta_2$

$28.9° = \theta_2$

What if the light speeds up?



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$(1.5)\sin(40°) = (1)\sin\theta_2$

$0.9642 = \sin\theta_2$

$74.6° = \theta_2$

What if? $\theta_1 = 50°$

$(1.5)\sin(50°) = (1)\sin\theta_2$

$1.149 = \sin\theta_2$

no valid θ_2
no refraction

Total Internal Reflection - When light in a material hits the outer surface at a glancing angle, it can get stuck inside the original material.

The threshold condition is called the critical angle. set $\theta_2 = 90°$

$n_1 \sin\theta_c = n_2$

$\sin\theta_c = n_2/n_1$

For glass-air: $\sin\theta_c = \frac{1.0}{}$ $\theta_c = 41.8°$

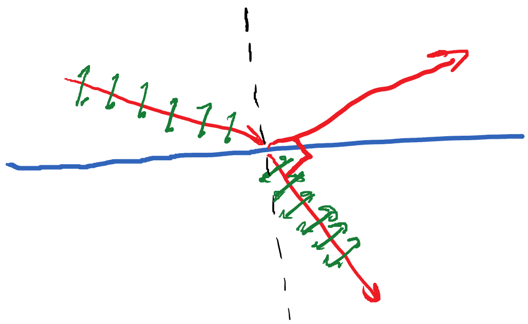
For glass-air:

$\sin\Theta_c = \frac{1.0}{1.5}$ $\Theta_c = 41.8^\circ$

Brewster's Angle

Tuesday, November 19, 2019 10:10 AM

Polarization by reflection



If the reflected and transmitted rays are perpendicular, then no light gets reflected. Why?

- Reflected ray is actually formed by oscillations in the second material. If the oscillations are parallel to the reflected ray, there is no transverse oscillation of E to generate the reflected ray.

$\theta_1 + \theta_2 = 90^\circ$ $\sin\theta_2 = \cos\theta_1$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$n_1 \sin\theta_1 = n_2 \cos\theta_1$

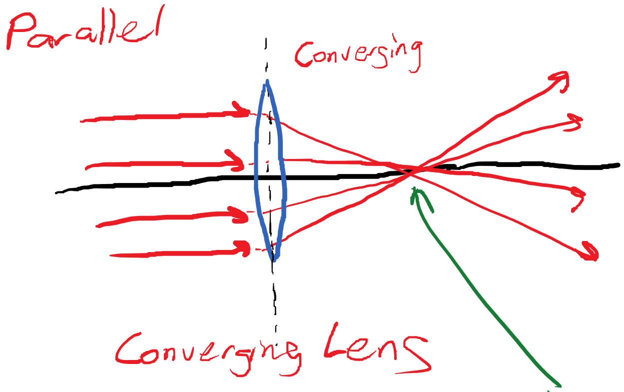
$\tan\theta_1 = n_2/n_1$ Condition to cause no reflection.

This only happens when the polarization is in the plane of incidence.
For horizontal surfaces, this means vertical polarization can be excluded.
This is why glare from water and roads tends to be horizontally polarized.
Vertically polarized light gets filtered out by the reflection.

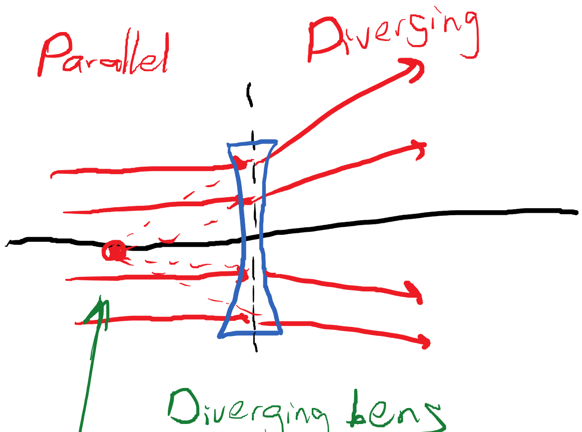
Lenses

Tuesday, November 19, 2019 10:20 AM

Lenses use refraction to change the pattern of rays and fool the observer into seeing things differently.



Real Image: Observe by
- Allowing rays to cross and viewing after.
- Place a screen at the crossing point.
- Intercept rays with another lens.



Virtual Image - can only observe by looking into lens

Calculations:

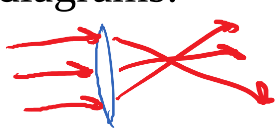
Lens Formula: $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

p = object location

q = image location

f = focal length

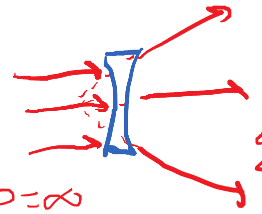
In the above diagrams:



$\cancel{\frac{1}{p}}^{0} + \frac{1}{5cm} = \frac{1}{f}$

$p = \infty$ $q = 5cm$ $f = 5cm$ Converging

$p=\infty$ $\quad q=5\text{ cm}$ $\quad f=5\text{ cm}$ $\quad$ Converging



$q=-5\text{ cm}$ $\quad f=-5\text{ cm}$ $\quad$ Diverging

$p=\infty$

What happens if p=15 cm with the converging lens? $(f=5\text{ cm})$

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \qquad \frac{1}{q}=\frac{1}{5}-\frac{1}{15}=\frac{2}{15}$$

$$\frac{1}{15}+\frac{1}{q}=\frac{1}{5} \qquad q=\frac{15}{2}=7.5\text{ cm}$$

Bringing the object closer made the image go further away.

What happens if p = 2.5 cm with the converging lens?

$$\frac{1}{2.5}+\frac{1}{q}=\frac{1}{5} \Rightarrow \frac{1}{q}=\frac{1}{5}-\frac{1}{2.5}=\frac{-1}{5}$$

smaller than focal length $\quad q=-5\text{ cm}$ why?

Placing the object closer than the focal distance causes q to go negative.

Vision

Thursday, November 21, 2019 9:24 AM

Our eyes are basically biological cameras, consisting of a converging lens and a screen.

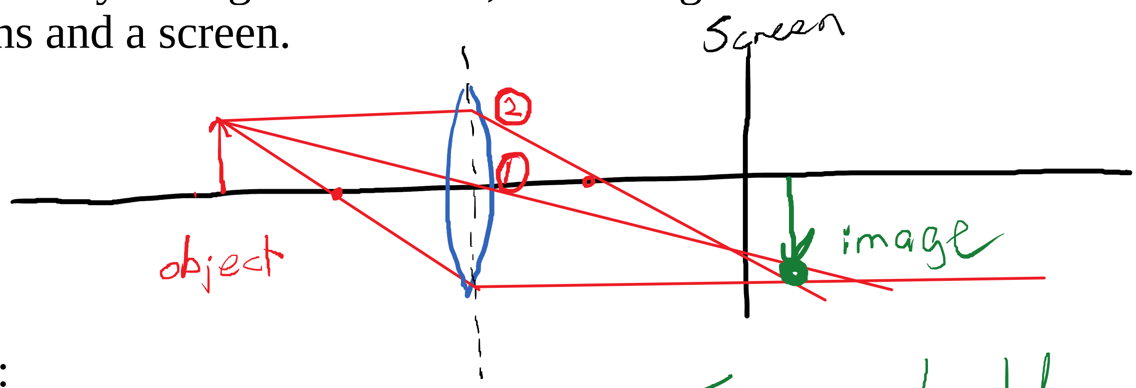


Image should be on screen.

Principle rays:

1. Central ray goes through middle of lens.
2. Parallel ray refracts thru focal point.
3. Focal ray refracts parallel.

We can view actual objects at a range of distances.

- Near point: Closest focusable object distance.
- Far point: Furthest focusable object distance.

good $d_{np} = 25$ cm

$d_{fp} = \infty$

Nearsightedness is the inability to see infinitely far objects. $d_{fp} < \infty$

Farsightedness is an inability to see near things. $d_{np} > 25$ cm

What is the range of focal distances of the eye?

Near: $p = 25$ cm $q = 3$ cm

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$2.68\text{cm} = \left(\frac{1}{25} + \frac{1}{3}\right)^{-1} = f$$

Far: $p = \infty$ $q = 3$cm

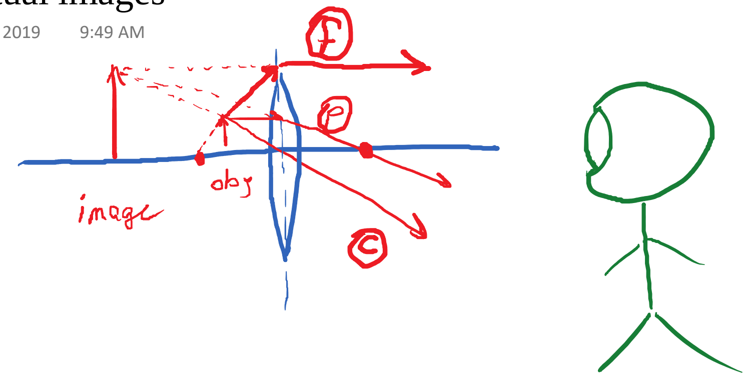
$$\left(\frac{1}{\infty} + \frac{1}{3}\right)^{-1} = f$$

$$3\text{cm} = f$$

To figure out how to correct vision problems, we have to look at how to view images.

Viewing virtual images

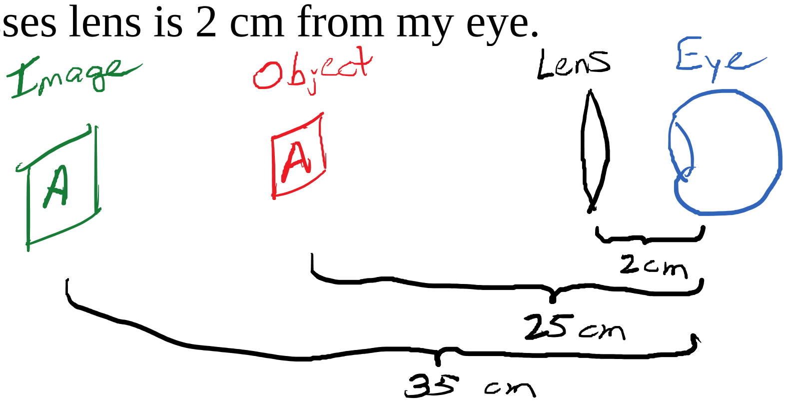
Thursday, November 21, 2019 9:49 AM



To view a virtual image, we look into the lens, viewing the image rays that are refracted and appear to come from the image.

The image from the magnifier becomes the object as far as our eyes are concerned. Since the image is further than the object, a farsighted person might find this useful. Note: q doesn't become p.

Ex: Trying to read something 25 cm away, with a near point of 35 cm. The eyeglasses lens is 2 cm from my eye.



Lens: $p = 23\text{ cm}$ $\qquad f = \left(\frac{1}{p} + \frac{1}{q}\right)^{-1} = 75.9\text{ cm}$

$q = -33\text{ cm}$

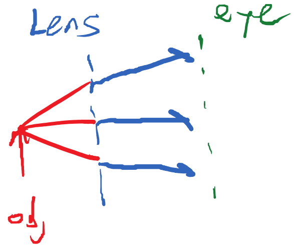
Lens power is the inverse of the focal length (in meters).

$$\text{Power} = \frac{1}{f} = \frac{1}{0.759} = 1.32\text{ diopters}$$

With contact lenses, the distance between the lens and the eye is practically zero.
The converging lens corrects farsightedness.

- My eye needs gently diverging rays (from a far object).

- Nearby objects make strongly diverging rays.
- The converging eyeglasses give those rays a nudge so that they're not strongly diverging any longer.



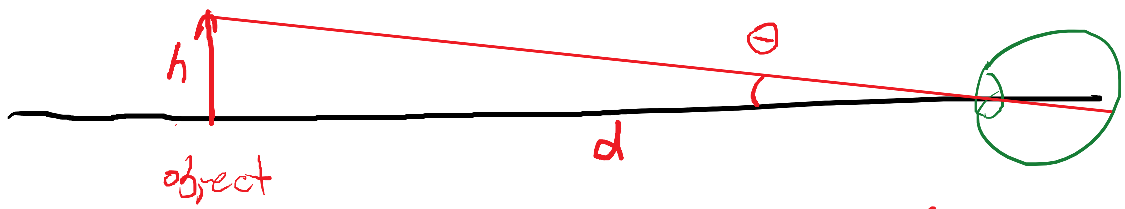
For a nearsighted person, the same thing can be done, but using a diverging lens.

Magnifying Glass

Thursday, November 21, 2019 10:06 AM

Practically, we use a magnifier to increase the size of an image on the back of our eye, not just the image of a lens.

Angular size is the most useful measure.



$\tan\theta = \frac{h}{d} \approx \theta$ ← radians

Small Angle Approx:

$\sin\theta \approx \theta$

$\cos\theta \approx 1$

$\tan\theta \approx \theta$

When viewing details, the small angle approx is okay.

A lens's magnification is its ability to increase the angular size of a detail, as compared to what we can do without the lens.

Without Lens: $\theta_0 = \frac{h}{25\,cm}$

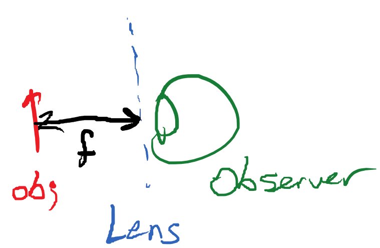
With Lens: Place image at infinity (Relaxed viewing)

$q = -\infty$

Given f, where is p = object location?

$\frac{1}{p} + \cancel{\frac{1}{q}} = \frac{1}{f}$ ($\frac{1}{q} \to 0$) $\quad p = f$

$\theta = \frac{h}{f}$



Angular magnification:

$$M_\theta = \frac{\theta}{\theta_0} = \frac{h/f}{h/25\,cm} = \frac{25\ cm}{f}$$

If we move the object closer to the lens, we can squeeze out one more factor of magnification.

$$_{max}M_\theta = \frac{25\,cm}{f} + 1$$

Ex: Lens sold as having a magnification of 5 times:

$$5 = \frac{25\,cm}{f} + 1$$

$$4 = \frac{25\,cm}{f}$$

$$f = \frac{25\,cm}{4} = 6.25\ cm$$

$$\text{Power} = \frac{1}{0.0625} = 16\ \text{diopters}$$

Compound Microscope and Telescope

Thursday, November 21, 2019 10:22 AM

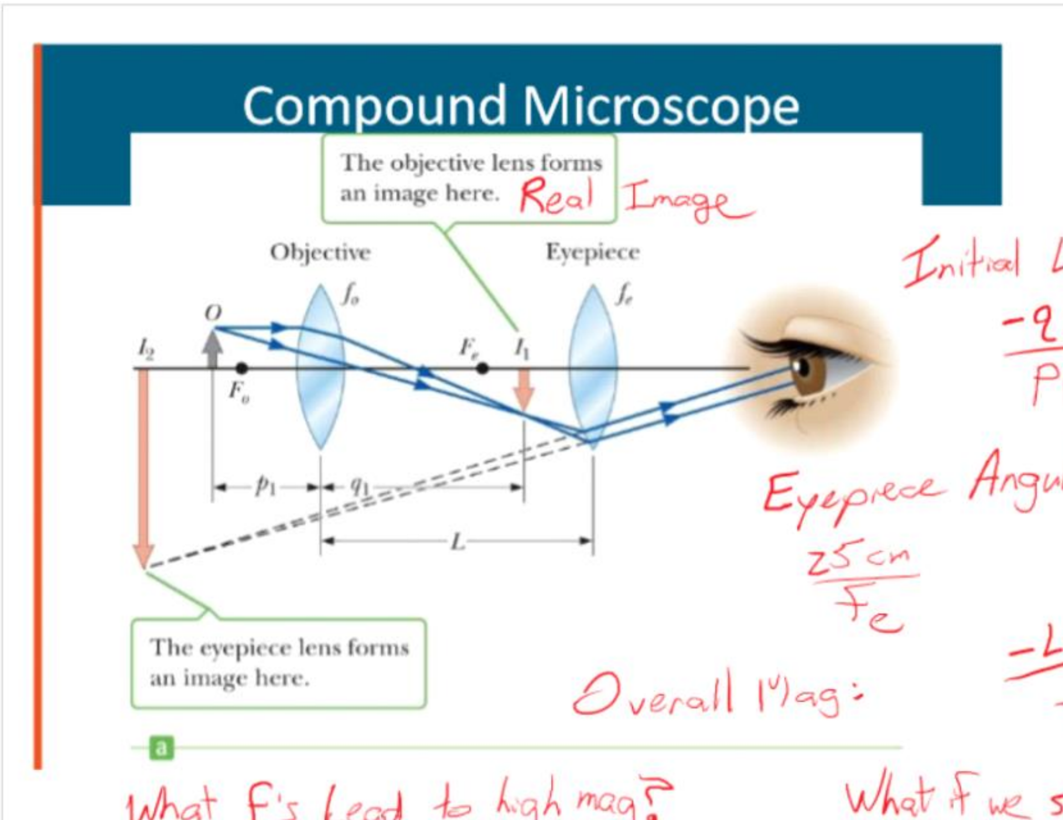
Both of these devices involve generating and viewing a real image.

- Microscope: Object is close, but must be outside focal length of objective lens.
- Telescope: Object is at infinity.

The Objective lens is a converging lens that is projecting a real image.

How can we view the real image?

- Move back to a point further than the image.
- Place a screen at the image location.
- Intercept the image rays with another lens. This is the eyepiece.



Initial Linear Mag.

$$\frac{-q_1}{p_1} \approx \frac{-L}{f_o}$$

Eyepiece Angular Mag

$$\frac{25\text{ cm}}{f_e}$$

Overall Mag: $\dfrac{-L(25\text{ cm})}{f_o f_e}$

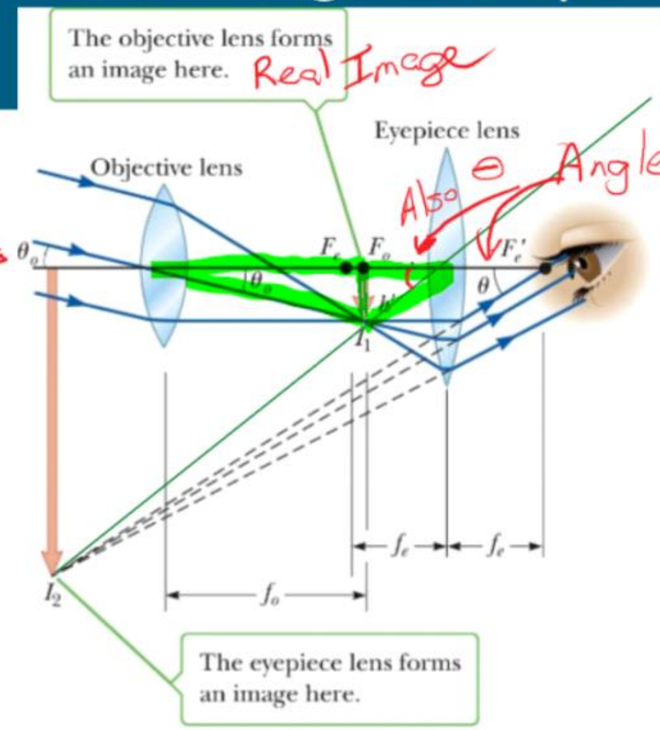
What f's lead to high mag?

What if we swap the lenses

Linear mag: $\dfrac{h'}{h} = \dfrac{-q}{p}$ $\dfrac{h_i}{h_o} = \dfrac{-d_i}{d_o}$

Refracting Telescope

Given →



Also θ Angle with telescope

$$\theta = \frac{h'}{f_e} \qquad \theta_o = \frac{-h'}{f_o}$$

$$\frac{\theta}{\theta_o} = \frac{-h'/f_e}{h'/f_o} = \frac{-f_o}{f_e}$$

Interference

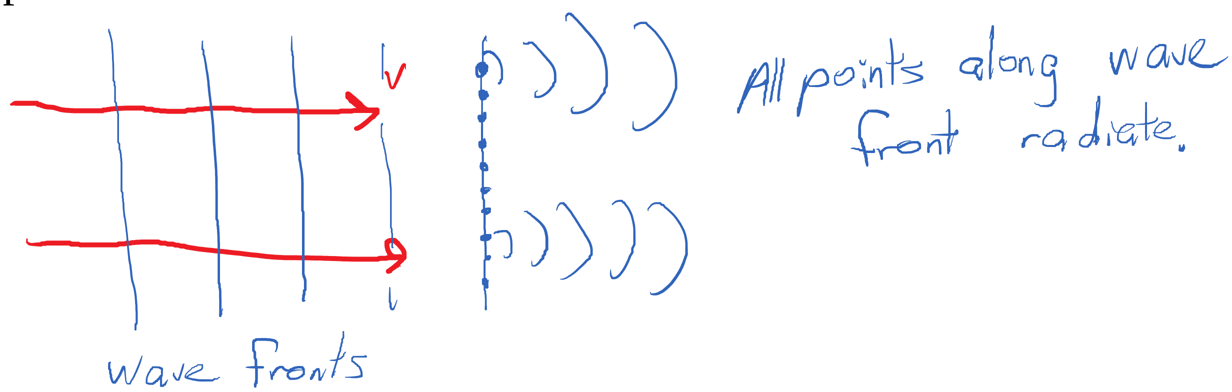
Tuesday, November 26, 2019 9:25 AM

What makes a wave propagate?

- Waves are governed by **<u>local</u>** differential equations.
- A wave is a propagating disturbance in a series of coupled oscillators.
- Every oscillator radiates waves in every direction. Thinking about this is way too much.

Huygens Principle

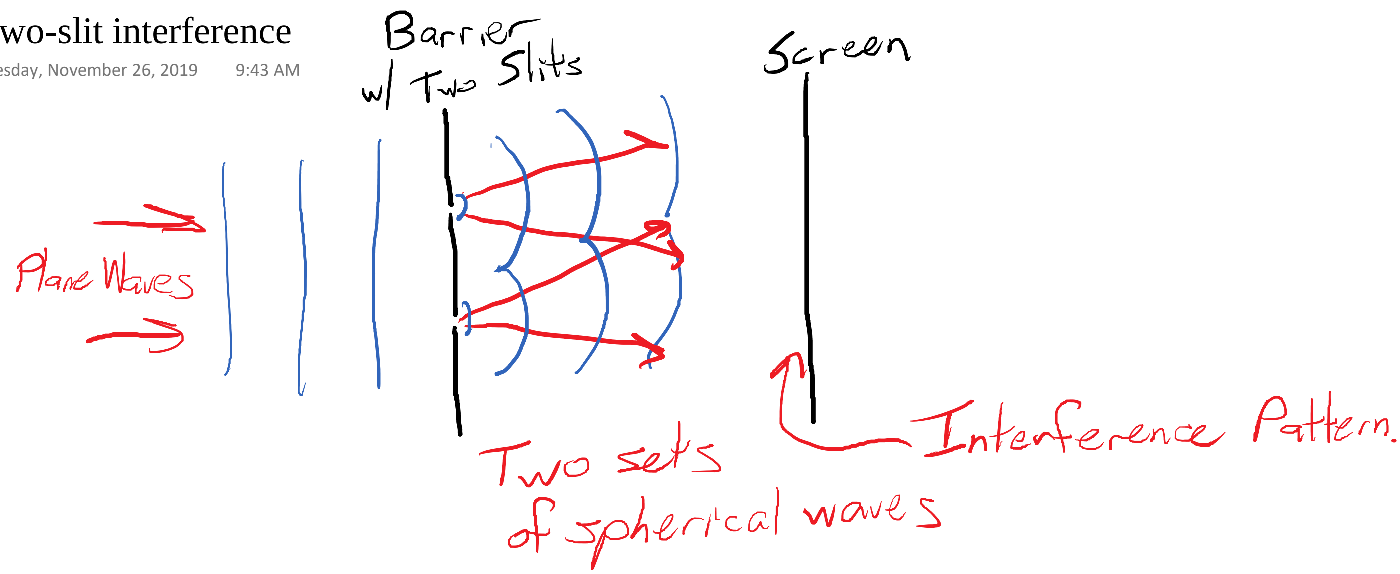
- Consider every point along a wave front.
- These points radiate in all "forward" directions.



- If we don't interrupt the wave, it will continue to propagate as normal.
- If we block most of the wave, except a little hole, that hole will radiate like a wave source. This is diffraction.

Two-slit interference

Tuesday, November 26, 2019 9:43 AM



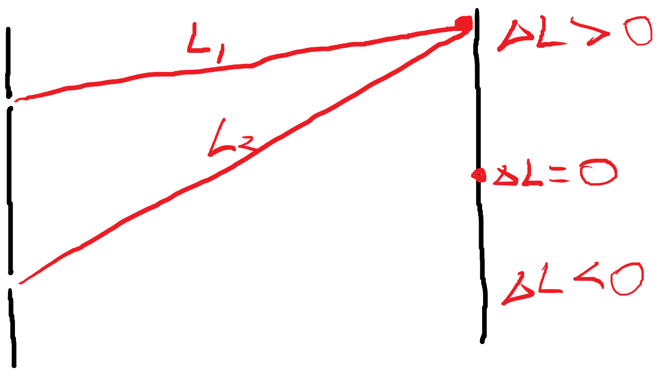
The interference pattern is determined by the relative phase of the two waves.

$$f_1 = A\sin(2\pi f t + \phi_1)$$
$$f_2 = A\sin(2\pi f t + \phi_2)$$

$\Delta\phi = 0,\ 2\pi,\ 4\pi,\ \ldots$ Constructive

$\Delta\phi = \pi,\ 3\pi,\ 5\pi,\ \ldots$ Destructive

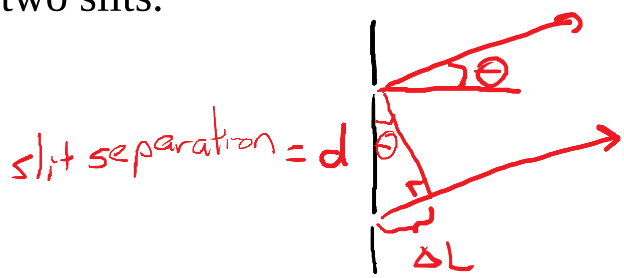
In the two-slit experiment, the phase difference is caused by a path length difference.



$$\Delta L = m\lambda$$

Integer?

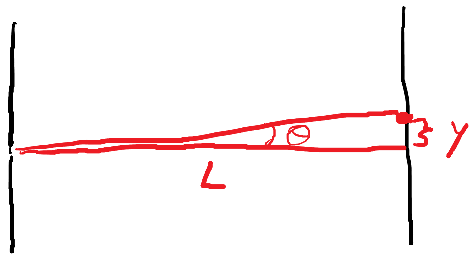
If the path length difference (ΔL) is an integer number of wavelengths, the interference is constructive. To find a formula for it, zoom in on the two slits.



$$\Delta L = d\sin\theta$$

$$m\lambda = d\sin\theta$$

So how do we measure theta?



$\tan\theta = \frac{y}{L}$

What does the pattern look like?

- If y causes theta to cause m to be an integer, there is a bright area.

A diffraction grating is actually many slits, called lines.
Since they're so close, they're described by a density.

$m\lambda = d\sin\theta$

$\rho = 250$ lines/mm

$d = \frac{1\text{ mm}}{250} = 0.004\text{ mm} = 4\ \mu\text{m} = 4000\text{ nm}$

Green light has a wavelength of about 530 nm.

$m(530\text{ nm}) = (4000\text{ nm})\sin\theta$

m	θ
0	0
±1	±7.6°
±2	±15.4°
⋮	⋮
±7	±68°

$\frac{530}{4000} = \sin\theta_1$

$2\left(\frac{530}{4000}\right) = \sin\theta_2$

⋮

$7\left(\frac{530}{4000}\right) = \sin\theta_7$

$\cancel{8\left(\frac{530}{4000}\right) = \sin\theta_8}$ Invalid

There are a total of 15 bright green spots for a
250 line/mm diffraction grating.

If a combination of colors (different wavelengths) hits a diffraction grating, which are deflected to the largest angles?

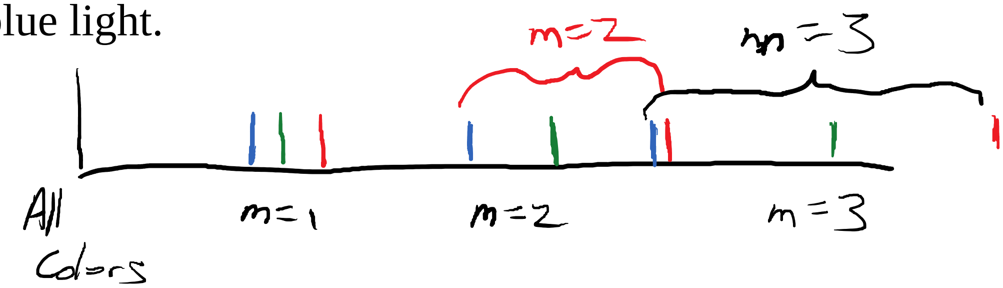
$m\lambda = d\sin\theta$

Bigger wavelength = bigger theta

It's possible for the nth order red light to overlap with the (n+1)st order blue light.

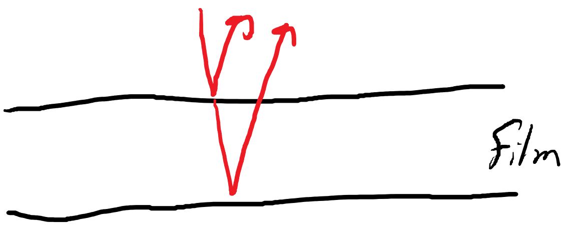
	$m=1$	2	3	4
$\lambda_{red} = 650$	$m\lambda = 650$	1300	1950	
$\lambda_{blue} = 410$	$m\lambda = 410$	820	1230	1640

m=2 red light reaches out to higher angles than the m=3 blue light.



Thin-film interference

Tuesday, November 26, 2019 10:21 AM



$\Delta\phi = 0, 2\pi, \dots$ Constructive

Two contributions to the phase difference:

- Path length difference $\quad \Delta\phi = \frac{2\pi \Delta L}{\lambda}$
- Type of reflection $\quad \Delta\phi = \begin{cases} 0 & \text{Same type} \\ \pi & \text{Opposite types.} \end{cases}$

$$m\, 2\pi = \Delta\phi = \frac{2\pi \Delta L}{\lambda} \overset{?}{+} [\pi]$$

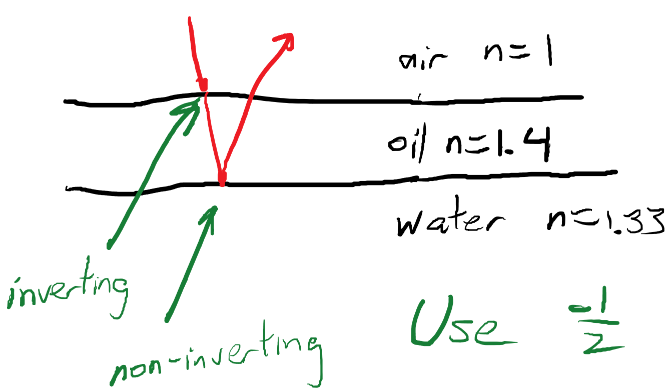
$$\left(m\, 2\pi = \frac{2\pi \Delta L}{\lambda} \overset{?}{+} \pi\right) \frac{\lambda}{2\pi}$$

$$m\lambda = \Delta L \overset{?}{+} \frac{\lambda}{2}$$

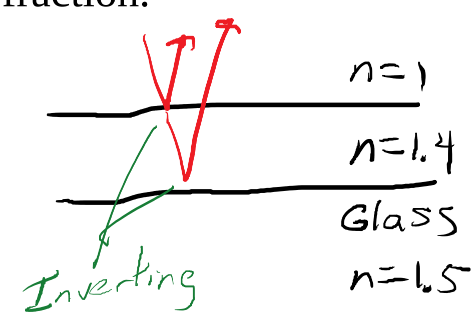
$$\left(m \overset{?}{-} \frac{1}{2}\right)\lambda = \Delta L$$

If the path length difference makes m=integer, the interference is constructive.

- The path length difference is 2 times the film thickness.
- The wavelength must be measured in the film. $\quad \frac{c}{n} = v = f\lambda \qquad \frac{\lambda}{n} = \lambda_n$
- The reflection type is determined by the indices of refraction.



Use $-\frac{1}{2}$



No $-\frac{1}{2}$

$$(m \overset{?}{-} \tfrac{1}{2})\lambda_n = 2t$$

t = thickness

$\lambda_n = \lambda_0 / n_{film}$

A film of oil (n=1.4) is on water (n=1.33).
What is the minimum thickness that will cause constructive interference of red light?

Yes, use $-\frac{1}{2}$

$$(m - \tfrac{1}{2})\lambda_n = 2t$$

$$\left(1 - \tfrac{1}{2}\right)\left(\frac{650\text{ nm}}{1.4}\right) = 2t$$

$$232\text{ nm} = 2t$$

$$116\text{ nm} = t$$

Any thickness less than this will cause destructive interference. That would make the film invisible.

Review for Final

Tuesday, December 3, 2019 9:28 AM

Final Exam Info

- Thu 12/12, 8am, EN-108 (i.e. here)
- Bring a pencil(s) and a calculator with fresh batteries.

Office hours: 11-4 each day except Friday 12/6 and Monday 12/9.

Last-minute HW Extensions: Use WebAssign Extension Request
See Syllabus for policy. Absolute deadline is day of Final.

Diffraction grating

Tuesday, December 3, 2019 9:44 AM

Physical law: $m\lambda = d\sin\theta$ $m = 0, \pm 1, \pm 2, \ldots$

Described by density: $\rho = \frac{1}{d}$ Ex: $\rho = 500$ lines/mm

$d = \frac{1mm}{500} = 0.002mm$

$= 2\mu m = 2000$ nm

Limit of m values: $\sin\theta < 1$

$\frac{m\lambda}{d} < 1$ $m < \frac{d}{\lambda}$

Ex: $d = 2000$ nm $\frac{d}{\lambda} = 4.88$

$\lambda = 410$ nm

$m = 0, \pm 1, \pm 2, \pm 3, \pm 4 = 9$ dots

Trend in angles:

$m\lambda = d\sin\theta$

Smaller $d \Rightarrow$ larger θ

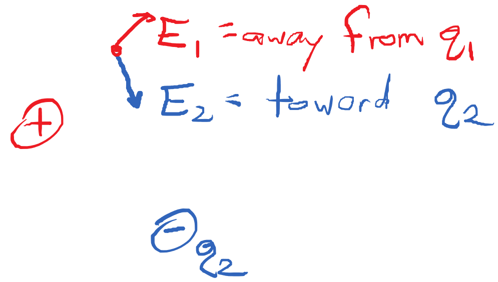
Electrostatics

Tuesday, December 3, 2019 9:53 AM

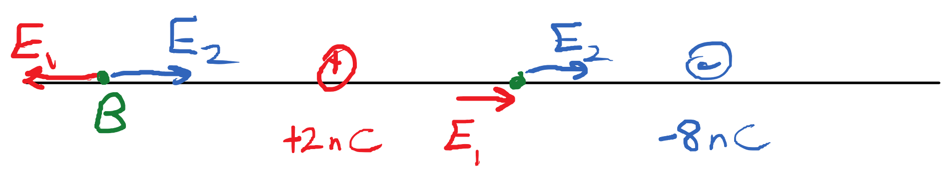
Charges generate E fields.
E fields are "linear": Each charge's contribution is added to the total E field.

$$E_r = \frac{kq}{r^2}$$

radial



Ex: Charges along a line:



At midpoint:

magnitude

$$E = E_1 + E_2 = \left|\frac{kq_1}{r_1^2}\right| + \left|\frac{kq_2}{r_2^2}\right| = \frac{(9\times10^9)}{(0.025)^2}\left(2\times10^{-9} + 8\times10^{-9}\right)$$

$$= 144000 \text{ N/C}$$

Where could the electric field be zero?

At B: E_1 and E_2 opp. dir.

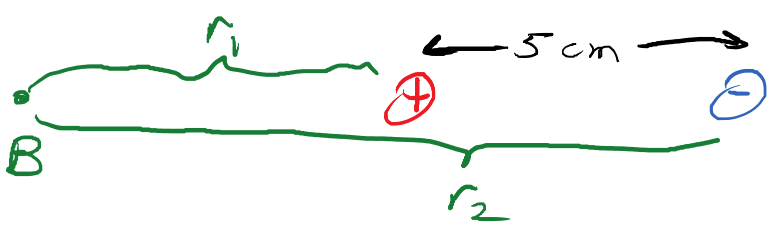
Check: $|E_1| = |E_2|$

$$\frac{k|q_1|}{r_1^2} = \frac{k|q_2|}{r_2^2}$$

Since point B is closer to the weaker charge,
there is a point B where E = 0.

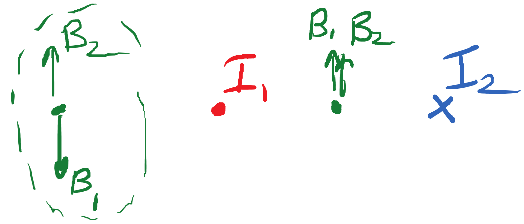
$$\left(\frac{r_2}{r_1}\right)^2 = \frac{|q_2|}{|q_1|} = \frac{8\,nC}{2\,nC} = 4$$

$$\frac{r_2}{r_1} = 2 \rightarrow r_2 = 2r_1$$



$$r_1 + 5\text{ cm} = r_2$$

(Similar situations happen in magnetostatics. There are locations near a pair of parallel wires where the net magnetic field is zero.)



This is how balanced transmission lines and twisted pair reduce noise.

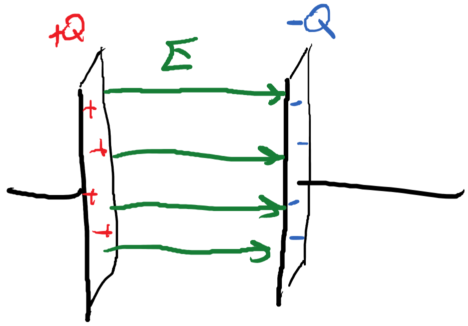
The other source of electric fields is fluctuating B fields. (Faraday's Law)

Results of electric fields:

- Exert forces on charges (Coulomb force)
- Fluctuating E generates B (Displacement current)

$$\vec{F}_E = q_0 \vec{E}$$

Big example of electrostatics: Parallel plate capacitor



$$\sigma = \frac{Q}{A}$$

$$E = 4\pi k \sigma$$

$$E = \frac{\Delta V}{d}$$

$$Q = \frac{1}{4\pi k}\frac{A}{d}\Delta V$$

$$Q = \frac{\varepsilon_0 A}{d}\Delta V$$

$$Q = CV_c$$

$$C = \frac{\varepsilon_0 A}{d}$$

Why capacitors? Gathering +Q without the -Q would create HUGE electric fields that would tear our lab apart.

- The net charge of a capacitor is always zero.
- The capacitance doesn't depend on voltage or the rest of the circuit.

Decibels and intensity

Tuesday, December 3, 2019 10:27 AM

- Decibels always measure a ratio of energy-like quantities (energy, power, or intensity).

$$I = I_0 \, 10^{\beta/10} \qquad \frac{I}{I_0} = 10^{\beta/10}$$

in dB

An "absolute" sound level is referenced to: $I_0 = 10^{-12}\ W/m^2$

Normal Conversation: $\beta = 60$ dB $\quad \frac{I}{I_0} = 10^6$

Normal conversation has an intensity of a million times the reference level.

When comparing two sounds via a relative level, one of the sounds is the reference.

Ex: 40 people talking simultaneously. How much louder is this than one person?

$$\frac{I}{I_0} = 40 = 10^{\beta/10}$$

$$\log 40 = \beta/10$$

$$10 \log 40 = \beta = 16 \text{ dB}$$

Easy dB values: 0 dB = factor of 1 3 dB = factor of 2 10 dB = factor of 10
Increasing or decreasing the intensity by multiplication and division corresponds to adding or subtracting the dB value.

Filters tend to be rated by a dB value.

Sound tends to radiate outward in all directions, so the intensity is inversely proportional to distance squared.

$$I = \frac{P}{4\pi r^2}$$

Ex: Increase r by doubling -> I decreases by 4 times -> level decreases by 6 dB.

General topics

Tuesday, December 3, 2019 10:40 AM

- Electrostatics
- DC Circuits
- Magnetism
- AC Circuits
- Waves and Optics

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