Lec 1 - Intro, Charge

Monday, August 26, 2019 1:17 PM

Why study electricity and magnetism (E&M)?

- Our society is electric
  - Energy
  - Information: Communication, storage, computation, Measurement, Control
- Light and Radio waves
- Chemistry
- Practice with math and "learning new things".

Electrostatics

Tuesday, August 27, 2019 12:54 PM

Study of stationary charges. What are charges?

Generally, "charge" is a measure of how electric something is.

"Charge" can refer to an object or particle that has charge.

All matter is made of atoms, which are in turn made of protons, electrons, and neutrons.

Proton: $q_p = 1.6 \times 10^{-19}\ C = +e$

Electron $q_e = -1.6 \times 10^{-18}\ C = -e$

How much charge is present in a 27 gram aluminum piece?

$M = 0.027\ kg$ $m = 27\ u = 27(1.67 \times 10^{-27}\ kg)$

$M = Nm$ $N = \frac{0.027\ kg}{27(1.67 \times 10^{-27}\ kg)} = 6 \times 10^{23}$

Each aluminum atom has 13 protons and 13 electrons.

$Q_p = q_p N_p = (1.6 \times 10^{-19})(6 \times 10^{23})(13) = 1.25 \times 10^{6}\ C$

$Q_e = -1.25 \times 10^{6}\ C$

$Q_T = 0$

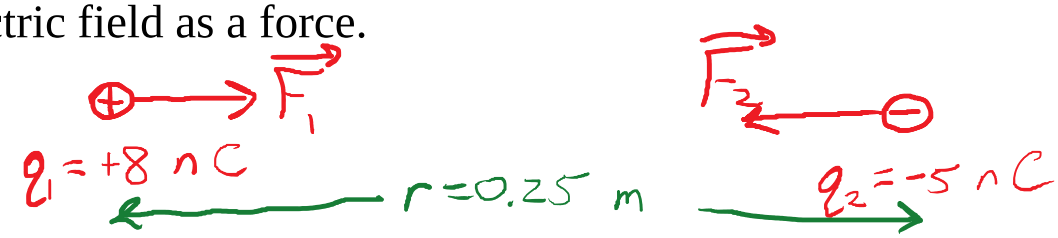
If we wanted to use the aluminum in an electric way, we could do a few things:

- Steal or donate electrons. Maybe only 1 nC to 1 micro-C worth. This is an electrostatic charge.
- Shift electrons around. About one electron per atom is mobile. As long as the total number remains constant, we can move massive amounts of charge.
- Shift electrons around, temporarily. This is called polarizing the material.

Electric Effects

Tuesday, August 27, 2019 1:22 PM

- Charges exert forces on each other. How?
- Charges generate "electric field" all around themselves. Other charges "feel" the electric field as a force.



Net result: Opposites attract.
Magnitude:

$$E_2 = \frac{k|q_2|}{r^2} \qquad E_1 = \frac{k|q_1|}{r^2}$$

$$F_1 = |q|_1 E_2 \qquad F_2 = |q|_2 E_1$$

$$k = 9\times10^9 \frac{Nm^2}{C^2}$$

$$F_1 = \frac{k|q_1||q_2|}{r^2} \qquad F_2 = k\frac{|q_1||q_2|}{r^2}$$

$$F_1 = F_2 = \frac{(9\times10^9 \frac{Nm^2}{C^2})(8\times10^{-9} C)(5\times10^{-9} C)}{(0.25 m)^2}$$

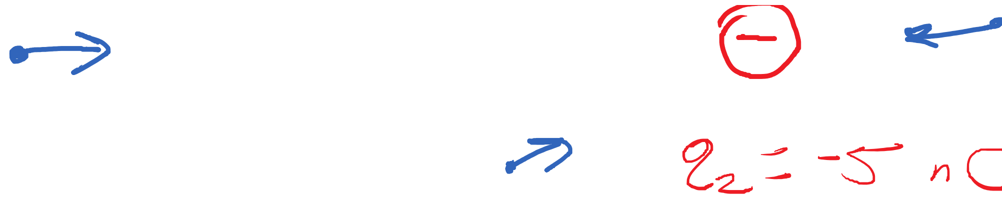
$$= 5.76\times10^{-6} N = 5.76 \mu N$$

How can we incorporate the direction into the formula?

$$\vec{F}_1 = q_1 \vec{E}$$

IF $q_1 = \oplus$ then $\hat{F}_1 = \hat{E}$ (same direction)

The electric field direction is the same as the direction of the force that would be felt by a positive charge. The positive charge doesn't need to actually be there. It is called a "test charge".



$q_2 = -5$ nC

The electric field generated by a negative q2 always points toward q2.

$$\vec{E}_2 = (\text{magnitude})(\text{direction})$$

$$= E_2 \, \hat{E}_2$$

$$= \frac{k\,|q_2|}{r_2^{\,2}} \left(-\hat{r}_2\right)$$

sign of q_2

$\hat{r}_2$ points away from q_2

$$\vec{E}_2 = \frac{k q_2}{r_2^{\,2}} \, \hat{r}_2$$

This is the vector formula for the electric field of a point charge.

2. E-Fields

Thursday, August 29, 2019 12:26 PM

Electric field describes how electrified space is.

$$\vec{F}_0 = q_0 \vec{E}$$

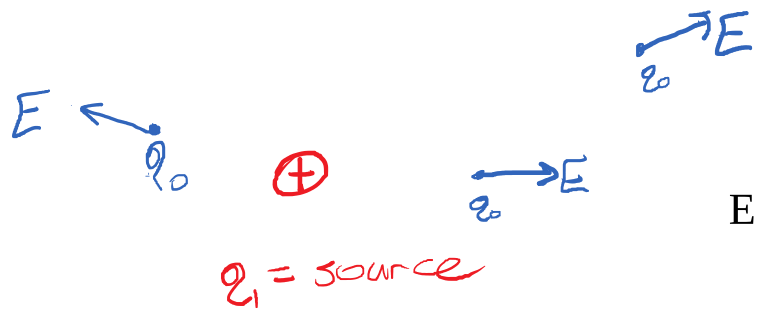
The result of electric field is forces on charges.

- If q0 is +, the force is in the same dir as E.
- If q0 is -, the force is opposite to E.
- Any q0 we place there will have the same E.

There are two sources of electric fields:

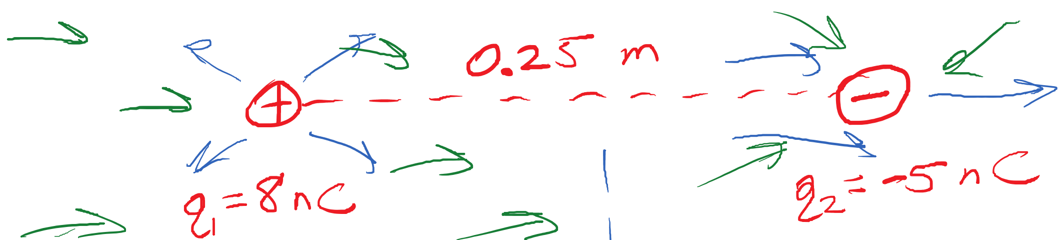
- Source charges.
- Fluctuating magnetic fields.

$$E = \frac{k|q_1|}{r^2}$$



E points away from a + source.

Often, it's easy enough to worry about the magnitude and direction of E separately.



q_2 = source

$$E_2 = \frac{(9\times10^{9})(5\times10^{-9})}{(0.25)^2}$$

$$= 720\ N/C \rightarrow$$

$$F_1 = q_1 E = (8\times10^{-9})(720)$$

$$= 5.76\times10^{-6}\ N \rightarrow$$

q_1 = source

$$E_1 = \frac{kq_1}{r^2} = \frac{(9\times10^{9})(8\times10^{-9})}{(0.25)^2}$$

$$= 1152\ N/C \rightarrow$$

$$F_2 = q_2 E = (5\times10^{-9})(1152)$$

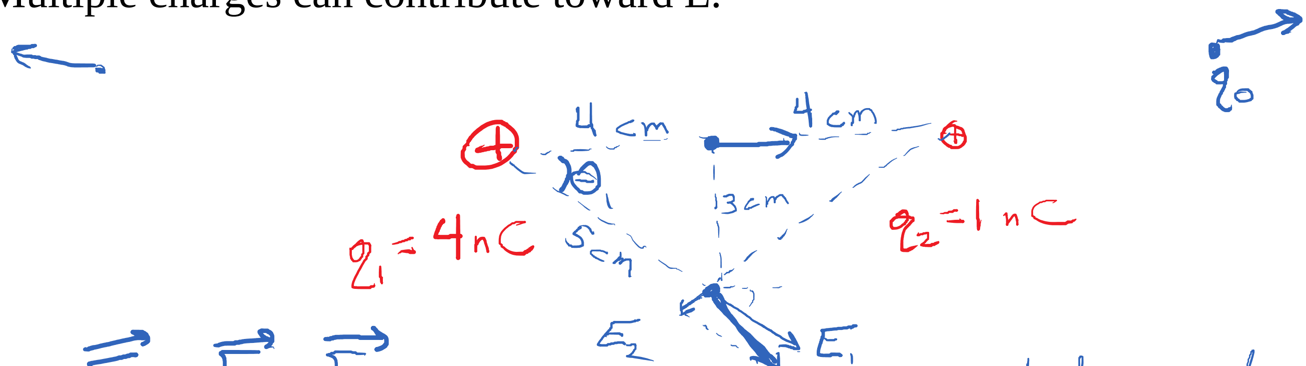
$$= 5.76\times10^{-6}\ N \leftarrow$$

Obeys Newton's 3rd Law

Charge Distributions

Thursday, August 29, 2019 12:56 PM

Multiple charges can contribute toward E.



$\vec{E} = \vec{E}_1 + \vec{E}_2$ $\qquad$ E = rightward, downward

$$E_1 = \frac{kq_1}{r_1^2} = \frac{(9\times10^{9})(4\times10^{-9})}{(0.05)^2} = 14400 \text{ N/C}$$

Dir of E_1? $\hat{E}_1$ ← "Hat" = direction

$$\hat{r} = \text{unit vector from source to point} = \frac{\vec{r}}{|\vec{r}|} = \frac{(+4\text{ cm})\hat{i} + (-3\text{ cm})\hat{j}}{(5\text{ cm})}$$

cos θ_1

$$\hat{r} = \frac{4}{5}\hat{i} + \frac{-3}{5}\hat{j}$$

$$\vec{E}_1 = \frac{kq_1}{r^2}\hat{r} = (14400 \text{ N/C})\left(\frac{4}{5}\hat{i} + \frac{-3}{5}\hat{j}\right)$$

$$E_{1x} = (14400 \text{ N/C})\left(\frac{4}{5}\right) = 11520 \text{ N/C}$$

$$E_{2x} = \underbrace{\frac{(9\times10^{9})(1\times10^{-9})}{(0.05)^2}}_{3600}\left(\frac{-4}{5}\right) = -2880 \text{ N/C}$$

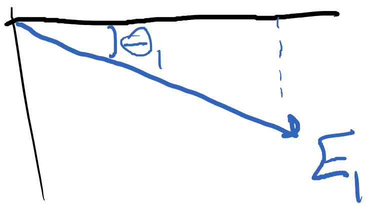
$$E_x = 8640 \text{ N/C}$$

$$E_{1y} = (14400)\left(\frac{-3}{5}\right) = -8640 \text{ N/C} \quad \text{(coincidence)}$$

$$E_{2y} = (3600)\left(\frac{-3}{5}\right) = -2160 \text{ N/C}$$

$$E_y = -10800 \text{ N/C}$$

Alternative way to find the components of E1:



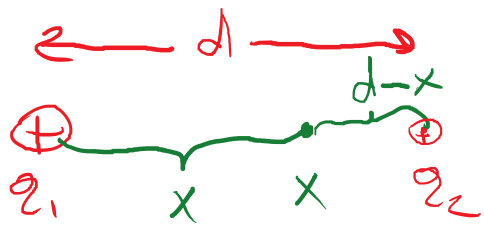
$$E_{1x} = E_1 \cos\theta_1$$
$$= (14400)\left(\frac{4}{5}\right)$$

Where is the electric field equal to zero?

$\vec{E}_1 + \vec{E}_2 = 0 \qquad \vec{E}_1 = -\vec{E}_2$

$\rightarrow E_1 = E_2$ magnitude

$\rightarrow \hat{E}_1 = -\hat{E}_2$ direction



$$E_1 = E_2$$

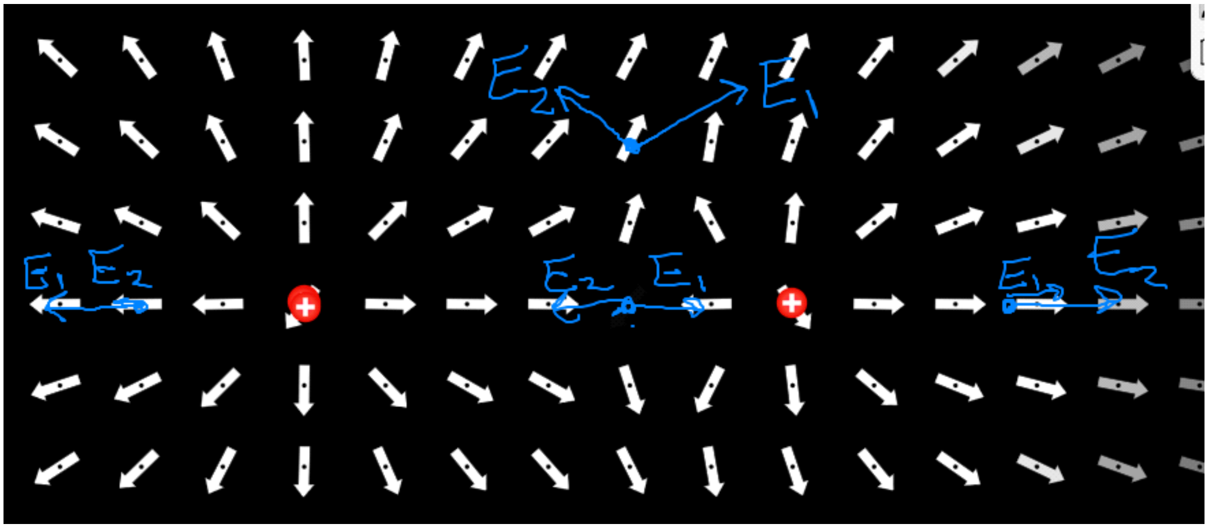
Ex: $q_1 = 4\,nC$
$q_2 = 1\,nC$

$$\frac{kq_1}{x^2} = \frac{kq_2}{(d-x)^2}$$

$$\frac{\cancel{k}(4\cancel{nC})}{x^2} = \frac{\cancel{k}(1\cancel{nC})}{(d-x)^2}$$

$$\frac{\sqrt{4}}{x} = \frac{\sqrt{1}}{(d-x)} \Rightarrow 2(d-x) = x$$

$$2d - 2x = x$$
$$2d = 3x$$
$$\frac{2}{3}d = x$$



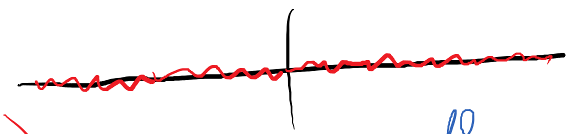
E_2
E_1
E_1 E_2
E_2 E_1
E_1 E_2

Continuous charge distributions

Thursday, August 29, 2019 1:28 PM

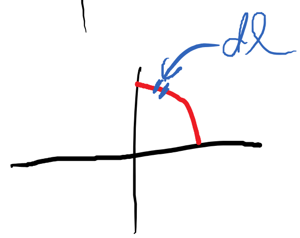
Line charge: $\lambda = \text{linear charge density} = \frac{Q}{\ell}$

$$Q = \lambda \ell \quad (\text{if } \lambda = \text{const})$$



$$Q = \int \lambda\, dx \quad (\text{IF along x-axis})$$

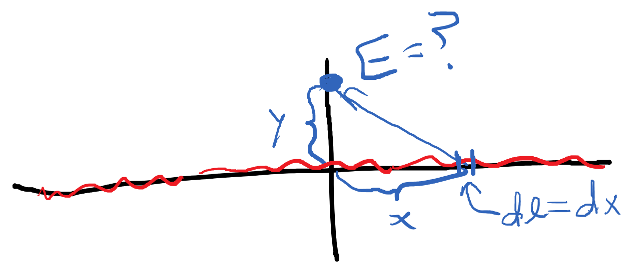
$$Q = \int \lambda\, d\ell \quad (\text{Arbitrary curve}) \quad \text{Line Integral}$$



$$Q = \int dq = \sum_i q_i$$

What if we want to know the electric field due to the line charge distribution?

$$\vec{E} = \int d\vec{E} = \int \frac{k\, dq}{r^2}\hat{r}$$



$$\vec{r} = -x\hat{i} + y\hat{j}$$

$$r^2 = x^2 + y^2$$

$$\hat{r} = \frac{-x\hat{i} + y\hat{j}}{\sqrt{x^2+y^2}}$$

$$E_x = \int_{-\infty}^{\infty} \frac{k\lambda(-x)\, dx}{(x^2+y^2)^{3/2}} = 0$$

$$E_y = \int_{-\infty}^{\infty} \frac{k\lambda y\, dx}{(x^2+y^2)^{3/2}} = \frac{2k\lambda}{y}$$

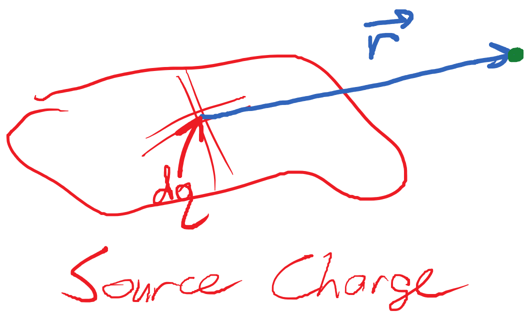
This is the "Green's Function Method" of solving the Maxwell's equation.

3. Gauss's Law

Tuesday, September 3, 2019 12:29 PM

This is a form of Stokes' Theorem.
Alternative way of finding Electric Field.

Integration method (Green's Function based):

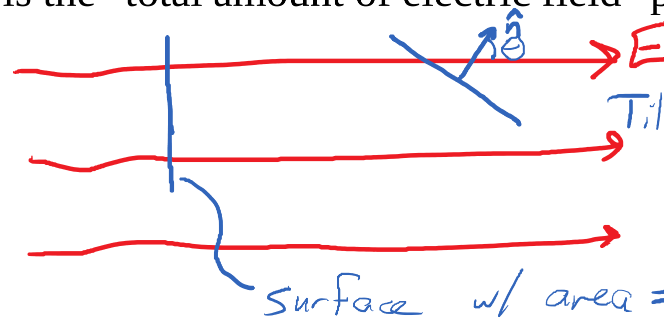


$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$

$d\vec{E} = \frac{k\,dq}{r^2}\hat{r}$

Gauss's Law Method is based on electric flux.

- Electric flux is the "total amount of electric field" pointing through a surface.



Tilt makes less E point "through" surface.

$\Phi_E = EA\cos\theta$

Flux: $\Phi_E = EA$

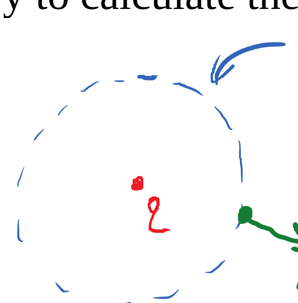
A large surface may have a variety of angles and a variety of E values.

$\hat{n}$ = normal vector to surface

$$\Phi_E = \iint \vec{E}\cdot\hat{n}\,dA$$
$$= \iint E_\perp\,dA$$

$E_\perp$ = component of $\vec{E}$ perpendicular to the surface.

Let's try to calculate the flux "emitted" by a point charge.



$E_\perp = \frac{kq}{r^2}$

$$\Phi_E = \iint E_\perp\,dA = \iint \frac{kq}{r^2}\,dA$$
$$= \frac{kq}{r^2}\iint dA$$

$E_\perp = \frac{kq}{r^2}$

No matter what sphere we choose, the calculated flux is the same.

$$\Phi_E = E_\perp A = \frac{kq}{r^2} A$$
$$= \frac{kq}{r^2}(4\pi r^2)$$
$$\Phi_E = 4\pi kq$$

In fact, the flux is the same for *any* surface that surrounds the point charge.

Integration is a linear operation.

$$\int [af(x) + bg(x)]\,dx = a\int f(x)\,dx + b\int g(x)\,dx$$

Gauss's Law in general:

$$\Phi_E = 4\pi k\, Q_{enc}$$

The flux pointing through any closed surface is equal to 4pi*k times the charge enclosed by the surface.

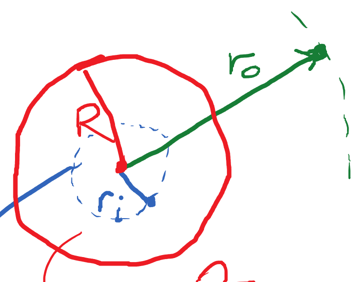
How does this help us with finding E?

- In symmetric cases, E "factors out" of the integral.

Spherical Symmetry

$$\Phi_E = \iint E_\perp\, dA = 4\pi r^2 E = 4\pi k Q_{enc}$$
$$E = \frac{kQ_{enc}}{r^2}$$

Example: Uniformly charged sphere

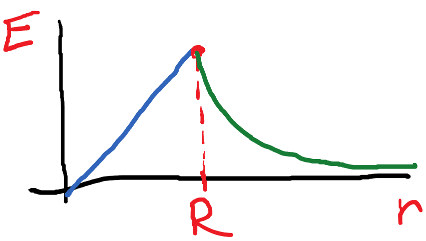


$$\rho = \frac{Q_T}{V_T}$$

Outside: $r_o > R$

$$E = \frac{kQ_T}{r_o^2}$$

Inside: $r_i < R$



$$Q_{enc} = \rho V_{enc} = \frac{Q_T}{V_T} V_{enc} = Q_T \frac{\frac{4}{3}\pi r_i^3}{\frac{4}{3}\pi R^3}$$

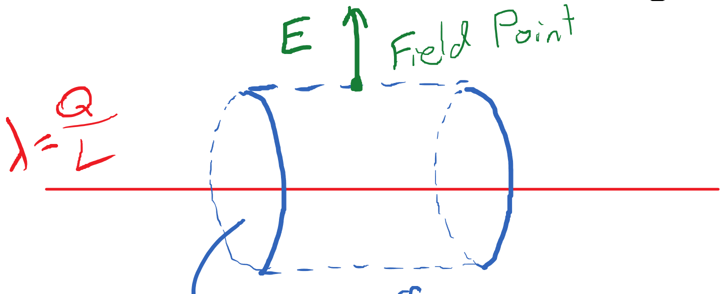
$$E = \frac{kQ_{enc}}{r_i^2} = \frac{k}{r_i^2}\left(Q_T \frac{r_i^3}{R^3}\right) = \frac{kQ_T r_i}{R^3}$$

$$E = \frac{kQ_{enc}}{r_i^2} = \frac{k}{r_i^2}\left(Q_T \frac{r_i^3}{R^3}\right) = \frac{kQ_T r_i}{R^3}$$

Other Symmetries: Cylindrical

Tuesday, September 3, 2019 1:16 PM

Source is a uniform infinite line charge.



$$\Phi_{end} = \iint E_{\perp}\, dA = 0$$

$$\Phi_E = \cancelto{0}{\Phi_{ends}} + \Phi_{shell}$$

$$= E A_{shell}$$

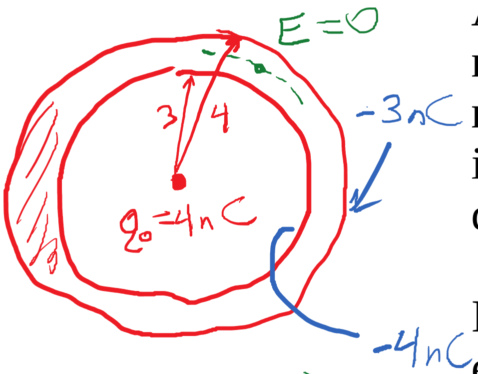
$$= E\, 2\pi r L = 4\pi k Q_{enc}$$

$$E = \frac{2k(Q_{enc}/L)}{r}$$

$$E = \frac{2k\lambda}{r}$$

Gauss's Law with spherical shells

Tuesday, September 3, 2019 1:24 PM



A 4 nC point charge is surrounded by a thick metal shell with inner radius 3 cm and outer radius 4 cm. The total charge of the metal shell is -7 nC. How is the charge of the shell distributed and what is E everywhere?

Important fact: E=0 in a conductor in electrostatics.

- Conductors allow charges to move.
- E causes forces on charges.
- If E != 0, the charges would move.

In the metal ($r=3.5$)

$$\cancel{E}A = 4\pi k Q_{enc}$$

$$0 = Q_{enc}$$

If I draw a Gaussian surface at 3 < r < 4, Q_enc = 0.
The only places on the metal where charge can reside are on the inner surface (r=3) and outer surface (r=4).

$$Q_{enc} = \underbrace{4\,nC}_{\text{in the hollow}} + Q_{inner} = 0 \qquad Q_{inner} = -4\,nC$$

$$Q_{metal} = Q_{inner} + Q_{outer}$$

$$-7\,nC = -4\,nC + Q_{outer} \qquad Q_{outer} = -3\,nC$$

To find the electric field, use Gauss's Law:

Inside: $\Phi_E = 4\pi r^2 E = 4\pi k Q_{enc}$ $\qquad E = \frac{k Q_{enc}}{r^2}$

$$E = \frac{k(+4\,nC)}{r^2}$$

Middle radii
$3 < r < 4$

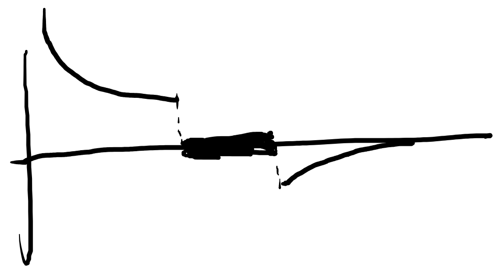
$$E = 0$$

Outside:

$$E = \frac{k(-3\,nC)}{}$$

Outside:

$$E = \frac{k(-3\,nC)}{r^2}$$



4. Potential and Capacitors

Thursday, September 5, 2019 12:25 PM

Physics I review: $W = \int \vec{F} \cdot \vec{d\ell} = \int F_x \, dx$

A good source of intuition for work and energy is free fall.

m

$F_g = -mg$

$W_g = \int F_y \, dy = \int (-mg) \, dy$

m $W_g = -mg\Delta y = +mgh$

(Note: only Δy is $\ominus$)

m $U_i = mgh$

$\Delta U_g = -mgh$

$\Delta K = +mgh$

m $U_f = 0$

$W_g = -\Delta U_g$

$\vec{F_g} = -\nabla U_g$

$F_{gy} = -\dfrac{dU_g}{dy}$

$\Delta U_g = -\int \vec{F_g} \cdot \vec{d\ell}$ (3D)

$\Delta U_g = -\int F_y \, dy$ (1D)

In electrostatics, we can do something similar:

$\vec{F_E} = q_0 \vec{E}$ ← Force per charge

$U_E = q_0 V$ ← Energy per charge

The relationship between E and V is the same as between force and potential energy:

$\vec{E} = -\nabla V$

$E_x = -\dfrac{dV}{dx}$ ← Elec Field is also $[V/m]$.

$\Delta V = -\int \vec{E} \cdot \vec{d\ell}$

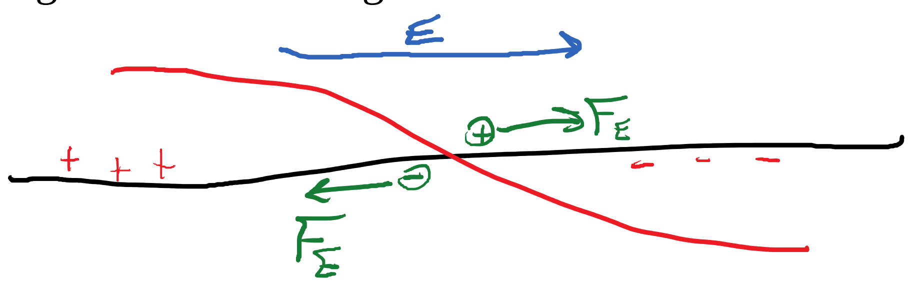
$\Delta V = -\int E_x \, dx$

V is a quantity with many names: electric potential, potential difference, voltage, electromotive force (EMF). V is measured in volts (V).

Just like E is force per unit charge, V is energy per unit charge.
The E and V fields are caused by source charges.
They describe the force and energy of a test charge.

What's the direction of the force on a test charge?

- A mass tends toward lower grav potential energy.
- A charge tends toward lower electric potential energy.
  - A + test charge tends toward lower V.
  - A - test charge tends toward higher V.

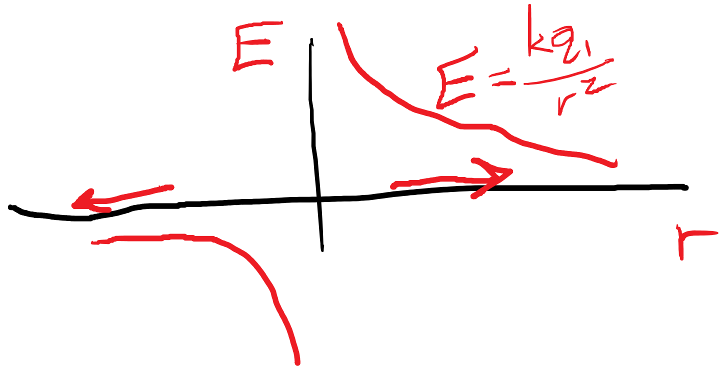


- The positive test charge is attracted toward low V.
- What kind of source charges would cause this?
- Positive source charges cause positive V,
  negative source charges cause negative V.

Potential of a point charge

Thursday, September 5, 2019 12:56 PM

$$\vec{E} = \frac{kq_1}{r^2}\hat{r}$$



Let $V(\infty) = 0$

$$V = -\int E_x \, dx$$
$$= -\int \frac{kq_1}{r^2} dr$$
$$= -kq_1\left(\frac{-1}{r}\right)$$

$$V = \frac{kq_1}{|r|}$$

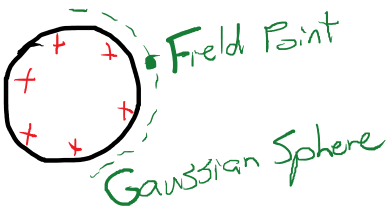
Storing charge - on a metal sphere

Thursday, September 5, 2019 1:02 PM

How much charge could we store on a BB?
Air can withstand E = 1,000,000 N/C

Dia = 3mm

R = 0.0015 m



$\Phi_E = 4\pi k Q_{enc}$

$EA = E\,4\pi R^2 = 4\pi k Q$

$E = \frac{kQ}{R^2}$

Solve for $Q = \frac{ER^2}{k} = \frac{(1\times10^6)(0.0015)^2}{(9\times10^9)} = 2.5\times10^{-10}\ C$
$= 0.25\ nC$

This charge creates an electric potential.
What is the potential at the surface?
Outside, the BB "looks like" a point charge.
The potential is also that of a point charge.

$V = \frac{kQ}{R} = \frac{(9\times10^9)(2.5\times10^{-10})}{(0.0015)} = 1500\ V$

There are two analogies that help with intuition:

- Voltage is like height.
- Voltage is like pressure in a fluid.

$Q = CV$

$(2.5\times10^{-10}\ C) = C\,(1500\ V)$

The metal sphere doesn't store a lot of charge considering how much voltage is needed.

$C = \frac{2.5\times10^{-10}}{1500} = 1.67\times10^{-13}\ F$

capacitance

$= 0.167\ pF$

Farads

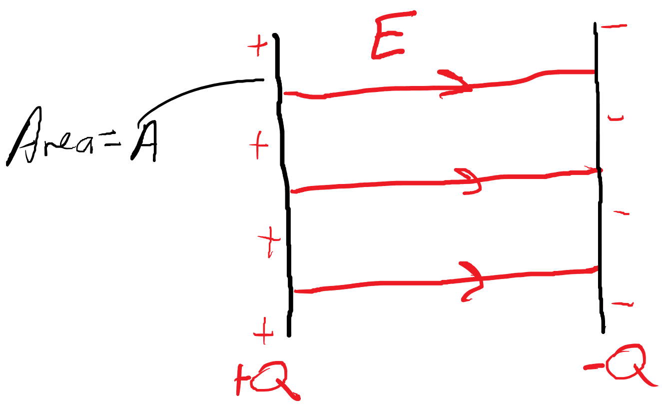
Parallel Plate Capacitor

Thursday, September 5, 2019 1:20 PM

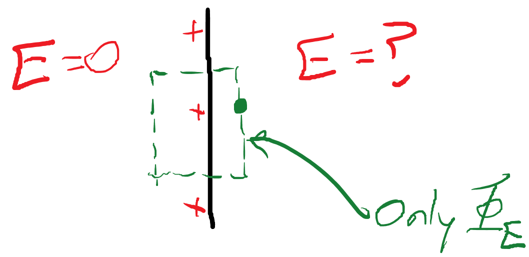
The reason we can't store much charge on the BB is that it's a monopole with all the same charge. The charges repel each other strongly.

Most practical capacitors are a dipole, with equal amounts of positive and negative charges. This significantly reduces the repulsion of the charges and allows us to store more charge.

Simplest case: Parallel Plate Capacitor



To find E, we use Gauss's Law in "slab geometry".



$$\Phi_E = 4\pi k Q_{enc}$$

$$E \cancel{A_{box}} = \frac{4\pi k Q \cancel{A_{box}}}{A}$$

Since E is uniform inside the capacitor, V simply depends on the location.

$$V = -(E)(x) + \text{const}$$

"Charging" the capacitor involves taking charges from one plate and moving them to the other. We want the Delta-V for the whole thing.

plate separation

$$|\Delta V| = Ed$$

Ex: Capacitor made from 1 m^2 sandwich of foil spaced 1 mm apart.

$$\Delta V = \frac{4\pi k Q d}{A}$$

$$\frac{\varepsilon_0 A}{d} \quad \Delta V = \left(\frac{1}{4\pi k} \frac{A}{d}\right) \Delta V = Q$$

$\frac{\varepsilon_0 A}{d}$ ΔV $\left(\frac{1}{4\pi k}\ d\right)$

Note: $k = \frac{1}{4\pi\varepsilon_0}$ $k = 9\times10^{9}$

$\varepsilon_0 = 8.85\times10^{-12}$

$\frac{1}{4\pi k} = \varepsilon_0$

Ex: $C = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12})(1)}{0.001} = 8.85\times10^{-9}$

$= 8850\ pF$

There are several ways to create larger capacitors:

- Large area (A)
- Smaller spacing (d)
- Different dielectric (epsilon_0)

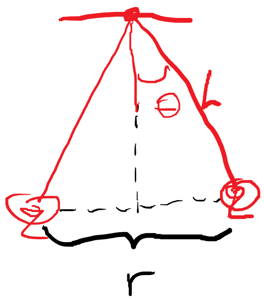
dielectric constant Kappa

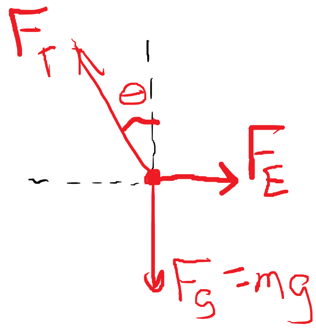
$\varepsilon_0 \Rightarrow \varepsilon = K\varepsilon_0$

5. V, I, R, P

Tuesday, September 10, 2019 12:26 PM

Electrostatic Charge Detector





x: $\frac{kq^2}{r^2} - F_T \sin\theta = 0$

y: $F_T \cos\theta - mg = 0$

$$F_T = \frac{mg}{\cos\theta}$$

$$\frac{r}{2} = L\sin\theta$$

q, m, L, θ

$$\frac{kq^2}{(2L\sin\theta)^2} - mg\frac{\sin\theta}{\cos\theta} = 0$$

It would be difficult to solve for theta.

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

DC Circuits

Tuesday, September 10, 2019 12:44 PM

There are 4 common electrical quantities in DC:

- Voltage (*V*) in volts (V): This is both the electrical "pressure" and the amount of energy per unit charge.
- Current (*I*) in amps (A): This is the actual stuff flowing in a circuit. It's the rate of charge motion in coulombs per second.
- Resistance (*R*) in ohms (Ω): The "difficulty" of pushing current through a part of a circuit.
- Power (*P*) in watts (W): The rate of energy transfer in joules per second.

DC Circuits are always constructed in continuous loops. This is because electrostatic charge is difficult to gather.

Wires: Used to route current with ease.

- In electrostatics, there is zero electric field in a conductor. $\Delta V = -\int E_x dx = 0$
- All points on a conductor are at the same potential = same voltage.
- With flowing current, this is still approximately true.

Battery: Most fundamental motivation for DC current.

- Chemical reactions maintain a (roughly) constant potential difference, called the EMF ("electromotive force") of the battery.

- The terminals of a battery form a capacitor. But unlike a regular capacitor, the battery's capacitor is self-charging. The chemical reactions replenish any used charge.
- Ideally, the battery can provide many coulombs of charge per second.

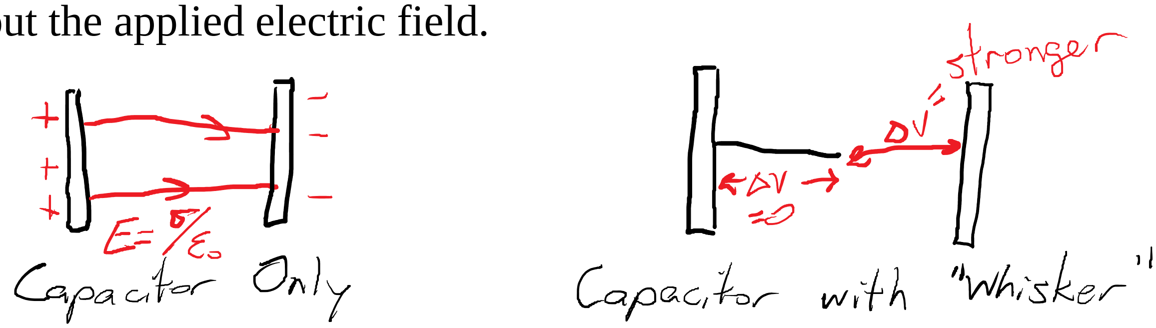
Resistor: Provides a semi-difficult path for current to follow.

- An ideal resistor always has the same current-to-voltage ratio.
  Ohm's Law $V = IR$

What would happen if you put a resistor in a strong electric field, like 10000 V/m? $\Delta V = \int E_x dx$? $I = \frac{\Delta V}{R}$

If the resistor isn't part of a circuit, it will gather just enough charges to cancel out the applied electric field.

If the resistor isn't part of a circuit, it will gather just enough charges to cancel out the applied electric field.

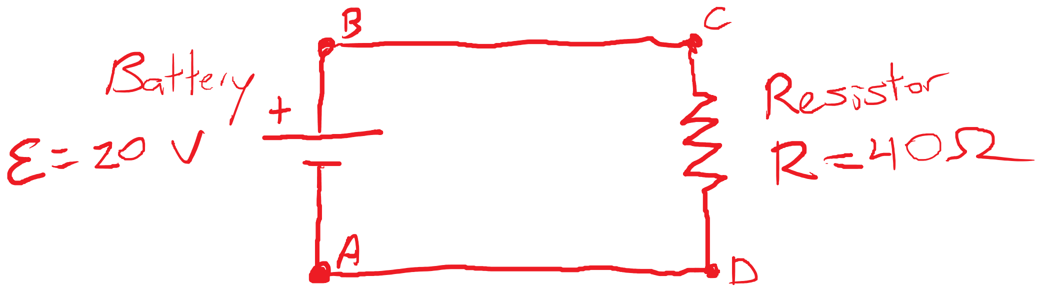


$$E = \frac{\Delta V}{d}$$

$$E = \frac{\Delta V}{d_2}$$

Less distance

Most basic electrical circuit:



$$\Delta V_{Batt} = \mathcal{E} = V_B - V_A \qquad \Delta V_{wire} = V_A - V_D = 0$$

$$\Delta V_{wire} = 0 = V_C - V_B \qquad \Delta V_R = V_C - V_D$$

The wires cause their ends to be at the same voltage, but the voltage DOES NOT FLOW DOWN THE WIRE.

Connecting the resistor and battery at both ends causes the resistor's voltage to be equal to the battery's EMF.

Then:

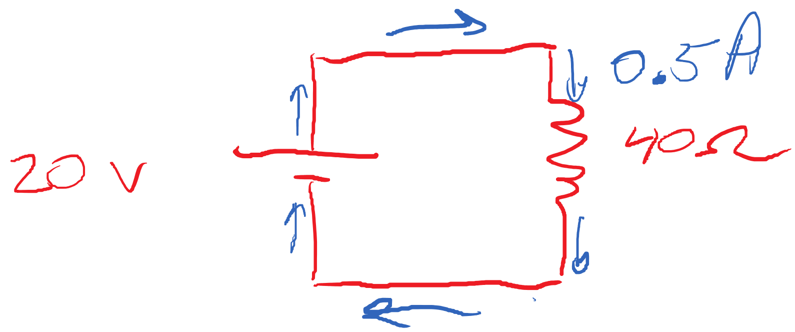
$$V = IR$$

$$(20\,V) = I(40\,\Omega)$$

$$0.5\,A = I$$

The resistor has 20 V applied to it.
The resistor is in parallel with the battery
(connected at both ends).

Where is the current flowing? Everywhere



The current follows the path of the circuit, without any branches or dead-ends.

With no "forks" or "merges", this is a series circuit.

Speed of the current:

- Voltage is conducted very quickly.
- When the current starts to flow in one place, it flows everywhere basically right away.
- The actual charges move slowly.

Power, Energy, and Cost

Tuesday, September 10, 2019 1:28 PM

Voltage is energy per unit charge.

$$\text{Power} = \frac{\text{Energy}}{\text{Time}} = \frac{\text{Energy}}{\text{Charge}} \cdot \frac{\text{Charge}}{\text{Time}}$$

$$P = V \cdot I$$

Battery: $P = \mathcal{E} I$ is power generated

Resistor: $P = VI = (IR)I = I^2 R$ used

In any circuit, the total power used must equal the total power generated. Any imbalance would cause an imbalance of energy, requiring the storage of energy somewhere in the circuit. Only capacitors and inductors can do that.

Ex: $P_{Batt} = (20\text{ V})(0.5\text{ A}) = 10\text{ W}$

$P_R = (0.5\text{ A})^2 (40\,\Omega) = 10\text{ W}$

Cost of Electricity: We Buy Energy

$(\text{cost}) = (\text{rate})(\text{amount})$

rate = $\$0.12/\text{kWh}$

$1\text{ kWh} = (1000\text{ W})(3600\text{ s}) = 3.6 \times 10^6\text{ J}$

$$\frac{1}{\text{rate}} = \frac{3.6\times10^6\text{ J}}{\$0.12} = 30{,}000{,}000\text{ J}/\$$$

How does this compare to buying energy in the form of gasoline?
1 GGE = 1 Gallon of Gasoline Equivalent

$1\text{ GGE} = 33.7\text{ kWh}$

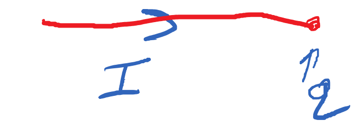
$$\text{rate} = \frac{\$2.20}{\quad} = \$0.065/\text{kWh}$$

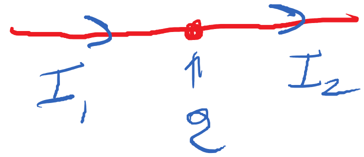
33.7 kWh ~ 1 KWh

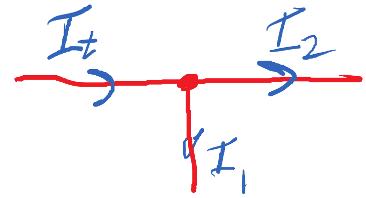
Gasoline only provides thermal energy, not work.
Engines turn heat into work, but with limited efficiency.

car engine ~25% - 33%
power plant ~ 50%

Charge conservation: Charge cannot even "build up" anywhere in a circuit.







$$I = \frac{dq}{dt} = 0$$

$$I_1 - I_2 = \frac{dq}{dt} = 0$$

$$I_1 = I_2$$

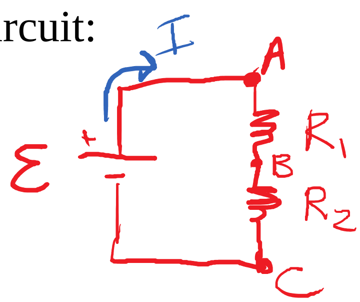
$$I_t = I_1 + I_2$$

This is Kirchoff's Current Law.

Energy conservation: Energy cannot "build up" anywhere in a circuit.

- If a charge follows a complete loop around a circuit, the energy of that charge gets back to its original value.
- There is no kinetic energy of these charges, so this means the voltage changes along the loop must bring the potential back to its original value.
- This is the same as assigning a voltage to every node in the circuit.

Series Circuit:

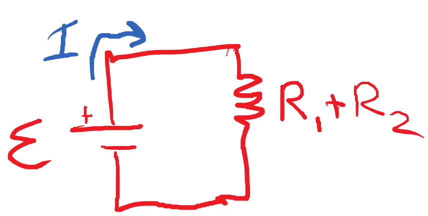


Calculate $V_A - V_C = V_A - V_C$

$$V_A - V_C = V_A - V_B + V_B - V_C$$

$$\mathcal{E} = V_1 + V_2$$

In a series circuit, voltage adds.
In a series, the current is the same.
What about resistance?



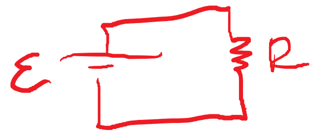
$$\mathcal{E} = IR_1 + IR_2$$

$$\mathcal{E} = I(R_1 + R_2)$$

$$R_{eq} = R_1 + R_2 \text{ (Series)}$$

Ex: Two light bulbs are designed to be hooked individually to a 12 V battery. One is a 12 W bulb, and the other is a 6 W bulb. What happens if they are attached in series to the battery? Which is brighter?

Individual



$P_1 = V_1 I_1$

$(12\,W) = (12\,V)\, I_1$

$1.0\,A = I_1$

$V_1 = I_1 R_1$

$(12\,V) = (1.0\,A) R_1$

$12\,\Omega = R_1$

$P_2 = V_2 I_2$

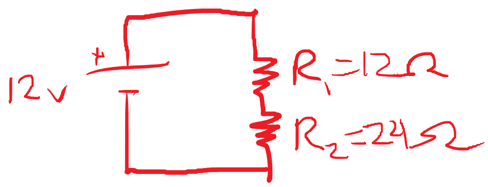
$(6\,W) = (12\,V)\, I_2$

$0.5\,A = I_2$

$(12\,V) = (0.5\,A) R_2$

$24\,\Omega = R_2$

Series



$V_T = I R_T$

$12\,V = I\,(36\,\Omega)$

$\frac{12\,V}{36\,\Omega} = 0.333\,A = I$

$P_1 = V_1 I_1 = (I_1 R_1) I_1 = I_1^2 R_1$

12 W Bulb: $P_1 = (0.333)^2\, 12 = 1.333\,W$

6 W Bulb: $P_2 = (0.333)^2\, 24 = 2.667\,W$

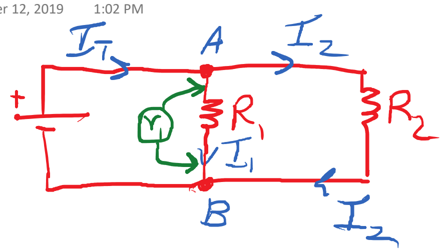
Voltages:

$V_1 = I R_1 = (0.333)(12) = 4\,V$

$V_2 = (0.333)(24) = 8\,V$

Parallel Circuits

Thursday, September 12, 2019 1:02 PM



Current branches @ A and merges @ B.

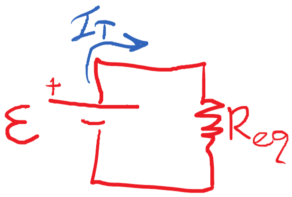
$$I_T = I_1 + I_2$$

$$V_A - V_B = \mathcal{E} = V_1 = V_2$$

What are the consequences of these rules and Ohm's Law?

$\mathcal{E} = V_1$

$\mathcal{E} = I_1 R_1$ Same as if R_1 is attached by itself.

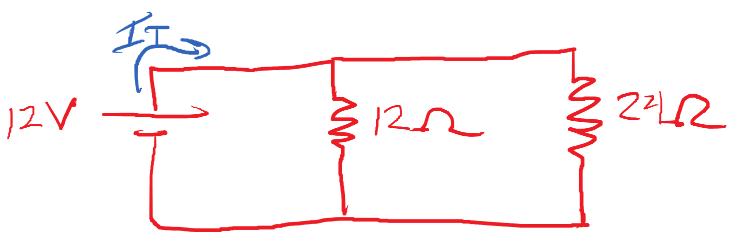


$$I_T = I_1 + I_2$$

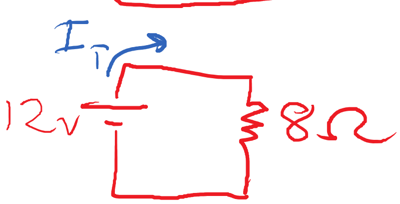
$$\frac{\mathcal{E}}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots\right)^{-1}$$

Let's hook those light bulbs up to the 12 V battery, now in parallel.



$$R_{eq} = \left(\frac{1}{12} + \frac{1}{24}\right)^{-1} = \left(\frac{3}{24}\right)^{-1} = 8\ \Omega$$



$$I_T = \frac{12\text{V}}{8\Omega} = 1.5\text{ A}$$

$$P_T = (12\text{V})(1.5\text{A}) = 18\text{ W}$$

Note: $I_T = 1.0\text{A} + 0.5\text{A} = 1.5\text{A}$

$P_T = 12\text{W} + 6\text{W} = 18\text{W}$

With this ideal battery, each bulb in parallel doesn't know that the other one exists.

Note that 12 ohm || 24 ohm = 8 ohm. The result of a parallel resistance calc is always lower than the individual R values.

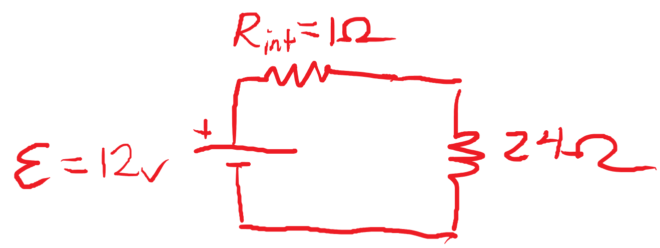
What happens if we put two identical resistors in parallel?

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_1}\right)^{-1} = \left(\frac{2}{R_1}\right)^{-1} = \frac{R_1}{2}$$

Internal Resistance

Thursday, September 12, 2019 1:22 PM

A realistic battery tends to provide less voltage when it's under load.
A simple model is to imagine an internal resistor in series with the circuit.



6-W Bulb, real (lousy) battery

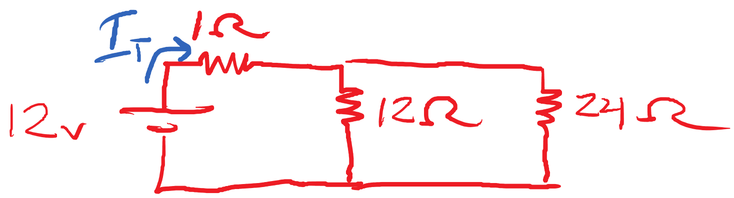
$$I = \frac{\varepsilon}{R_{eq}} = \frac{12\,\text{v}}{25\,\Omega} = 0.48\,\text{A}$$

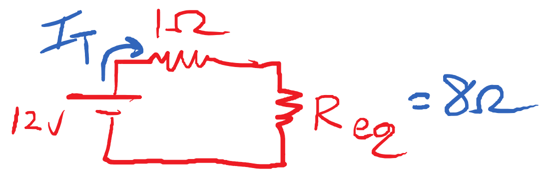
$$V_2 = I_2 R_2 = (0.48\,\text{A})(24\,\Omega) = 11.52\ \text{V}$$

$$P_2 = V_2 I_2 = (11.52)(0.48) = 5.53\ \text{W}$$

$$V_{int} = I R_{int} = (0.48\,\text{A})(1\,\Omega) = 0.48\ \text{v}$$

Combination Circuit: Both light bulbs in parallel, with a realistic battery.





$$R_{eq} = \left(\frac{1}{12} + \frac{1}{24}\right)^{-1} = 8\,\Omega$$

$$I_T = \frac{\varepsilon}{R_{eq2}} = \frac{12\,\text{v}}{(1+8)\,\Omega} = 1.333\,\text{A}$$

Since R_eq was a parallel equivalent, its voltage is the same as V_12 or V_24.

$$V_{eq} = I_T R_{eq} = (1.333\,\text{A})(8\,\Omega) = 10.67\ \text{V}$$

So the 12 ohm and 24 ohm resistors "feel" 10.67 V.

$$I_{24} = \frac{V_{24}}{R_{24}} = \frac{10.67\,\text{v}}{24\,\Omega} = 0.444\,\text{A}$$

$$P_{24} = I_{24} V_{24} = (0.444\,\text{A})(10.67\,\text{v}) = 4.74\ \text{W}$$

This 6-W light bulb uses different amounts of power in different circuits:

- Ideal 12-V battery: P = 6 W
- Realistic 12-V battery (1 ohm internal): P = 5.53 W

- Realistic also with 12 ohm in parallel: P = 4.74 W

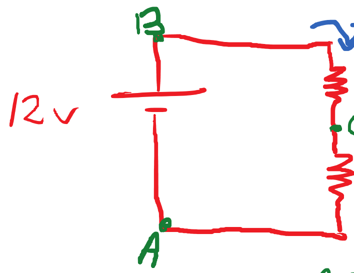
Tuesday, September 17, 2019 12:24 PM

(Useful website: All About Circuits)

To analyze anything: Pick variables, apply laws, solve.

Kirchhoff's Current Law: At any node: $\sum I_{in} = \sum I_{out}$

Kirchhoff's Voltage Law: For any Loop: $\sum \Delta V = 0$



$I = \frac{12v}{2\Omega + 4\Omega} = 2.0\ A$

$V_2 = I_2 R_2 = 4.0\ V$

$V_4 = I_4 R_4 = 8.0\ V$

Loop ABC: $\sum \Delta V = 0$

$V_{AB} + V_{BC} + V_{CA} = 0$

$(+12v) - (4.0v) - (8.0v) = 0$

Alternative

$12.0v = 4.0v + 8.0v$

$\mathcal{E} = IR_2 + IR_4$

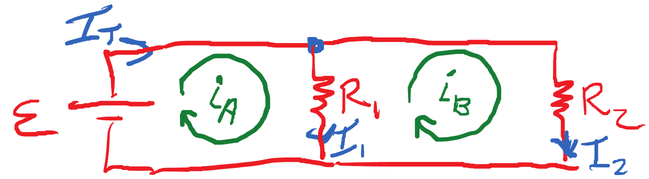
2 things to note: We picked the current *I* as our variable.
The alternative equation gives another loop law: Any Loop: $\sum \mathcal{E} = \sum IR$

Picking actual currents in wires is called "Branch Current Analysis".

$I_T = I_1 + I_2$

$\mathcal{E} = I_1 R_1$ (Left)

$\mathcal{E} = I_2 R_2$ (Outer)



Mesh currents = "sub-currents"

$i_A + i_B = i_A + i_B$

$\mathcal{E} = (i_A - i_B) R_1$

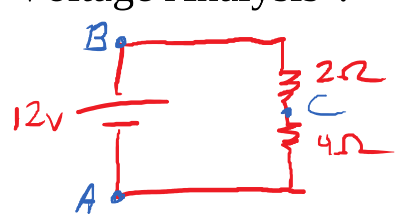
$\mathcal{E} = i_B R_2$

Be aware of "Mesh Current Analysis".
It's different, subtle, and tricky. Just do Branch analysis.

Another technique is "Node Voltage Analysis".

$V_A = 0$

$V_B = V_A + 12v$



At C: $\sum I_{in} = \sum I_{out}$

$\frac{V_B - V_C}{2} = \frac{V_C - V_A}{4}$

$\frac{12 - V_C}{2} = \frac{V_C}{4}$

$4(12 - V_C) = 2V_C$

2Ω voltage: $V_B - V_C = 12 - 8 = 4$

4Ω voltage: $V_C - V_A = 8 - 0 = 8$

$2\,\Omega$ voltage: $V_B - V_C = 12 - 8 = 4$

$4\,\Omega$ voltage: $V_C - V_A = 8 - 0 = 8$

$$4(12 - V_C) = 2V_C$$

$$4(12) = 6V_C$$

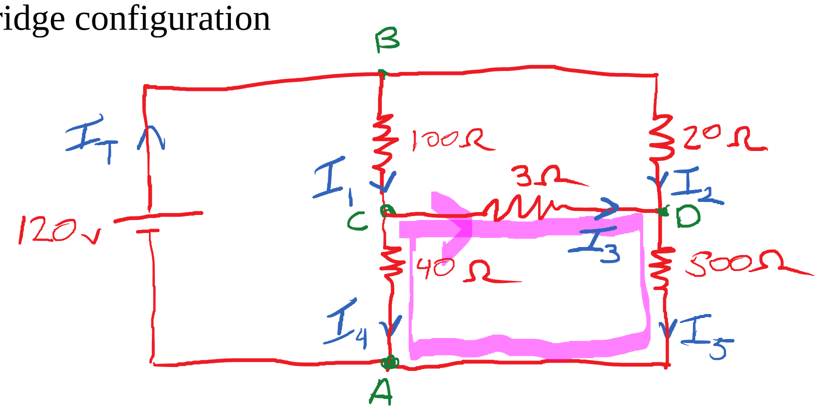
$$8 = V_C$$

Examples

Tuesday, September 17, 2019 1:05 PM

When do we NEED to use Kirchhoff's Laws?

- Multiple batteries
- Bridge configuration



$I_T = I_1 + I_2$

$I_1 = I_3 + I_4$

$I_4 + I_5 = I_T$

Loop ABC: $120 = I_1(100) + I_4(40)$ ①

ABD: $120 = I_2(20) + I_5(500)$

ABCD: $120 = I_1(100) + I_3(3) + I_5(500)$ ②

② − ①: $0 = I_3(3) + I_5(500) - I_4(40)$ Loop CDA

(Loop CDA follows I4 backwards, so it appears as a negative IR drop.)

To solve in calculator, write in standard form:

$$120 = 100 I_1 + 40 I_4$$
$$120 = 20 I_2 + 500 I_5$$
$$120 = 100 I_1 + 3 I_3 + 500 I_5$$
$$0 = I_1 + I_2 - I_T$$
$$0 = -I_1 + I_3 + I_4$$
$$0 = -I_4 - I_5 + I_T$$

$$\begin{pmatrix} 120 \\ 120 \\ 120 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 100 & 0 & 0 & 40 & 0 & 0 \\ 0 & 20 & 0 & 0 & 500 & 0 \\ 100 & 0 & 3 & 0 & 500 & 0 \\ 1 & 1 & 0 & 0 & 0 & -1 \\ -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_T \end{pmatrix}$$

These matrices go into a calculator to solve for all variables at once.

In Wolfram Alpha, use single-letter variables. Result:

$a \approx 0.40104, \quad b \approx 1.7657, \quad c \approx -1.5964, \quad d \approx 1.9974, \quad f \approx 0.16937, \quad x \approx 2.1668$

$I_1 \qquad I_2 \qquad I_3 \qquad I_4 \qquad I_5 \qquad I_T$

Calculating voltages between various points in the circuit.

$$V_{CD} = V_D - V_C = -I_3 R_3 = -(-1.6)(3\Omega) = +4.8\text{ V}$$

$$V_{DA} = -I_5 R_5 = -(0.17)(500) = -85\text{ V}$$

$$V_{AB} = 120\text{ V}$$

$$V_{BC} = -I_1 R_1 = -(0.40)(100) = -40\text{ V}$$

Loop $\quad 120 - 85 - 40 + 5 = 0$ ✓

If you put all voltages on the left, resistors go in as -IR.
If you put the voltage drops on the right, resistors go in as +IR.

There are some other equivalent techniques:

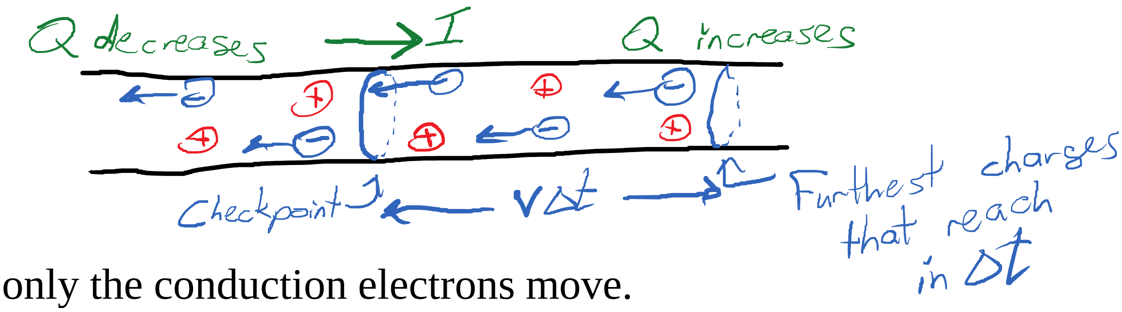
- Delta-Wye
- Thevenin Equivalent (EMF+R)

8. Microscopic Current and RC Circuits

Thursday, September 19, 2019 12:25 PM

Reminder: Exam 1 Thursday

$$I = \frac{dQ}{dt} = \frac{\Delta Q}{\Delta t}$$



In solid materials, only the conduction electrons move.
For the current to point rightward, the electrons must go left.
To figure out a relationship between *I* and speed:

How many charges ΔN make it across?

Count = Density • Volume

$$\Delta N = n \cdot A v \Delta t$$

$$I \Delta t = \Delta Q = \Delta N q$$

$$I \cancel{\Delta t} = n A v \cancel{\Delta t} q$$

$$I = n A v_d q$$

Ex: Drift velocity in a cell phone charging cable.

Material: Copper

Mass Density $= \rho = 8.92\ g/cm^3 = 8920\ kg/m^3$

Atomic Mass $= m = 63.5\ u = 1.05 \times 10^{-25}\ kg$

$u = 1.66 \times 10^{-27}\ kg$

$$n = \frac{\rho}{m} = 8.41 \times 10^{28}\ \text{atoms}/m^3$$

Wire: 24 AWG $\quad A = 0.20\ mm^2 = 0.2 \times 10^{-6}\ m^2$

Current $\quad I = 1.5\ A$

$$I = n A v_d q$$

$$(1.5\ C/s) = (8.41 \times 10^{28}\ m^{-3})(0.2 \times 10^{-6}\ m^2)\, v_d\, (-1.6 \times 10^{-19}\ C)$$

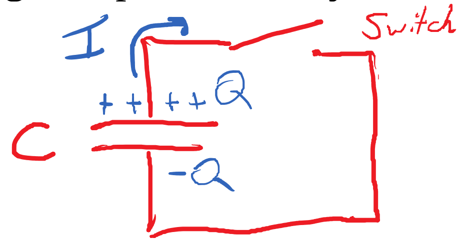
$$-5.6 \times 10^{-4}\ m/s = v_d = -0.56\ mm/s$$

RC Circuits

Thursday, September 19, 2019 1:02 PM

RC Circuits are about HOW a capacitor gets charged or discharged.

If I took a charged capacitor, and just connected a wire to it:



Initially: $Q = Q_0$

$V_c = Q/C$

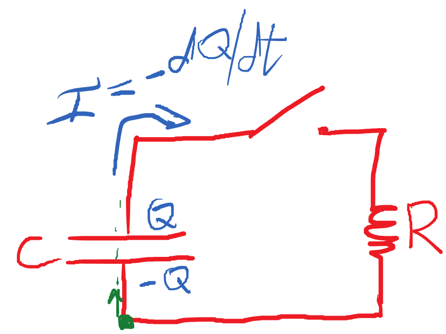
When connected:

Kirchhoff's Voltage Law

$0 = V_c$

$$I = \frac{-dQ}{dt}$$

Satisfying this equation requires Q to decrease instantly. That is a huge dQ/dt.
Practically, our wires will have some resistance. Infinite current would require infinite voltage. We only have so much voltage, so the current is limited. To represent this, re-draw the circuit with some resistance.



$$+V_c = IR$$

$$\frac{1}{C} Q = \frac{-dQ}{dt} R$$

$$\frac{dQ}{dt} = \frac{-1}{RC} Q$$

$$\frac{dQ}{dt} = \frac{-1}{\tau} Q_0 e^{-t/\tau}$$

True if $\tau = RC$

Differential Equation

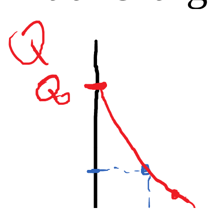
$f = e^t \qquad \frac{df}{dt} = e^t = f$

$g = e^{-t} \qquad \frac{dg}{dt} = -e^{-t} = -g$

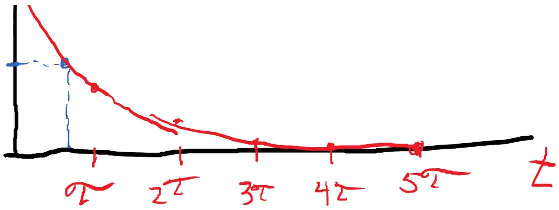
$h = e^{-t/\tau} \qquad \frac{dh}{dt} = \frac{-1}{\tau} e^{-t/\tau} = \frac{-1}{\tau} h$

$$Q = Q_0 e^{-t/\tau}$$

The exponential tells us what fraction of the initial charge is still remaining in the capacitor.



t	$e^{-t/\tau}$
0	1.0
τ	$e^{-1} = 0.37$
2τ	$e^{-2} = 0.14$
3τ	0.05



3τ	0.05
4τ	0.02
5τ	<0.01

By the time t=5 tau, the capacitor is > 99% drained.

At what time is the process 50% complete?

$$e^{-t/\tau} = 0.5$$
$$\frac{-t}{\tau} = \ln(0.5) = -0.7$$
$$t = 0.7\tau$$

What are the electrical variables and constants in this situation?

Constants	Variables
R	Q
C	I
τ	V

$\left.\begin{matrix} Q \\ I \\ V \end{matrix}\right\}$ All decay exponentially

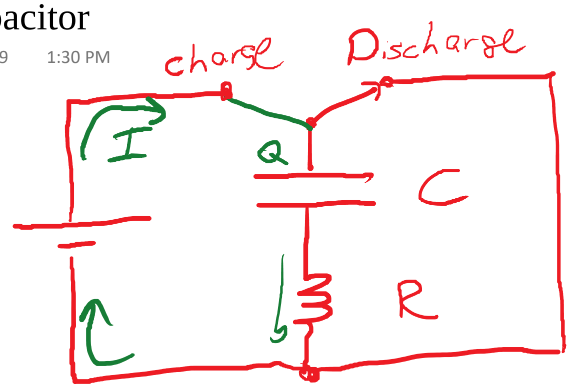
Ex: An unknown capacitor is charged up to 15 V, and attached to a 2 k-ohm resistor. After 15.0 s, the voltage is measured to be 0.75 V. What is the capacitance?

$$V = V_0 e^{-t/\tau} \qquad \tau = RC \qquad (5.0\,s) = (2000\,\Omega)C$$
$$\frac{V}{V_0} = \frac{0.75\,V}{15\,V} = 0.05 \qquad \ln(0.05) = -3 \qquad 0.0025\,F = C$$
$$2500\,\mu F = C$$
$$-t/\tau = -3 \qquad \tau = 5.0\,s$$
$$15.0/\tau = 3$$

Charging a capacitor

Thursday, September 19, 2019 1:30 PM

$I = +\frac{dQ}{dt}$



Charging KVL

$\mathcal{E} = V_C + V_R$

$\mathcal{E} = \frac{1}{C}Q + IR$

$\mathcal{E} = \frac{1}{C}Q + \frac{dQ}{dt}R$

$\frac{\mathcal{E}}{R} = \frac{1}{RC}Q + \frac{dQ}{dt}$

$I = -\frac{dQ}{dt}$ Discharging

$V_C = V_R$

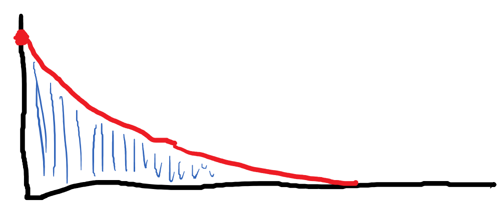
$\frac{1}{C}Q = -\frac{dQ}{dt}R$

$0 = \frac{1}{RC}Q + \frac{dQ}{dt}$

Discharging is like charging, but with a zero-volt battery.
Steady-state (dQ/dt = 0):

$\frac{\mathcal{E}}{R} = \frac{1}{RC}Q_{ss} \Rightarrow Q_{ss} = C\mathcal{E}$

$Q_{ss} = 0$



"Exponentially approaching a limit"

$Q_f - Q = Q_f e^{-t/\tau}$

$Q = Q_f(1 - e^{-t/\tau})$

$Q_f = Q + Q_f e^{-t/\tau}$

$Q = Q_0 e^{-t/\tau}$

$$Q_f = Q + \underbrace{Q_f e^{-t/\tau}}$$

"Full" "Actual" "Available Empty Space"

Exam 1 Thu 9/26. Bring with you:

- Pencil
- Non-internet calculator

I will provide:

- Exam, equation sheet, scantron

Topics: Electrostatics (Chap 22) thru DC Circuits (Chap 27).
(RC Circuits are also in Chap 27).

Dependence of resistance (and resistivity) on temperature.

Temperature Coefficient: relative change in R or ρ for each degree of ΔT,

standard → $$R = R_0(1+\alpha(T-T_0))$$

$$R = R_0 + R_0\alpha(T-T_0)$$

$$R-R_0 = R_0\alpha(T-T_0)$$

$$\frac{R-R_0}{R_0} = \alpha(T-T_0)$$

$$\frac{\Delta R}{R_0} = \alpha\Delta T$$

α is in $(^\circ C)^{-1} = K^{-1}$

When converting temperatures:

T_1: $T_{1f} = T_{1c}\left(\frac{9}{5}\right) + 32^\circ$

T_2: $T_{2f} = T_{2c}\left(\frac{9}{5}\right) + 32^0$

T_2: $T_{2F} = T_{2C}(9/5) + 32$

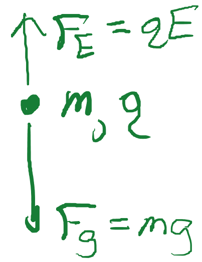
ΔT: $T_{2F} - T_{1F} = (T_{2C} - T_{1C})\,9/5$

$$\Delta T_F = \Delta T_C \left(9/5\right)$$

Electrostatic levitation

Tuesday, September 24, 2019 12:44 PM

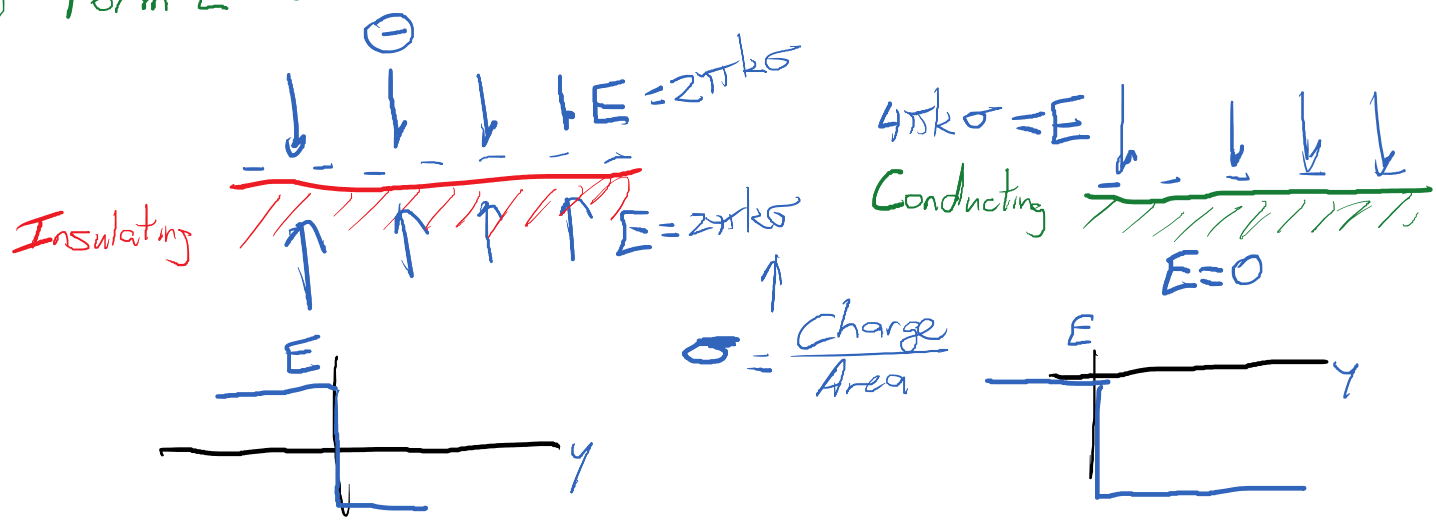
①



$qE = mg$

Note: $\vec{F_E}$ is up. If $q = \ominus$, $\vec{E}$ = down

② Form E w/ surface charge:

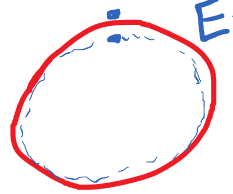


Gauss's Law in spherical symmetry

Tuesday, September 24, 2019 12:52 PM

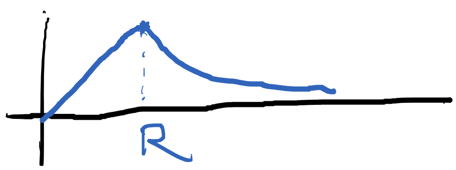
$E_{out} = \frac{kQ}{R^2}$

$E_{in} = \frac{kQ}{R^2}$



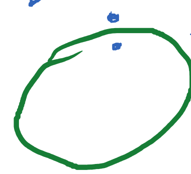
Insulator Uniform Q
Throughout volume

E is continuous



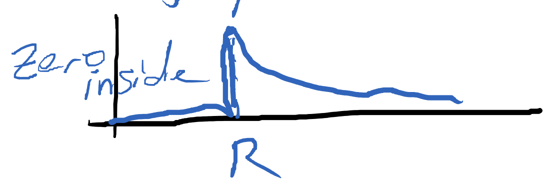
$E_{out} = \frac{kQ}{R^2}$

$E_{in} = 0$



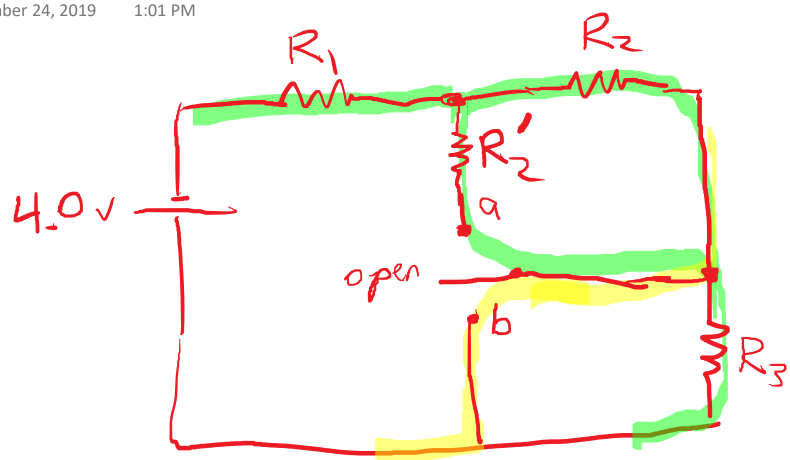
Metal, Uniform Q
On Surface

E jumps in value



Changing resistor network

Tuesday, September 24, 2019 1:01 PM



$I_{open} = 1.05$ mA

$I_a = 1.30$ mA

$I_B = 2.02$ mA

$\mathcal{E} = I_{batt} R_{eq}$

open: $R_{eq} = R_1 + R_2 + R_3$

a: $R_{eq} = R_1 + \frac{R_2}{2} + R_3$

b: $R_{eq} = R_1 + R_2$

Equal resistors in parallel:

$R_{eq} = \left(\frac{1}{R_2} + \frac{1}{R_2'}\right)^{-1} = \left(\frac{2}{R_2}\right)^{-1}$

$R_{eq} = R_2/2$

RC Circuit example (HW2-13)

Tuesday, September 24, 2019 1:11 PM

A 12 micro-F capacitor is charged by a 12 V battery.
After 3.0 s, it reaches 4.0 V. What's the resistance?

$$V = V_f\left(1 - e^{-t/\tau}\right)$$

$$4 = 12\left(1 - e^{-t/\tau}\right)$$

$$\frac{4}{12} = 1 - e^{-t/\tau}$$

$$e^{-t/\tau} = 1 - \frac{4}{12} = \frac{8}{12}$$

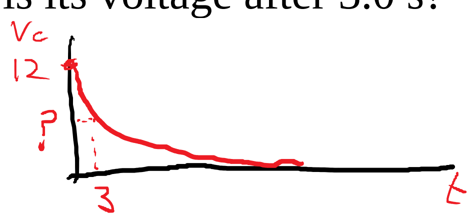
$$-t/\tau = \ln\left(\frac{8}{12}\right) = -0.405$$

$$RC = 7.40\,s = \frac{-3}{-0.405} = \frac{-t}{-0.405} = \tau$$

If the charged capacitor is now discharged through the same resistor, what is its voltage after 3.0 s?

Same t, τ

$$e^{-t/\tau} = \frac{8}{12}$$



$$V = V_0 e^{-t/\tau}$$

$$V = (12)\left(\frac{8}{12}\right) = 8\ V$$

When charging, this capacitor gained 4 out of 12 V in 3 s.
When discharging, it lost 4 out of 12 V in 3 s.

Physical Capacitor

Tuesday, September 24, 2019 1:22 PM

$$C = \frac{K \varepsilon_0 A}{d}$$

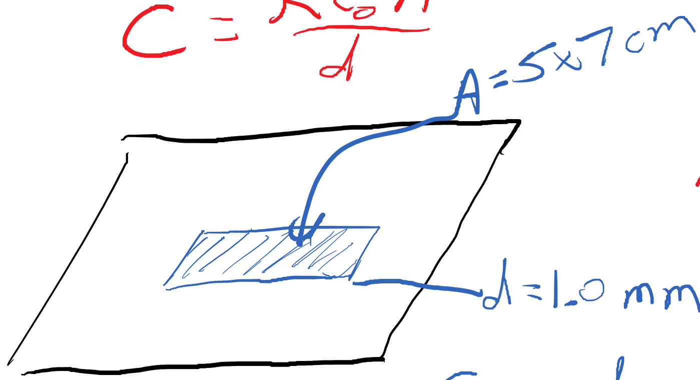
K = "Kappa" = Dielectric Constant

$K_{air} = 1$

$\varepsilon_0 = 8.85 \times 10^{-12}$

A = Overlap Area

d = Plate separation



$C \sim$ dozens of pF

Inside Capacitor: $E = \frac{V}{d}$

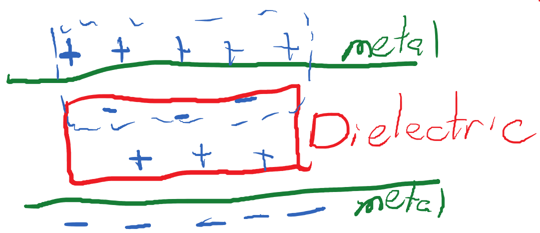
$$Q = CV$$

K "amplifies" C

For the same Q less V.

Same Q: Less E.

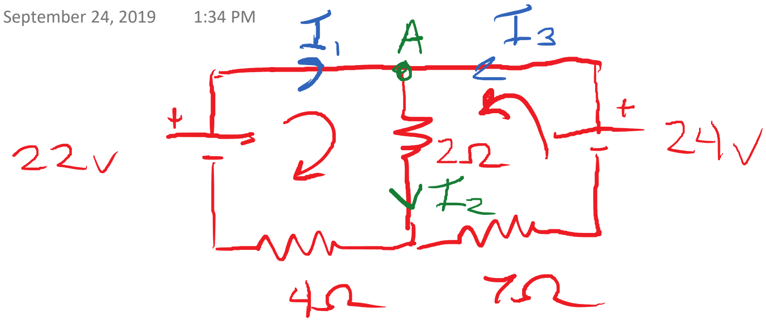
How?



E is as if Q is smaller.

Kirchoff's Laws - Branch Analysis

Tuesday, September 24, 2019 1:34 PM



If: $I_1 + I_3 = I_2$

What is I_2?

Leaving A, outward.

KVL Left Loop: $22 = 2I_2 + 4I_1$ $\quad 22 - 2I_2 - 4I_1 = 0$

Right Loop $24 = 2I_2 + 7I_3$ $\quad 24 - 2I_2 - 7I_3 = 0$

Subtract: $22 - 24 - 4I_1 + 7I_3 = 0$