Why study electricity and magnetism (E&M)?

- Our society is electric
  - Energy
  - Information: Communication, storage, computation, Measurement, Control
- Light and Radio waves
- Chemistry
- Practice with math and "learning new things".
Study of stationary charges. What are charges?

Generally, "charge" is a measure of how electric something is.

"Charge" can refer to an object or particle that has charge.

All matter is made of atoms, which are in turn made of protons, electrons, and neutrons.

\[
\text{Proton: } q_p = 1.6 \times 10^{-19} \text{ C} = +e \\
\text{Electron} \quad q_e = -1.6 \times 10^{-18} \text{ C} = -e
\]

How much charge is present in a 27 gram aluminum piece?

\[
M = 0.027 \text{ kg} \quad m = 27 \text{ u} = 27(1.67 \times 10^{-27} \text{ kg}) \\
M = Nm \quad N = \frac{0.027 \text{ kg}}{27(1.67 \times 10^{-27} \text{ kg})} = 6 \times 10^{-23}
\]

Each aluminum atom has 13 protons and 13 electrons.

\[
Q_p = q_p N_p = (1.6 \times 10^{-19})(6 \times 10^{23})(13) = 1.25 \times 10^6 \text{ C} \\
Q_e = -1.25 \times 10^6 \text{ C} \\
Q_T = 0
\]

If we wanted to use the aluminum in an electric way, we could do a few things:

- Steal or donate electrons. Maybe only 1 nC to 1 micro-C worth. This is an electrostatic charge.
- Shift electrons around. About one electron per atom is mobile. As long as the total number remains constant, we can move massive amounts of charge.
- Shift electrons around, temporarily. This is called polarizing the material.
• Charges exert forces on each other. How?
• Charges generate "electric field" all around themselves. Other charges "feel" the electric field as a force.

\[ \mathbf{F}_1 = \frac{k|q_1|q_2}{r^2} \]
\[ \mathbf{F}_2 = \frac{k|q_2|q_1}{r^2} \]

Net result: Opposites attract.

Magnitude:

\[ E_1 = \frac{k|q_1|}{r^2} \]
\[ E_2 = \frac{k|q_2|}{r^2} \]

\[ F_1 = q_1 E_2 \]
\[ F_2 = q_2 E_1 \]

\[ F_1 = E_2 = \frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2)(8 \times 10^{-9} \text{ C})(5 \times 10^{-9} \text{ C})}{(0.25 \text{ m})^2} = 5.76 \times 10^{-6} \text{ N} = 5.76 \text{ mN} \]

How can we incorporate the direction into the formula?

\[ \mathbf{F}_1 = q_1 \mathbf{E} \]
\[ \text{If } q_1 = + \text{ then } \mathbf{F}_1 = \mathbf{E} \text{ (same direction)} \]

The electric field direction is the same as the direction of the force that would be felt by a positive charge. The positive charge doesn't need to actually be there. It is called a "test charge".
The electric field generated by a negative $q_2$ always points toward $q_2$. 

\[ \mathbf{E}_2 = (\text{magnitude})(\text{direction}) \]
\[ = E_2 \hat{E}_2 \]
\[ = \frac{k |q_2|}{r_2^2} (-\hat{r}_2) \]

\[ \hat{r}_2 \text{ points away from } q_2 \]

This is the vector formula for the electric field of a point charge.
Electric field describes how electrified space is.

The result of electric field is forces on charges.
- If \( q_0 \) is +, the force is in the same dir as \( E \).
- If \( q_0 \) is -, the force is opposite to \( E \).
- Any \( q_0 \) we place there will have the same \( E \).

There are two sources of electric fields:
- Source charges.
- Fluctuating magnetic fields.

\[
E = \frac{k|q_0|}{r^2}
\]

Often, it's easy enough to worry about the magnitude and direction of \( E \) separately.

\[
q_1 = 8 \text{nC} \quad q_2 = -5 \text{nC}
\]

\[
E_1 = \frac{kq_1}{r^2} = \frac{(9 \times 10^9) \times (8 \times 10^{-9})}{(0.25)^2} = 720 \text{ N/C}
\]

\[
F_1 = q_1E_1 = (8 \times 10^{-9}) \times (720) = 5.76 \times 10^{-6} \text{ N}
\]

\[
E_2 = \frac{kq_2}{r^2} = \frac{(9 \times 10^9) \times (5 \times 10^{-9})}{(0.25)^2} = 1152 \text{ N/C}
\]

\[
F_2 = q_2E_2 = (5 \times 10^{-9}) \times (1152) = 5.76 \times 10^{-6} \text{ N}
\]

Obeying Newton's 3rd Law.
Multiple charges can contribute toward E.

\[ E = E_1 + E_2 \]

\[ E_1 = \frac{kq_1}{r_1^2} = \frac{(9 \times 10^9)(4 \times 10^{-9})}{(0.05)^2} = 14400 \ \text{N/C} \]

Direction of \( E_1 \): \( \hat{E}_1 \) = unit vector from source to point = \( \frac{\hat{r}}{|\hat{r}|} = \frac{(4 \text{ cm})\hat{i} + (-3 \text{ cm})\hat{j}}{\sqrt{5^2} \cos \Theta} \)

\[ \hat{r} = \frac{4}{5} \hat{i} + \frac{-3}{5} \hat{j} \]

\[ E_1 = \frac{kq_1}{r_1^2} \hat{r} = (14400 \ \text{N/C}) \left( \frac{4}{5} \hat{i} + \frac{-3}{5} \hat{j} \right) \]

\[ E_{1x} = (14400 \ \text{N/C}) \left( \frac{4}{5} \right) = 11520 \ \text{N/C} \]

\[ E_{2x} = \frac{(9 \times 10^9)(1 \times 10^{-9})}{(0.05)^2} \left( \frac{-4}{5} \right) = -2880 \ \text{N/C} \]

\[ E_2 = 3600 \]

\[ E_\text{x} = 8640 \ \text{N/C} \]

\[ E_{1y} = (14400) \left( \frac{-3}{5} \right) = -8640 \ \text{N/C} \quad \text{(coincidence)} \]

\[ E_{2y} = (3600) \left( \frac{-3}{5} \right) = -2160 \ \text{N/C} \]

\[ E_y = -10800 \ \text{N/C} \]

Alternative way to find the components of E1:
Where is the electric field equal to zero?

\[ \mathbf{E}_1 + \mathbf{E}_2 = 0 \]

\[ \mathbf{E}_1 = -\mathbf{E}_2 \]

\[ \mathbf{E}_1 = \mathbf{E}_2 \]

magnitude

direction

\[ \frac{kq_1}{x^2} = \frac{kq_2}{(d-x)^2} \]

\[ \frac{k(4nC)}{x^2} = \frac{k(1nC)}{(d-x)^2} \]

\[ \frac{\sqrt{4}}{x} = \frac{\sqrt{1}}{(d-x)} \]

\[ \frac{2(d-x)}{2d-2x} = x \]

\[ 2d - 2x = \frac{2}{3}d = x \]
Line charge:  \( \lambda = \text{linear charge density} = \frac{Q}{l} \)

Q = \( \lambda l \) (if \( \lambda \) = const)

Q = \( \int \lambda \, dx \) (IF along x-axis)

Q = \( \int \lambda \, dl \) (Arbitrary curve) Line Integral

Q = \( \int dq = \frac{2}{\pi} \, q_i \)

What if we want to know the electric field due to the line charge distribution?

\[ E = \int dE = \int \frac{k \lambda \, dq}{r^2} \]

\[ E_x = \int_{-\infty}^{\infty} \frac{k \lambda (-x) \, dx}{(x^2+y^2)^{3/2}} = 0 \]

\[ E_y = \int_{-\infty}^{\infty} \frac{k \lambda \, dx}{(x^2+y^2)^{3/2}} = \frac{2k \lambda}{y} \]

This is the "Green's Function Method" of solving the Maxwell's equation.
This is a form of Stokes' Theorem. Alternative way of finding Electric Field.

Integration method (Green's Function based):

Gauss's Law Method is based on electric flux.
• Electric flux is the "total amount of electric field" pointing through a surface.

\[ \Phi_E = EA \cos \theta \]

A large surface may have a variety of angles and a variety of E values.

Let's try to calculate the flux "emitted" by a point charge.
No matter what sphere we choose, the calculated flux is the same.

In fact, the flux is the same for *any* surface that surrounds the point charge.

Integration is a linear operation.

\[ \int [af(x) + bg(x)]dx = a\int f(x)dx + b\int g(x)dx \]

Gauss's Law in general:

\[ \Phi_E = 4\pi kQ_{enc} \]

The flux pointing through any closed surface is equal to 4pi*k times the charge enclosed by the surface.

How does this help us with finding E?

- In symmetric cases, E "factors out" of the integral.

Spherical Symmetry

\[ \Phi_E = \int E_1 dA = 4\pi r^2 E = 4\pi kQ_{enc} \]

\[ E = \frac{kQ_{enc}}{r^2} \]

Example: Uniformly charged sphere
\[ E = \frac{kQ_{\text{enc}}}{r_i^2} = \frac{k}{r_i^2} \left( Q_T \frac{r_i^3}{R^3} \right) = \frac{kQ_T}{R^3} r_i \]
Source is a uniform infinite line charge.

\[ \lambda = \frac{Q}{L} \]

\[ \Phi_E = \Phi_{\text{ends}} + \Phi_{\text{shell}} \]
\[ = EA_{\text{shell}} \]
\[ = E 2\pi r L = 4\pi k Q_{\text{enc}} \]
\[ E = \frac{2k \lambda}{r} \]
A 4 nC point charge is surrounded by a thick metal shell with inner radius 3 cm and outer radius 4 cm. The total charge of the metal shell is -7 nC. How is the charge of the shell distributed and what is E everywhere?

Important fact: E=0 in a conductor in electrostatics.
- Conductors allow charges to move.
- E causes forces on charges.
- If E ≠ 0, the charges would move.

If I draw a Gaussian surface at 3 < r < 4, Q_enc = 0. The only places on the metal where charge can reside are on the inner surface (r=3) and outer surface (r=4).

$$Q_{enc} = 4 \text{nC} + Q_{inner} = 0 \quad Q_{inner} = -4 \text{nC}$$

in the hollow

$$Q_{metal} = Q_{inner} + Q_{outer}$$

$$-7 \text{nC} = -4 \text{nC} + Q_{outer} \quad Q_{outer} = -3 \text{nC}$$

To find the electric field, use Gauss's Law:

Inside:  \( \Phi_E = 4\pi r^2 E = 4\pi k Q_{enc} \quad E = \frac{k Q_{enc}}{r^2} \)

Middle radii: 3 < r < 4

E = 0

E = k \left( \frac{-3 \text{nC}}{r^2} \right)
Outside:

\[ E = \frac{k \cdot (-3 \, \text{nC})}{r^2} \]