11. Exam1 Return, Intro Magnetism

Tuesday, October 1, 2019 12:25 PM

Average: 59%

It's the Course Average, updated later today, that is your best estimate of your status. (60% Exams, 15% HW, 25% Lab)

Equivalent unit to the farad (F)? measures capacitance (C).

$$C = \frac{K\varepsilon_0 A}{d}$$

$$Q = CV \qquad [C] = [F][V] \qquad \left[\frac{C}{V}\right] = [F]$$

$$\tau = RC \qquad [s] = [\Omega][F] \qquad \left[\frac{s}{\Omega}\right] = [F]$$

Mass is the only quantity where the fundamental unit has a prefix.

Kilogram (kg)

$$25\,\text{mg} = 25\times10^{-3}\,\text{g} = 25\times10^{-6}\,\text{kg}$$

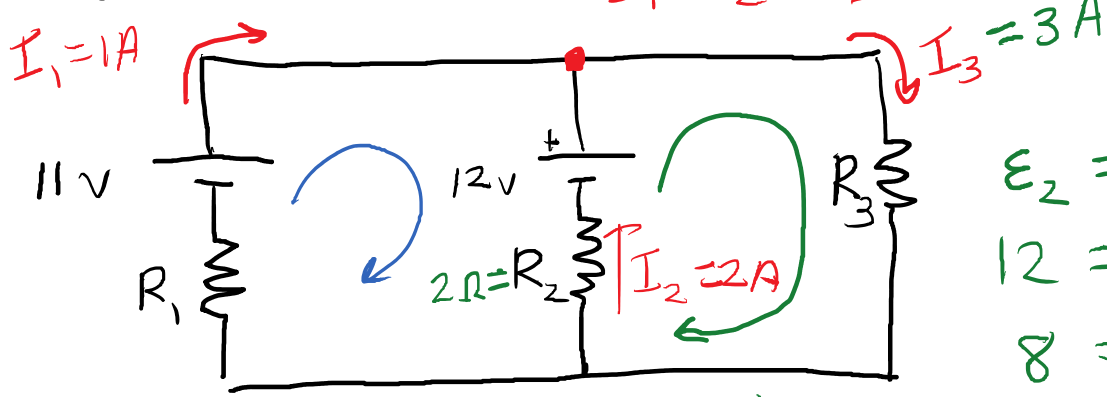
Cost of electricity

$$\frac{\$1.25}{\$0.12/\text{kWh}} = 10.4\ \text{kWh}$$

$$\frac{10400\ \text{Wh}}{50\ \text{W}} = 208\ \text{hours}$$

Two-battery Kirchhoff's Law circuit

$$I_1 + I_2 = I_3$$



$$\varepsilon_2 = V_3 + V_2$$

$$12 = V_3 + 4$$

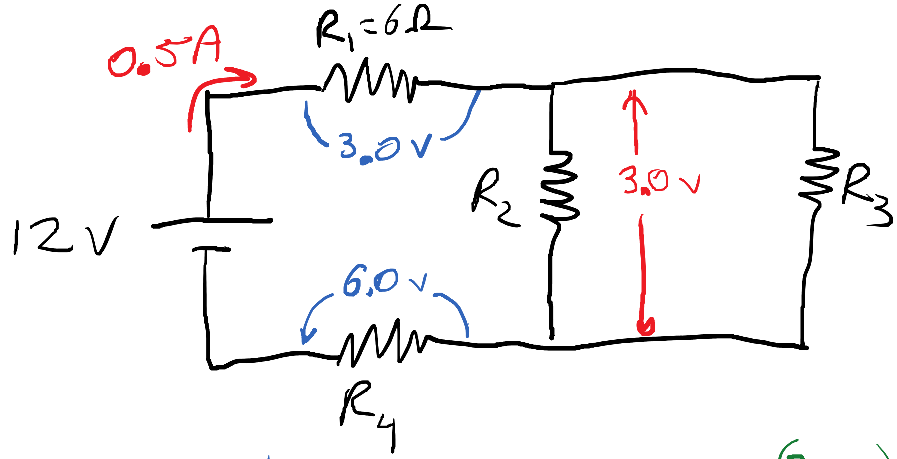
$$8 = V_3$$

$$11 - 12 + I_2R_2 - I_1R_1 = 0$$

$$V_3 = I_3R_3$$

$$8 = 3R_3$$

Combination resistor circuit



$I_2 + I_3 = (0.5A)$

$I_2 = 2I_3$

$2I_3 + I_3 = 0.5$

$I_3 = (0.167A)$

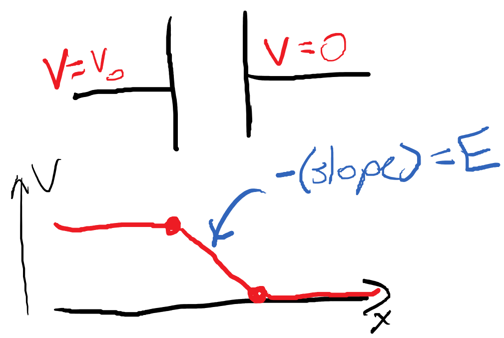
$(6.0v) = (0.5A)R_4$

$(3.0v) = (0.167A)R_3$

Capacitors in electrostatics and RC Circuits.

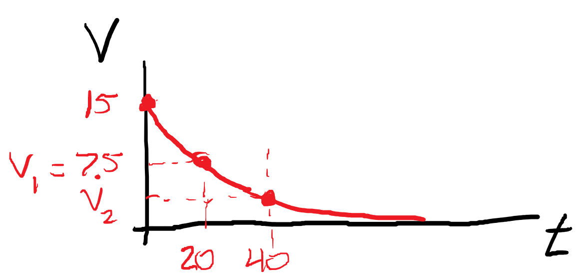
$C = 300\mu F \qquad V_0 = 15.0\ V \qquad Q_0 = CV_0 = 4500\mu C$

$E_x = \frac{-dV}{dx}$



$E = \frac{V}{d} = \frac{15 v}{0.1 \times 10^{-3} m}$

$= 150000\ V/m$



$V = V_0 e^{-t/\tau}$

$7.5 = 15 e^{-20/\tau} \qquad e^{-20/\tau} = 0.5$

$\frac{V_2}{V_1} = \frac{V_0 e^{-t_2/\tau}}{V_0 e^{-t_1/\tau}} = e^{-(t_2 - t_1)/\tau}$

$\frac{2^5}{2^2} = 2^3$

$\frac{V_2}{V_1} = e^{-20/\tau}$

If the voltage is cut in half in one 20 s interval,
it must be cut in half in any 20 s interval.

$e^{-20/\tau} = 0.5$

$\frac{-20}{\ln(0.5)} = \tau = 29 \longleftarrow \frac{-20}{\tau} = \ln(0.5)$

Intro to Magnetism

Tuesday, October 1, 2019 1:17 PM

Magnetic fields are a lot like electric fields.
Magnetic fields ($\vec{B}$) always form loops.

- No "sources" where B points away, or "sinks" where B points toward.

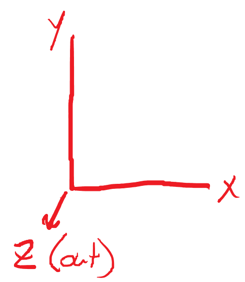
Magnetic fields are created by:

- Electric Currents
- Magnetic materials (dipoles)
- Fluctuating Electric Fields

Effects of magnetic fields:

- Forces on currents and moving charges
- Torques and forces on dipoles
- Generate electric fields and voltages

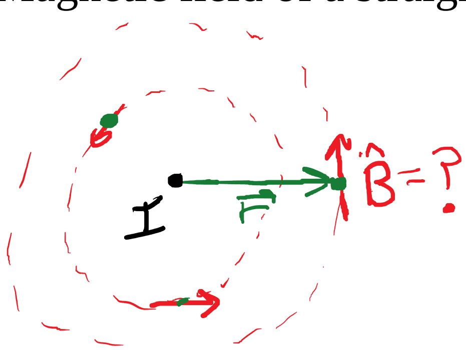
Magnetic geometries are always 3-D.



Paper	Relative	Map	Arrows
x = Right	x = Right	x = East	→
y = Up	y = Up	y = North	↑
z = Out	z = Backward	z = Up	⊙ (out) X (in)

By itself, "Up" is ambiguous.

Ex: Magnetic field of a straight wire:



$\vec{r}$ points from source to field point,

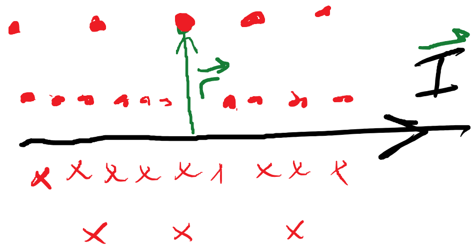
Find $\hat{B}$ by right-hand-rule

$\hat{I} = \hat{z}$ $\hat{r} = \hat{x}$ $\hat{B} = \hat{y}$

$\hat{I}$

$I = z$ $\quad r = x$

Side View:



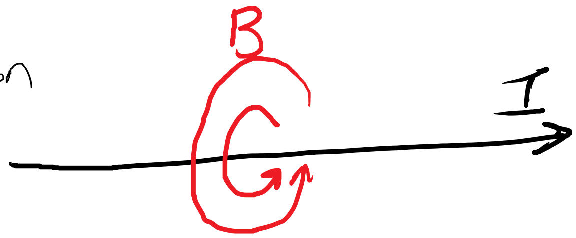
$\hat{I} = \hat{x}$

$\hat{r} = \hat{\varphi}$

$\vec{B} = \hat{z}$

(Only points in plane are shown above.)

3D Version



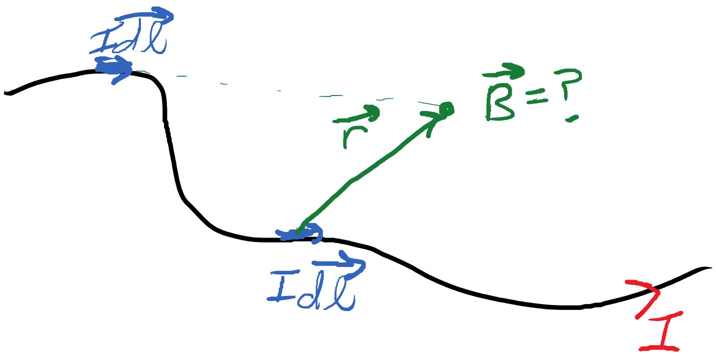
12. Magnetic Sources

Thursday, October 3, 2019 12:25 PM

Generally, $\vec{B}$ is caused by currents.

Biot - Savart Law

$$\vec{B} = \int \frac{\mu_0 I \, \vec{d\ell} \otimes \hat{r}}{4\pi r^2}$$



$$\hat{r} = \vec{r}/r$$

$\otimes$ = Cross Product

$$\mu_0 = 4\pi \times 10^{-7} \ \frac{T \cdot m}{A}$$

Vector Cross Product:

$$\vec{C} = \vec{A} \otimes \vec{B}$$

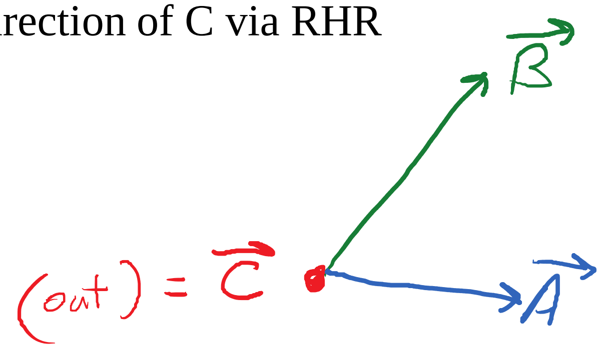
- C is perpendicular to A and B
- C has a magnitude of AB sin(theta), where theta is the angle between A and B.
- Pick direction of C via RHR

$$\theta = 0 \rightarrow \vec{A} \otimes \vec{B} = 0$$

C = thumb

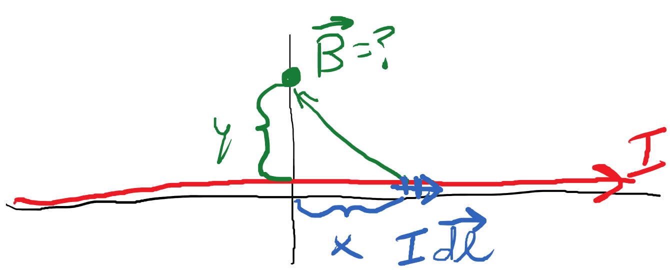
A = index finger

B = middle finger



Ex: Infinite, straight current

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{d\ell} \otimes \hat{r}}{r^2}$$



$$\vec{r} = -x\hat{i} + y\hat{j}$$

$$r^2 = x^2 + y^2$$

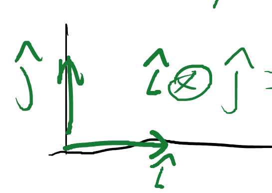
$$r = \sqrt{x^2 + y^2}$$

$$\hat{r} = \frac{-x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}}$$

$$\vec{d\ell} = dx\,\hat{i}$$

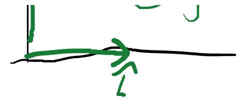
$$\vec{d\ell} \otimes \hat{r} = (dx\,\hat{i}) \otimes \left(\frac{-x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}} \right)$$

$$= \frac{y\,dx}{} \ \cancel{\hat{i} \otimes \hat{j}} \ \hat{k}$$

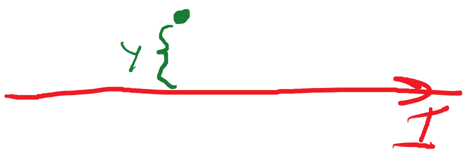


$$\hat{i} \otimes \hat{j} = \hat{k}$$

$$= \frac{y\,dx}{\sqrt{x^2+y^2}}\ \cancel{\hat{i} \otimes \hat{j}}\ \hat{k}$$

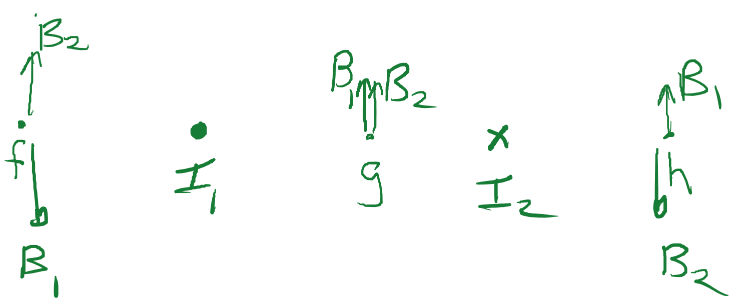


$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{y\,dx}{(x^2+y^2)^{3/2}}\ \hat{k} = \frac{\mu_0 I}{2\pi y}\ \hat{k}$$



Generally, we quote this formula as: $B_{wire} = \frac{\mu_0 I}{2\pi r}$

Example: Magnetic field due to two currents:



Point	$\hat{B}_1$	$\hat{B}_2$	$\lvert B \rvert$
f	$-\hat{j}$	$+\hat{j}$	$B_2 - B_1$
g	$+\hat{j}$	$+\hat{j}$	$B_1 + B_2$
h	$+\hat{j}$	$-\hat{j}$	$B_1 - B_2$

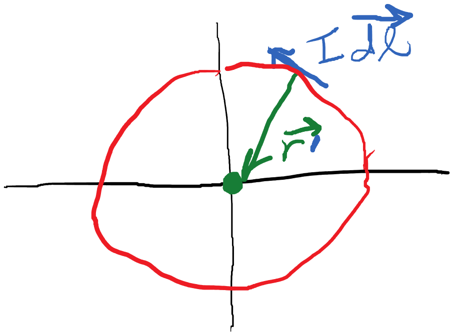
Ex: I1 = I2 = 1.5 A, with 4 cm between them.

$$g: \quad B_g = \frac{\mu_0 (1.5)}{2\pi(0.02)} + \frac{\mu_0 (1.5)}{2\pi(0.02)} = 3 \times 10^{-5}\ T$$

Hint: type mu_0 as (pi * 4e-7)

$$\mu_0 = 4\pi \times 10^{-7}\ \frac{Tm}{A}$$

Magnetic Field of a circle of current, at the origin.



$$\int \frac{\mu_0 I\, \vec{dl} \otimes \hat{r}'}{4\pi r'^2}$$

$\vec{r}/r'$

$$\vec{dl} = R\,d\theta\,\hat{\theta}$$

$$\vec{r}' = -R\,\hat{r}$$

outward

$$\vec{dl} \otimes \hat{r} = \frac{R^2\,d\theta\,\hat{k}}{R} = R\,d\theta\,\hat{k}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{R\,d\theta\,\hat{k}}{R^2} = \frac{\mu_0 I}{2R}\hat{k}$$

Usually quoted as: $B_{loop} = \frac{\mu_0 I}{2R}$ $\qquad B_{wire} = \frac{\mu_0 I}{2\pi r}$

Ampere's Law

Thursday, October 3, 2019 1:24 PM

This is akin to Gauss's Law for electrostatics.
For any imaginary looping path:

$$\int \vec{B} \cdot \vec{d\ell} = \mu_0 I_{enc} = \int (B\, d\ell \cos\theta) = \int B\, d\ell = B \int d\ell$$

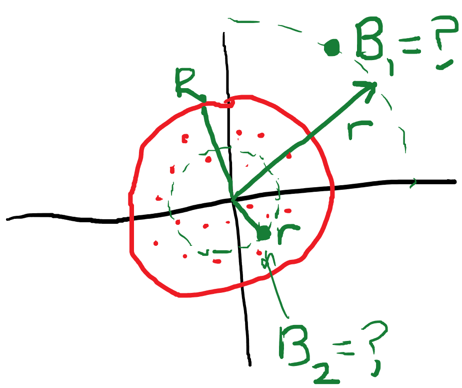
When applied, we make the integral easy.

- Make sure B = constant along the path.
- Keep theta (of the dot product) constant along the path.

$$B\ell = \mu_0 I_{enc}$$

Ex: Magnetic field inside and outside a thick wire:

$$B\ell = \mu_0 I_{enc}$$

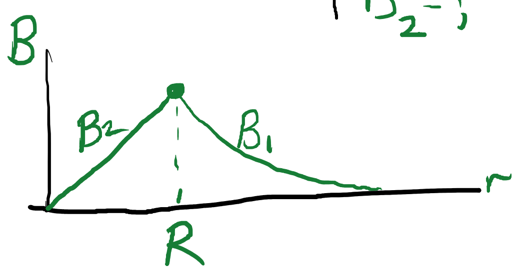


$$B_1\, 2\pi r = \mu_0 I$$

$$B_1 = \frac{\mu_0 I}{2\pi r}$$

$$B_2\, 2\pi r = \mu_0 I \left(\frac{r}{R}\right)^2$$

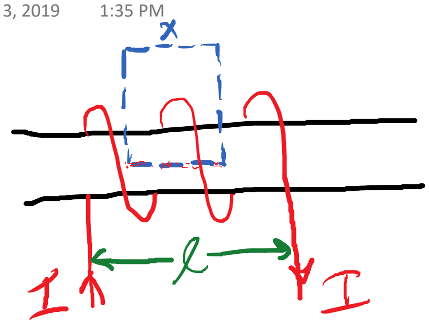
$$B_2 = \frac{\mu_0 I r}{2\pi R^2}$$



When $r = R$, $B_1 = B_2$

Solenoid Coil

Thursday, October 3, 2019 1:35 PM



Solenoid: $\oint \vec{B} \cdot d\vec{\ell} = B_o x = \mu_o I_{enc}$

$$I_{enc} = \left(\frac{N}{\ell}\right)(x)\, I$$

Inside: $B = B_o$

Outside: $B = 0$

$$B_o \cancel{x} = \frac{\mu_o I}{\ell} N \cancel{x}$$

13. Magnetic Forces and Torques

Tuesday, October 8, 2019 12:27 PM

Magnetic fields cause magnetic forces on moving charges.

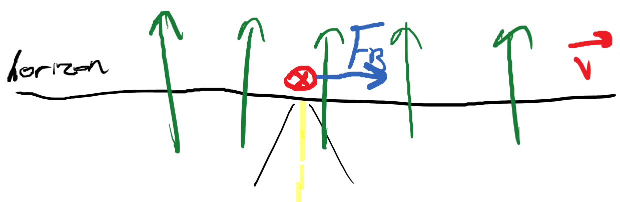
$$\vec{F}_B = q\vec{v} \otimes \vec{B}$$

The cross product does a few things beyond F_B = qvB:

- The velocity must have some component perpendicular to B.
- The direction of the force is by the RHR for cross products.
  - F_B is perpendicular to velocity
  - F_B is perpendicular to B

$$\vec{F}_B = q\vec{v} \otimes \vec{B}$$

$$\vec{B} = B_0\hat{j}$$



$$\vec{v} = v_0(-\hat{k})$$

into page

$$\vec{F}_B = q v_0 B_0 \hat{i}$$

Note: If the charge is negative, reverse the result after doing the RHR.

Since the magnetic force is always perpendicular to velocity:

- F_B cannot change the velocity magnitude, only its direction.
- F_B cannot transfer energy.

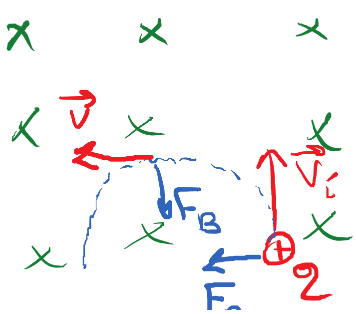
$$W_B = \int \vec{F}_B \cdot \vec{d\ell} = \int \vec{F}_B \cdot \vec{v}\,dt$$

$= 0$ because $\vec{F}_B \perp \vec{v}$

When the magnetic force is the only significant force acting on a particle:

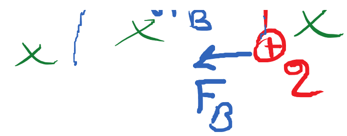
- The perpendicular magnetic force is also the centripetal force.

$$\vec{B} = B_0(-\hat{k})$$



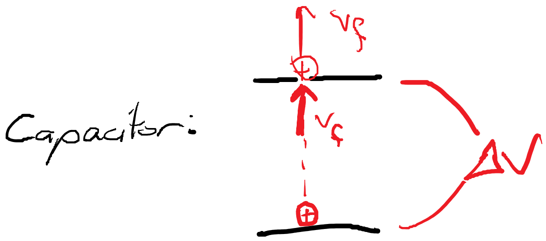
$$\vec{F}_{net} = m\vec{a}$$

$$qvB = F_B = \frac{mv^2}{R}$$



$$R = \frac{mv}{qB}$$

The radius of the path depends on the mass of the particle.
This allows building a mass spectrometer.
There are multiple methods of feeding the particles into the mass spec.
Method 1: Linear accelerator (like Lab 6)



Voltage

$$q\Delta V = \frac{1}{2}mv^2$$

speed

$$v = \sqrt{\frac{2q\Delta V}{m}}$$

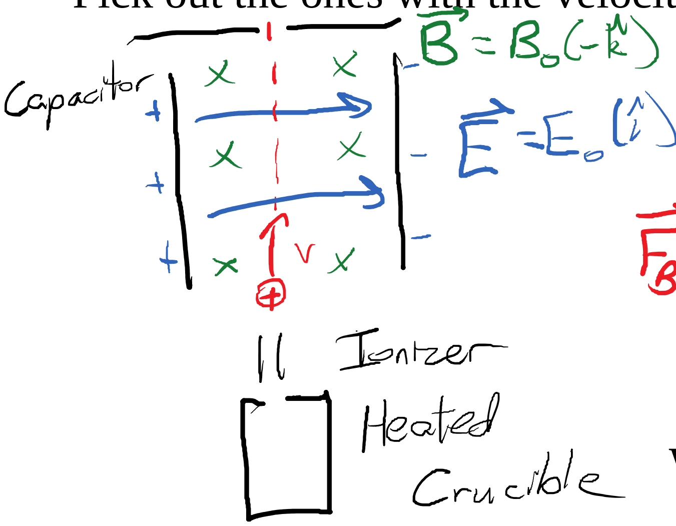
With this method, the speed also depends on mass.

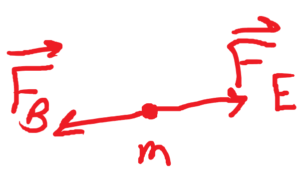
$$R = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \frac{1}{B}\sqrt{\frac{2m\Delta V}{q}}$$

Now it's clear that R is proportional to the square root of mass.
The only problem is this "compresses" large masses into similar R.

It would be nice to give all particles the same velocity.

Method 2: Velocity Selector
- Start with particles of various velocities.
- Pick out the ones with the velocity you want.





$$F_B = F_E$$
$$qvB = qE$$
$$vB = E$$
$$v = E/B$$

When v=E/B, the particles feel zero net force.
- Too fast: particles bend left.

- Too slow: particles bend right.

When a velocity selector is fed into a mass spectrometer, we get the simple:

$$R = \frac{mv}{qB} \qquad R \propto m$$

Magnetic force on current

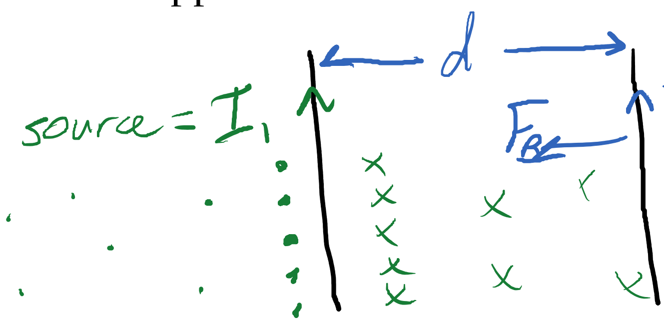
Tuesday, October 8, 2019 1:08 PM

$$\vec{F}_B = q\vec{v} \otimes \vec{B}$$

$$\vec{F}_B = I\vec{\ell} \otimes \vec{B}$$

└ Length and direction of wire.

Direct application: Force exerted by wires on each other.



ℓ

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

Recall:

$$F_E = \frac{k q_1 q_2}{r^2}$$

$$F_B = I_2 \ell B_1 = \frac{\mu_0 I_1 I_2 \ell}{2\pi d}$$

Unlike with electrostatics, here "opposites repel".

$\mu_0 = 4\pi \times 10^{-7}$

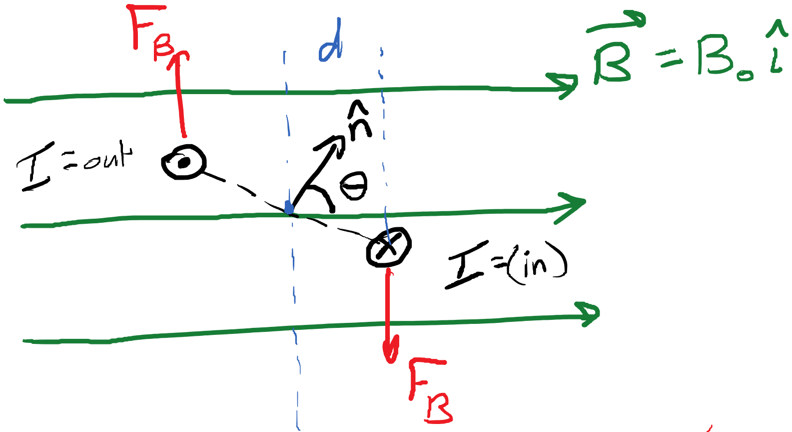
$$F_B = \frac{2\times10^{-7}\ell}{d} I_1 I_2$$

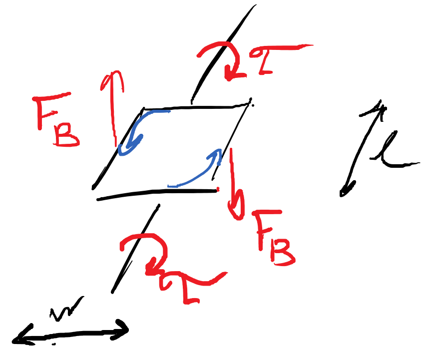
This served as the original definition of the ampere.

Magnetic torques

Tuesday, October 8, 2019 1:16 PM

(Torque = moment)





$$\tau = 2\tau_1 = 2(F_B\, d) = 2\left(I \ell B \tfrac{w}{2} \sin\theta\right)$$

$$\tau = NBAI \sin\theta$$ N = # turns in coil

$$\tau_{max} = \underbrace{NBA}\, I$$ Max motor/generator τ

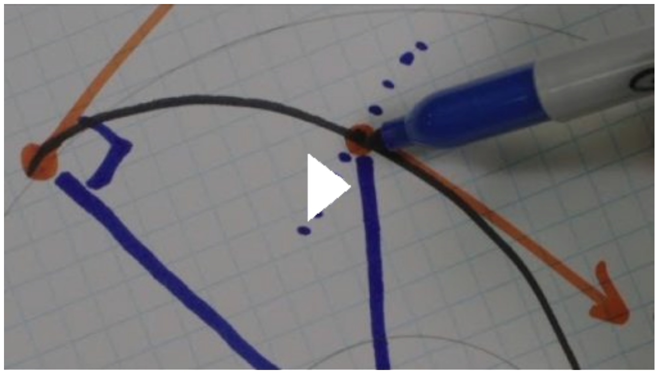
Φ_B = magnetic flux

14. Electromagnetic Induction

Thursday, October 10, 2019 12:22 PM

Chapter 30

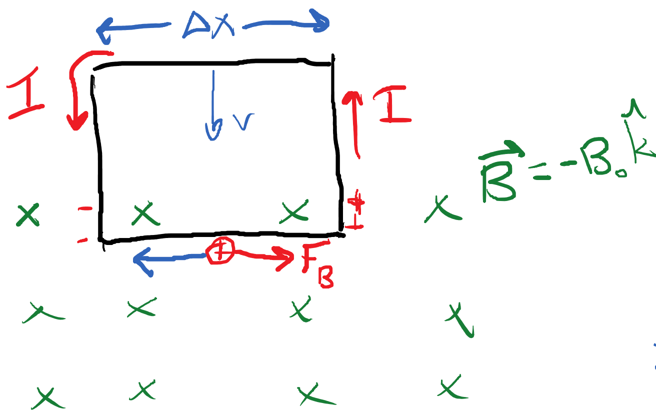
Laws & Causes - posted by Vsauce



Faraday's Law describes induced EMF, but doesn't explain it.
There are two separate causes that are both summarized by the law.

- Motional EMF
- Fluctuating magnetic fields

Motional EMF, by example: Drop a rectangular loop into a B field.



+ charges in the wire are falling downward, and feel a magnetic force.

$$\vec{F}_B = q\vec{v}\otimes\vec{B} = F_B\hat{i} = qvB_0\hat{i}$$

$$\vec{F}_E = q\vec{E} = qE(-\hat{i})$$

$$|E| = \left|\frac{dV}{dx}\right| = \frac{\mathcal{E}}{\Delta x}$$

Equilibrium:

$$\cancel{q}E = \cancel{q}vB$$

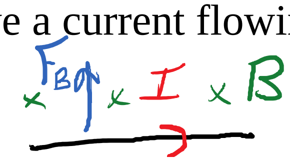
$$\frac{\mathcal{E}}{\Delta x} = vB$$

$$\mathcal{E} = Bv\Delta x$$

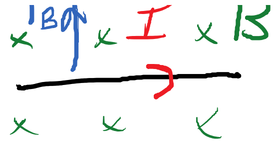
What does this voltage do? Drives current around the loop.

$$\mathcal{E} = IR$$

Now we have a current flowing across a magnetic field.



$$\vec{F}_B = I\vec{l}\otimes\vec{B}$$



$$F_B = I \ell \otimes B$$

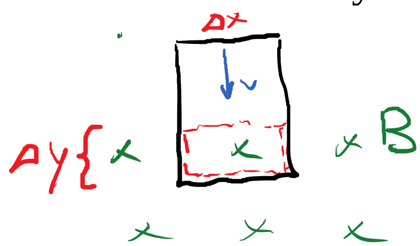
This upward force tends to oppose the velocity of the loop.
If we let go of the loop, the velocity will increase until the forces on the loop balance. The loop can then fall at constant velocity.

$$I \ell B = mg$$

A 25-gram loop that is 50 cm wide and 70 cm tall falls into a 0.4 T magnetic field, and it falls at 15 m/s. What is the resistance?

$$\mathcal{E} = Bv\Delta x = (0.4\,T)(15\,m/s)(0.5\,m)$$
$$= 3.0\,v$$
$$mg = (0.025\,kg)(9.8\,N/kg) = 0.245\,N$$
$$F_B = I\,\Delta x\,B$$
$$(0.245\,N) = I\,(0.5\,m)(0.4\,T)$$
$$1.225\,A = I$$
$$\mathcal{E} = IR$$
$$R = \frac{\mathcal{E}}{I} = \frac{3.0\,v}{1.225\,A} = 2.45\,\Omega$$

How is this related to Faraday's Law?



$$\mathcal{E} = Bv\Delta x \qquad v = \frac{\Delta y}{\Delta t}$$

$$\mathcal{E} = \frac{B\Delta y\Delta x}{\Delta t} = \frac{B\Delta A}{\Delta t}$$

Faraday's Law

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad \Phi_B = \iint \vec{B}\cdot d\vec{A}$$
$$= NBA\cos\Theta$$

loops

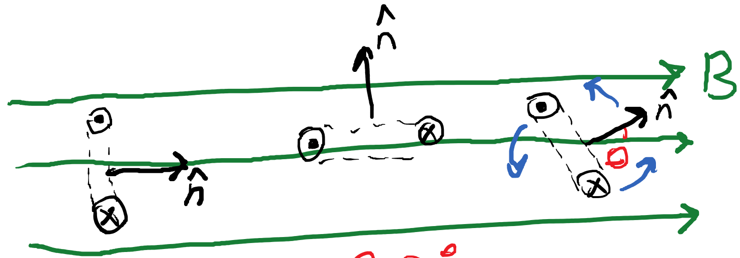
endicular

$$= NBA\cos\theta$$

Loops

Area of overlap

How perpendicular is B to loop?



$\theta = 0$ $\qquad$ $\theta = 90°$

$\Phi_B = NBA$ $\qquad$ $\Phi_B = 0$

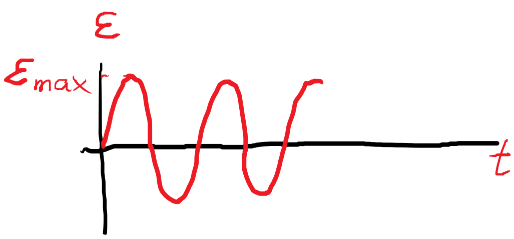
$$\Phi_B = NBA\cos\theta$$

$$\frac{d\Phi_B}{dt} = NBA(-\sin\theta)\frac{d\theta}{dt}$$

Electric Generator

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = NBA\,\omega\sin\theta$$

omega is angular velocity



$$\mathcal{E}_{max} = NBA\,\omega$$

Motors and generators are basically the same thing.

Motor

We apply voltage, which makes I.

Current makes torque, $\tau = NBAI\sin\theta$ which spins the coil.

Coil makes Back EMF.

This cancels some of our voltage, and reduces the current I.

$$P_{in} = P_{out}$$

$$VI = \tau\omega$$

Generator

We apply torque, which spins the coil.

Coil makes EMF, which drives current.

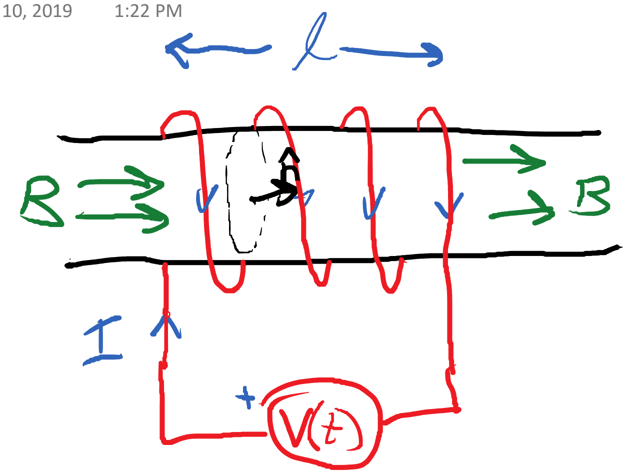
Current makes drag torque, which makes it harder to spin the coil.

$$P_{in} = P_{out}$$

$$\tau\omega = VI$$

We can have EMF without motion of the coil.

Thursday, October 10, 2019 1:22 PM



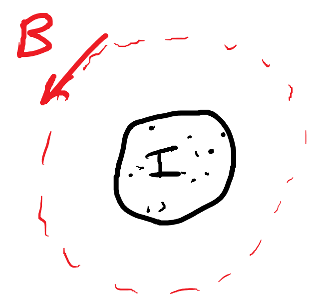
$$B = \frac{\mu_0 N I}{\ell}$$

$$\Phi_B = NBA\cos\theta$$

$$\Phi_B = \frac{\mu_0 N^2 A}{\ell} I$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 N^2 A}{\ell}\frac{dI}{dt}$$

Without motional EMF, what makes the EMF?
The fluctuating magnetic field generates electric field.
There's an analogue to Ampere's Law for electric fields.



$$\int \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$\dot{B} = \frac{dB}{dt}$$

Any time there is a fluctuating B, there are electric field lines looping around it.

15. Inductance

Tuesday, October 15, 2019 12:24 PM

Self-Inductance (L) is the proportionality between a coil's own magnetic flux (Φ_B) and its current (I).

Ex: Solenoid coil

$$B = \frac{\mu_0 N I}{\ell} \qquad \Phi_B = NBA\cancel{\cos\theta}^{\,1}$$

$$\Phi_B = \underbrace{\left(\frac{\mu_0 N^2 A}{\ell}\right)}_{\text{self-inductance}} I$$

$$L = \frac{\mu_0 N^2 A}{\ell}$$

$N = 1500$
$r = 0.03\ m$
$\ell = 0.11\ m$

$$L = \frac{(4\pi \times 10^{-7})(1500)^2(\pi(0.03)^2)}{(0.11)}$$

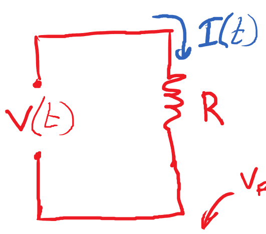
$$L = 0.0727\ H = 72.7\ mH$$

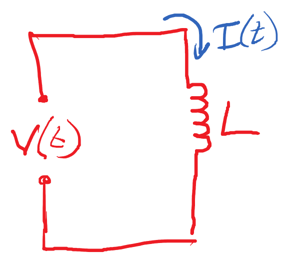
Inductance in henries

Why? Magnetic flux is related to EMF.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(LI) = -L\frac{dI}{dt}$$

Let's compare inductors vs. resistors.





$V_R = IR$ = voltage drops.

$$\Sigma\mathcal{E} = \Sigma V_R$$

$$V(t) = I(t)\,R$$

$$I = \frac{V}{R}$$

$$V(t) - L\frac{dI}{dt} = 0$$

$$V(t) = L\frac{dI}{dt}$$

$$\frac{V}{L} = \frac{dI}{dt}$$

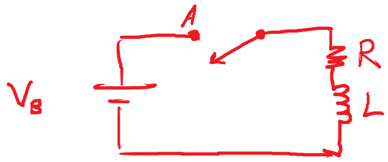
Having an eternal constant dI/dt isn't possible. Eventually, the current will be large enough that even the small resistance in the wires drops the rest of the voltage.

$$V(t) = L\frac{dI}{dt} + IR$$

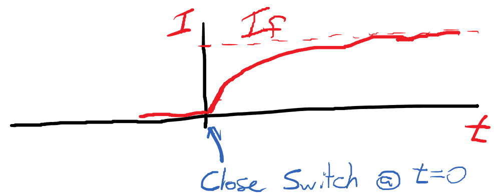
$$V_L = L\frac{dI}{dt} \qquad V_R = IR$$

- The resistor opposes current by "dropping" voltage.
- The inductor opposes changes in current the same way.

When an inductor is initially connected to a battery, we're requesting a quick change in the amount of current.



$$V_B = IR + L\frac{dI}{dt}$$



- The inductor's current cannot change instantaneously. (dI/dt is finite)
- At *t*=0 $\quad V_R = IR = 0 \qquad V_B = L\frac{dI}{dt} \qquad \frac{dI}{dt} = \frac{V_B}{L} = \text{slope}$

Ex: $L = 72.7\text{ mH}$, $V_B = 12\text{ V}$ $\qquad \frac{dI}{dt} = \frac{12}{0.0727} = 165.1\ A/s$

- Eventually, at *t*=infinity:

$$\frac{dI}{dt} = 0 \qquad V_B = IR$$

Ex: $I_f = 0.24\text{ A}$ $\qquad R = \frac{12}{0.024} = 500\,\Omega$

- In between, the current is a shifted decaying exponential.

$$I = I_f + (\overset{0}{\cancel{I_i}} - I_f)e^{-t/\tau}$$

$$I = I_f\left(1 - e^{-t/\tau}\right)$$

Let's check dI/dt:

$$\frac{dI}{dt} = I_f\left(-e^{-t/\tau}\right)\left(\frac{-1}{\tau}\right) = \frac{V_B}{L} = \frac{I_f R}{L} \qquad V_B = I_f R$$

$$\text{at } t=0 \qquad \frac{I_f}{\tau} = \frac{I_f R}{L} \qquad \tau = \frac{L}{R}$$

This is the time constant in an RL circuit.

What is my RL time constant in the above example:

$$\tau = \frac{L}{R} = \frac{0.0727}{500} = 1.45\times10^{-4}\text{ s}$$
$$= 145\ \mu\text{s}$$

Frequently, we care about the "halfway" point.

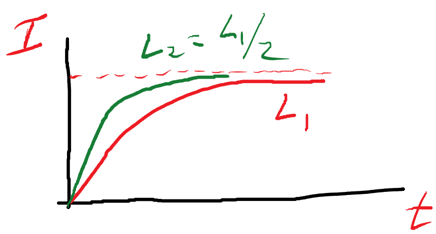
$$I = I_f\left(1-e^{-t/\tau}\right)$$
$$I = I_f\left(\tfrac{1}{2}\right)$$
$$\Big\} \quad e^{-t/\tau} = 0.5$$
$$\frac{-t}{\tau} = \ln(0.5)$$
$$t_{1/2} = -\tau \ln(0.5)$$
$$t_{1/2} = \tau \ln(2)$$

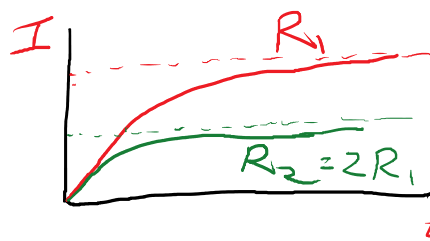
Ex: $\tau_{1/2} = (145\,\mu s)\ln(2) = 101\,\mu s$

For the coil from lab, hooking it up to a 12 V car battery will cause the current to rise to an eventual 0.24 A with a half-life of 101 micro-s.

How does the circuit change when R or L is adjusted? $\tau = L/R$

$t=0$: $V_B = L\frac{dI}{dt}$





$t=\infty$: $V_B = IR$

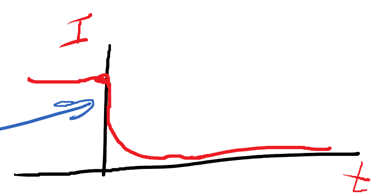
Smaller L --> Faster "charging".

Larger R --> Initial charging rate is the same, but the target current is less, so we get there sooner.

One problem with inductors is "inductive kick".

- When disconnecting an inductor, R -> infinity. This makes tau = L/R very tiny.

$\frac{dI}{dt}$ = Huge!



$\mathcal{E} = -L\frac{dI}{dt}$ = Huge!

This can be a huge problem, which is why there is often a "flyback diode" on relays.

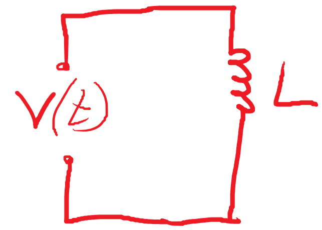
It can also be a huge advantage. "Buck-Boost converter".

Inductors in DC Circuits:

- Steady-state: No opposition to current.
- Transient: Changing the current requires voltage.

Inductors in AC Circuits

Tuesday, October 15, 2019 1:27 PM



$V(t) = V_0 \sin(\omega t)$

$V(t) = L\frac{dI}{dt}$ $\quad I = -I_0 \cos(\omega t)$

$V_0 \sin(\omega t) = L\frac{d}{dt}\left(-I_0 \cos(\omega t)\right)$

$= L\left(-I_0\left(-\sin(\omega t)\omega\right)\right)$

$V_0 = I_0(\omega L)$

Reactance

$X_L = \omega L$

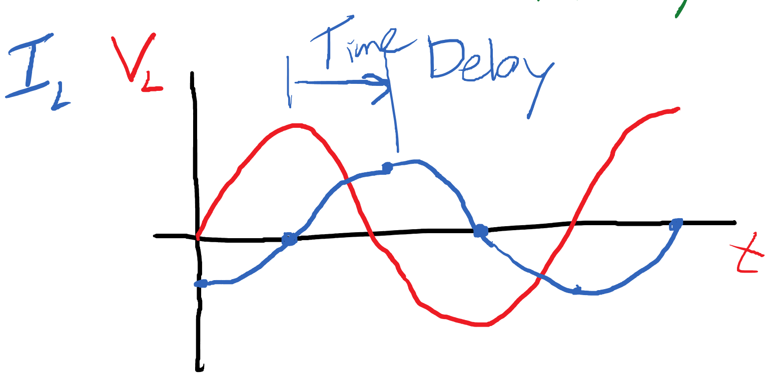
This equation looks like Ohm's Law.

- Reactance is like resistance, but for AC.
- It relates the AMPLITUDES of voltage and current.
- Remember, the voltage and current aren't the same sinewave.
- The reactance is not constant, it depends on the frequency of the circuit.

Ex: $L = 72.7\ mH$

$\omega = 377\ s^{-1}$ radians per sec.

$X_L = (377)(0.0727) = 27.4\,\Omega$



$V_L = V_0 \sin(\omega t)$

$I_L = -I_0 \cos(\omega t)$

Same Frequency

The current "lags" behind the voltage by a quarter cycle.
This is the applied voltage or the voltage drop of the inductor, not the EMF of the inductor.

16. AC Circuits

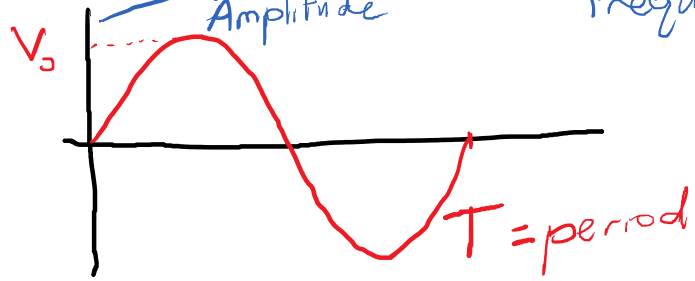
Thursday, October 17, 2019 12:24 PM

Please complete Teamwork Evaluation 1 on Blackboard.

AC electricity involves sinusoidally-varying voltage and current.

Ex: $V = V_0 \sin(2\pi f t)$ or $V = V_0 \cos(2\pi f t)$

Amplitude Frequency



$T = \frac{1}{f}$

Sin(theta) repeats every 2*pi

$$\sin(\theta) = \sin(\theta + 2\pi)$$
$$\sin(2\pi f t) = \sin(2\pi f t + 2\pi)$$
$$= \sin(2\pi f (t + T))$$

$$2\pi = 2\pi f T$$
$$1 = fT$$

Angular frequency vs. frequency

$$\sin(\omega t) = \sin(2\pi f t)$$

$$\omega = 2\pi f$$

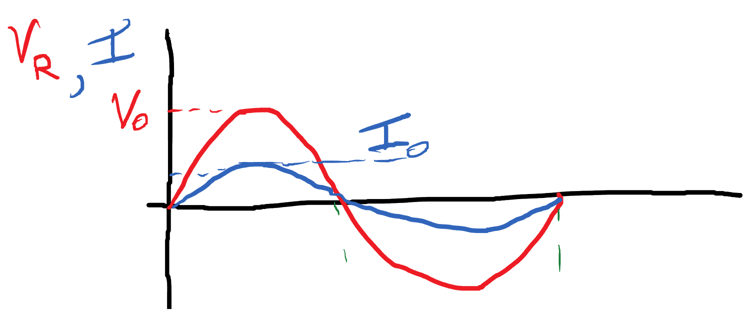
- ω = angular freq in rad/s = s^{-1}
- f = frequency in cycles/s = Hz

Resistors in AC Circuits:

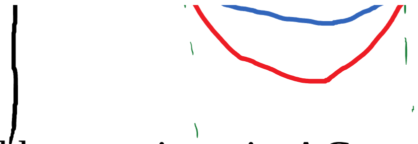
$$V_R = I R$$
$$V_0 \sin(\omega t) = I R$$

$$I = \frac{V_0}{R} \sin(\omega t)$$
$$I = I_0 \sin(\omega t)$$
$$V_0 = I_0 R$$

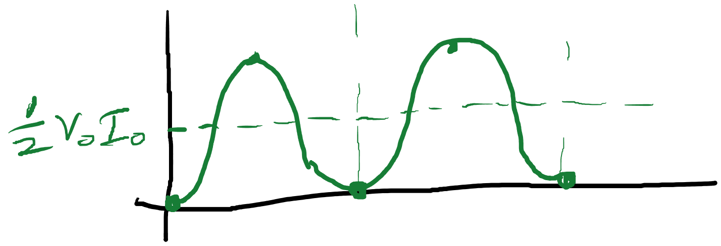
- Ohm's Law for resistors works for amplitudes just as well as for the functions.



$P = V I$



Power used by a resistor in AC.



$$P = VI$$
$$P = V_0 \sin(\omega t)\, I_0 \sin(\omega t)$$
$$P = V_0 I_0 \sin^2(\omega t)$$
$$= V_0 I_0 \tfrac{1}{2}\left(1 - \cos(2\omega t)\right)$$

The graph shows that a resistor gets 2 pulses of power for every AC cycle.
The average power is actually half of the peak power.

DC: $P = VI$

$P = I^2 R$

$$P_{avg} = \tfrac{1}{2} V_0 I_0$$
$$P_{avg} = \tfrac{1}{2} I_0^{\,2} R$$
$$= \left(\frac{I_0}{\sqrt{2}}\right)^2 R$$
$$P_{avg} = I_{rms}^{\,2} R$$

RMS Current and Voltage are average-ish values that make
it easy to compare DC and AC circuits.

$$V_{rms} = \frac{V_0}{\sqrt{2}} \qquad I_{rms} = \frac{I_0}{\sqrt{2}}$$

Ex: Household electricity: V_rms = 120 V f = 60 Hz

$$V_0 = \sqrt{2}(120) = 170\text{ V}$$
$$\omega = 2\pi(60) = 372\ s^{-1}$$
$$V = (170\text{ V}) \sin\left((372\,s^{-1})\, t\right)$$

Inductors in AC

Thursday, October 17, 2019 12:57 PM

$$V_L = L \frac{dI}{dt}$$

$$V_0 \sin(\omega t) = L \frac{d}{dt}\left(-I_0 \cos(\omega t)\right)$$

$$= I_0 \, \omega L \sin(\omega t)$$

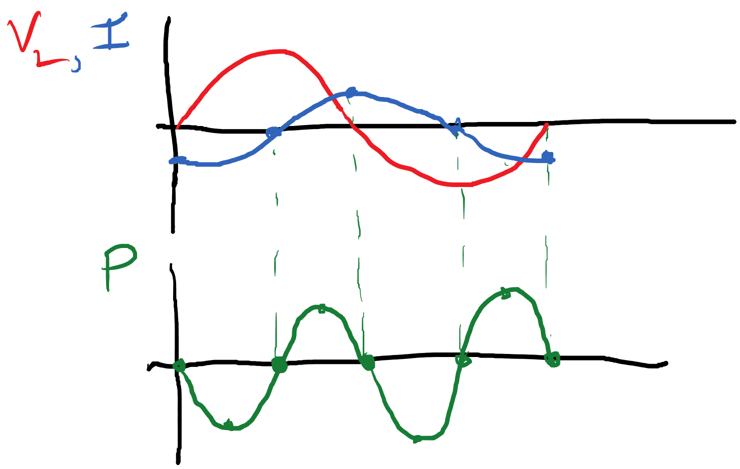
$$V_0 = I_0 \, \omega L$$

$$X_L = 2\pi f L$$

This looks like Ohm's Law, but there are some differences:

- Proportionality depends on frequency.
- The voltage and current are slightly out of phase.

Power of an inductor in AC:



$$P = VI$$

$$= V_0 \sin(\omega t) \, I_0(-\cos \omega t)$$

$$= -V_0 I_0 \sin(\omega t) \cos(\omega t)$$

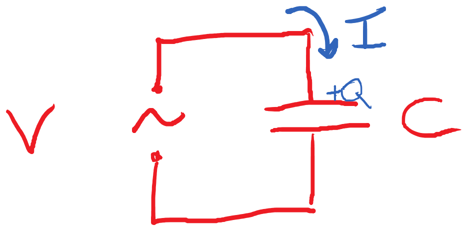
$$P_{L\,avg} = 0$$

On average, the inductor doesn't use power in an AC circuit.

Capacitors in AC Circuits

Thursday, October 17, 2019 1:05 PM

$$Q = CV \rightarrow V_c = \frac{Q}{C}$$



$$I = \frac{dQ}{dt}$$

$$Q = C(V_o \sin(\omega t))$$

$$I = \frac{dQ}{dt} = CV_o \cos(\omega t)\, \omega$$

$$I = I_0 \cos(\omega t)$$

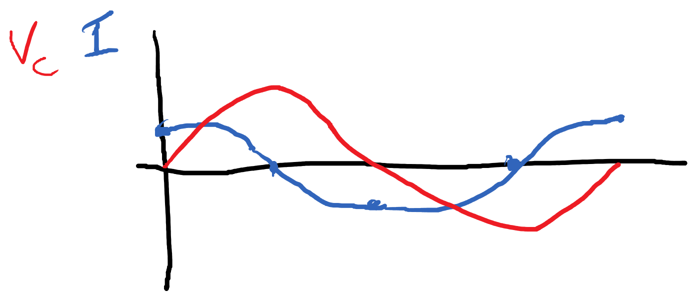
$$I_o = V_o \omega C$$

$$V_o = I_o \left(\frac{1}{\omega C}\right)$$

$$X_c = \frac{1}{2\pi f C}$$

The capacitor obeys AC Ohm's Law, but:

- The reactance gets smaller with higher frequency.
- The voltage and current are out-of-phase by 1/4 cycle.



- In a capacitor, the voltage "lags" behind the current.

AC Ohm's Law works just as well for RMS as for amplitudes.

$$V_{rms} = I_{rms} Z \qquad Z = \text{Impedance}$$

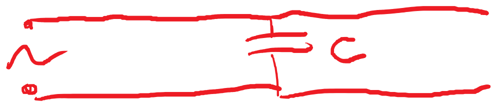
$$Z = R \qquad Z = X_L = 2\pi f L \qquad Z = X_c = \frac{1}{2\pi f C}$$

$$P_{R\,avg} = I_{rms}^2 R \qquad P_{L\,avg} = 0 \qquad P_{c\,avg} = 0$$

What capacitor could be connected to a wall outlet and only allow 1 mA (RMS) to flow?

$V_{rms} = 120\text{ V}$

$f = 60\text{ Hz}$



$$V_{rms} = I_{rms} Z$$

$$120 = 0.001\, Z$$

$$120000\,\Omega = Z$$

$$X_C = \frac{1}{2\pi f C}$$

$$120000 = \frac{1}{2\pi(60)C}$$

$$C = \frac{1}{2\pi(60)(120000)} = 2.21 \times 10^{-8}\text{ F}$$

$$C = 22.1 \times 10^{-9}\text{ F} = 22.1\text{ nF}$$

Why would we connect a capacitor across the power input?

- All power sources have effective internal resistance.
- Current drawn reduces the available voltage.
- The capacitor easily conducts current at high frequencies.
- So our device is exposed to less high-frequency voltage.

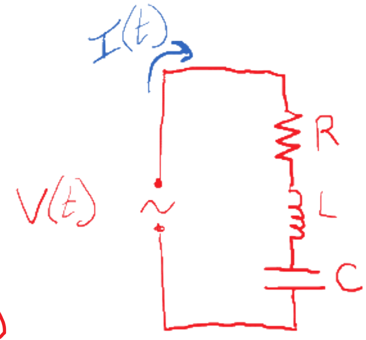
$$X_C = \frac{1}{2\pi f C}$$

Tuesday: Series AC Circuit

- Same current in each component.
- Voltages add, but as functions, not amplitudes.

Exam 2 Thu 10/31

What do we know about series circuits?



- Same current everywhere.

$$I(t) = I_R(t) = I_L(t) = I_C(t)$$

$$I_0 = I_R = I_L = I_C \quad \text{(Amplitudes or RMS)}$$

- Voltages add to get the total

$$V(t) = V_R(t) + V_L(t) + V_C(t)$$

Not true for amplitudes.

Since the components all share the same current, let's start there.

$$I(t) = I_0 \cos(\omega t)$$

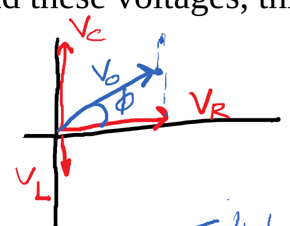
$$V_R(t) = I(t)R = I_0 R \cos(\omega t)$$

$$V_L(t) = L\frac{dI}{dt} = I_0 \omega L\,(-\sin(\omega t)) = V_L\,(-\sin(\omega t))$$

$$V_C(t) = \frac{Q}{C} = I_0 \frac{1}{\omega C}(\sin(\omega t)) = V_C\,(\sin(\omega t))$$

$$V(t) = V_R \cos(\omega t) + (V_C - V_L)\sin(\omega t)$$

To add these voltages, think of them as vectors, called phasors.



$$V(t) = V_0 \cos(\omega t + \phi)$$

Mag and Dir of vector sum.

$$\left.\begin{array}{l}\text{Total } x:\ V_R \\ \text{Total } y:\ V_C - V_L\end{array}\right\} V_0 = \sqrt{V_R^2 + (V_C - V_L)^2}$$

Since each of these voltages is proportional to the current, factor that out.

$$V_R = I_0 R$$

$$\left.\begin{array}{l} V_L = I_0 \omega L = I_0 X_L \\ V_C = I_0 \frac{1}{\omega C} = I_0 X_C \end{array}\right\} V_C - V_L = I_0 \underbrace{(X_C - X_L)}_{X = X_C - X_L} = I_0 X$$

The overall series amplitude is:

$$V_0 = \sqrt{(I_0 R)^2 + (I_0 X)^2} = I_0\sqrt{R^2 + X^2}$$

$$Z = \sqrt{R^2 + X^2}$$

Impedance (Z) is like resistance, but for AC.

- Resistor: $Z = R$
- Inductor or capacitor: $Z = X$
- Series RLC: Z = sqrt(R^2 + X^2)
- Using impedance reminds us that the voltages aren't in phase. Some sines some cosines.
- There is some frequency dependence.

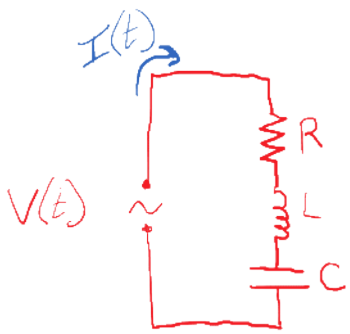
$$V_0 = I_0 Z$$

$$\omega = 2\pi f$$

$$V(t) = V_0 \cos(\omega t + \phi) \qquad I(t) = I_0 \cos(\omega t)$$

Series RLC Example

Tuesday, October 22, 2019 12:56 PM



$V_{rms} = 120\ \text{V}$ $f = 318.3\ \text{Hz}$

$R = 300\ \Omega$ $L = 0.3\ \text{H}$ $C = 2.5\ \mu\text{F}$

Want current, all voltages, and average power.

$$X_L = \omega L = 2\pi(318.3)(0.3) = 600\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(2\pi(318.3)(2.5\times10^{-6}))} = 200\ \Omega$$

$$X = X_C - X_L = -400\ \Omega = 400\ \Omega,\ \text{Inductive}$$

$$Z = \sqrt{R^2 + X^2} = \sqrt{(300^2 + 400^2)} = 500\ \Omega$$

AC Ohm's Law:

$$V_{rms} = I_{rms} Z$$

$$(120\ \text{v}) = I_{rms}(500\ \Omega) \longrightarrow I_{rms} = 0.24\ \text{A}$$

Only the resistor uses power.

$$P_{avg} = I_{rms}^2 R = (0.24)^2(300) = 17.3\ \text{W}$$

Note: $V_{rms} I_{rms} = (120)(0.24) = 28.8\ \cancel{\text{W}}\ \text{VA}$ is more,

Now we can calculate each component voltage:

$V_{rms} = 120\ \text{V}$

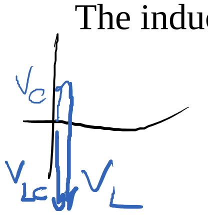
$I_{rms} = 0.24\ \text{A}$

$$V_R = IR = (0.24)(300) = 72\ \text{V}$$

$$V_C = IX_C = (0.24)(200) = 48\ \text{V}$$

$$V_L = IX_L = (0.24)(600) = 144\ \text{V}$$

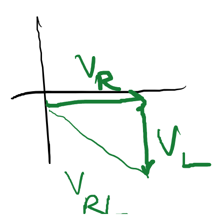
The inductor has more voltage than the power supply!

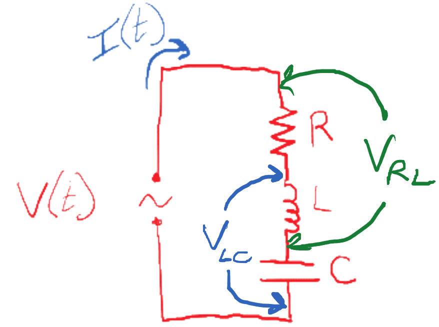


$$V_{LC} = |V_C - V_L| = |48 - 144| = 96\ \text{V}$$

$$V_{RL} = \sqrt{V_R^2 + V_L^2} = \sqrt{72^2 + 144^2}$$

$$= 161\ \text{V}$$





V_{RL}

Frequency trends

Tuesday, October 22, 2019 1:16 PM

At very low frequencies: $X_C = \frac{1}{2\pi f C} = \text{Huge} = Z$

The capacitor blocks current at low frequencies.

At very high frequencies: $X_L = 2\pi f C = \text{Huge} = Z$

The inductor blocks current at high frequencies.

At some special middle frequency, called the resonant frequency:

$X_L = X_C \qquad X = 0 \qquad Z = \sqrt{R^2 + 0^2} = R$

$$\omega_R L = \frac{1}{\omega_R C}$$

$$\omega_R = \frac{1}{\sqrt{LC}} = 2\pi f_R \qquad f_R = \frac{1}{2\pi\sqrt{LC}}$$

At resonance:

- The series impedance is as low as possible.
- The current is as high as possible.
- The power delivered is as high as possible.

$R = 300\ \Omega \qquad L = 0.3\ H \qquad C = 2.5\times10^{-6}\ F$

$V_{rms} = 120\ V$

$$P_{avg} = I_{rms}^2 R = \left(\frac{120}{300}\right)^2 (300) = 48\ W$$

$$f_R = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.3)(2.5\times10^{-6})}} = 183.8\ Hz$$

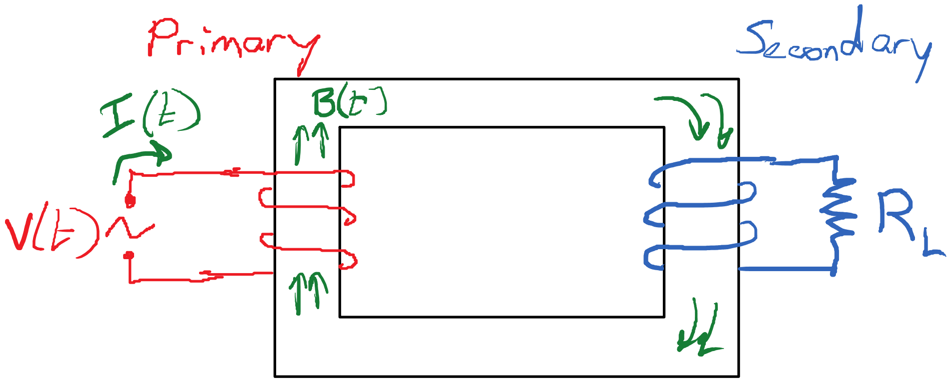
$$X_L = 2\pi(183.8)(0.3) = 346\ \Omega$$

$$X_C = \frac{1}{2\pi(183.8)(2.5\times10^{-6})} = 346\ \Omega$$

18. Transformers

Thursday, October 24, 2019 12:29 PM

A transformer is two coils that share the same magnetic flux (per loop).



The power supply drives current, which causes magnetism in the iron core.
The fluctuating magnetism causes EMF in each coil.

- In the primary, the coil EMF serves to limit the current.
- In the secondary, the coil EMF acts as a power supply for the load.
- Since both coils share the same magnetic flux (per loop): $\Phi_1 = BA$

$$\frac{V_s}{V_p} = \frac{d\Phi_s/dt}{d\Phi_p/dt} = \frac{N_s}{N_p}\frac{dBA/dt}{dBA/dt} \qquad \Phi_s = N_s BA$$

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

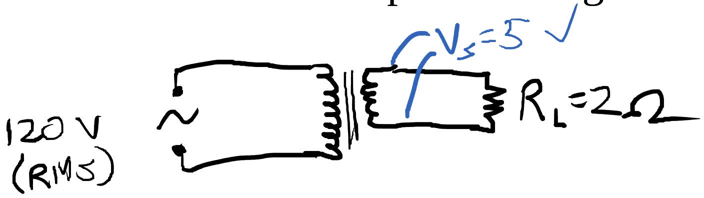
Ideally, a transformer would not lose any energy.

$$P_{out} = P_{in}$$

$$V_s I_s = V_p I_p \qquad \text{(Ideal)}$$

The current ratio is the inverse of the voltage ratio (in an ideal transformer).

Ex: Transformer for a phone charger



$$\frac{V_s}{V_p} = \frac{5\,V}{120\,V} = \frac{1}{24}$$

If there are 100 turns in the secondary, there must be 2400 turns in the primary.

The load is connected to the secondary voltage:

$$V_s = I_s R_L$$
$$5V = I_s (2\Omega)$$
$$2.5A = I_s$$

Assuming an ideal transformer:

$$V_s I_s = V_p I_p$$
$$(5V)(2.5A) = (120V) I_p$$
$$0.1042\ A = I_p$$

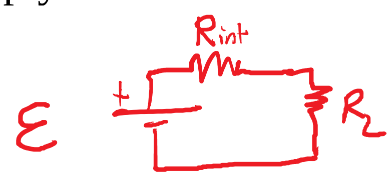
1. The power supply "sees" a different effective resistance.

$$V_p = I_p R_{eq}$$
$$120 = 0.1042\ R_{eq}$$
$$1152\ \Omega = R_{eq}$$

Ratio of R's: $\dfrac{R_L}{R_{eq}} = \dfrac{2}{1152} = \dfrac{1}{576} = \left(\dfrac{1}{24}\right)^2 = \left(\dfrac{N_s}{N_p}\right)^2$

2. Using a step-down transformer like above allows a low-current power supply to feed a high-current device.

If the power supply has internal resistance, this increases the efficiency.



$$I = \frac{\mathcal{E}}{R_{int} + R_L}$$

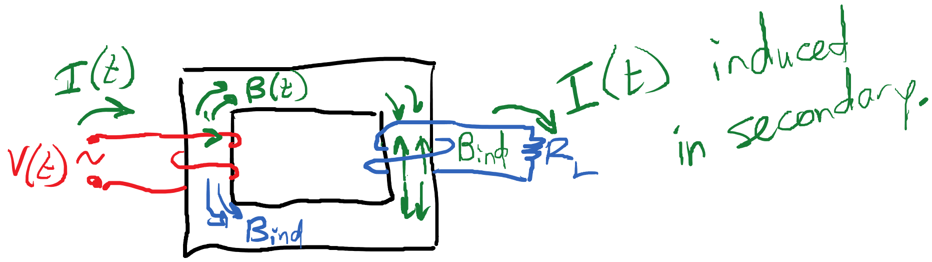
$$P_{waste} = V_{R_{int}} I = I^2 R_{int}$$

In power distribution systems, we use a variety of voltages for different stages:

- Long-distance: 500 kV transmission lines
- Medium-distance: 135 kV distribution lines
- Short-distance: 35 kV local distribution lines
- Household: 240 V and 120 V household electricity.

Lenz's Law

Thursday, October 24, 2019 1:01 PM



Lenz's Law: EMF tries to oppose the change in flux causing the EMF.

If B(t) is increasing, EMF tries to make B (induced Bind) point the other way.

- How? By pushing current in the proper direction.
- Above, this would make a voltage opposing the I(t). This induced voltage merely manages to limit the amount of current. This is the source of X_L.

In the secondary:

- B(t) points down and is increasing.
- The EMF tries to generate magnetism pointing up.
- If the secondary current actually flows (because R_L allows it), it ends up cancelling out some of the original magnetism. So more primary current is needed to generate the primary voltage to match the power supply. This is the source of

$$V_s I_s = V_p I_p$$

Inefficiencies in transformers:

- Lost magnetic flux reduces the secondary voltage, as compared to the ideal equation.
- Current in the coils must pass through metals, which have resistance.

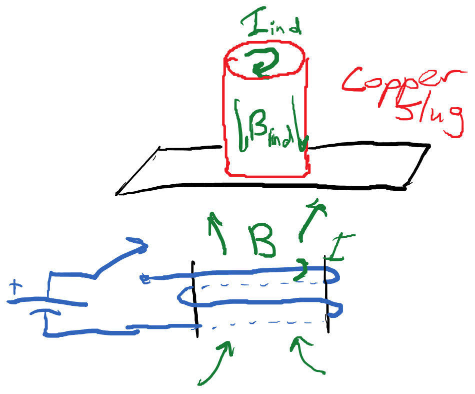
$$\text{efficiency} = \epsilon = \frac{P_{out}}{P_{in}} \qquad \epsilon = \text{"epsilon"}$$

$$\epsilon P_{in} = P_{out}$$

$$\epsilon V_p I_p = V_s I_s$$

19. Review

Tuesday, October 29, 2019 12:29 PM

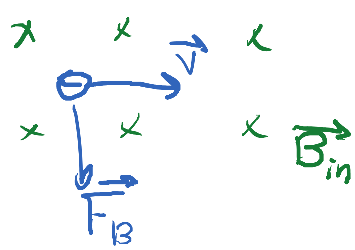


A copper slug sits on a plastic shelf, over top of a solenoid coil. When the current is suddenly turned on in the solenoid, what happens?

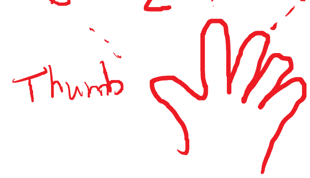
- Current Direction: R across Front.
- Magnetic Field: Up (top of page)
- Magnetic Flux: Increasing magnitude.
- Lenz's Law: Fight change by forming new B_{ind} opposite old B.

The original coil has its North upward. The copper slug has an induced magnet with North downward. The two magnets will repel, and the copper slug could jump up off the shelf.

Other RHR for magnetic force:



$\vec{F}_B = q\vec{v} \otimes \vec{B}$



- Index = $\vec{v}$ = right
- Middle = $\vec{B}$ = in
- Thumb = Top of page
- charge is ⊖, so F_B = Bottom of page

Quick checks: F_B must be perpendicular to both velocity and B.
The velocity and B can't be parallel or anti-parallel.

AC Circuits

Tuesday, October 29, 2019 12:55 PM

$L = 0.592\ H$ $\quad V(t) = (120\ V)\sin\left(\frac{31\pi}{s}\ t\right)$

Generic $\quad v(t) = V_0 \sin(\omega t)$

Amplitude $\quad V_0 = 120\ V$

$\omega = 2\pi f = 31\pi$

$f = \frac{31\pi}{2\pi} = 15.5$

Inductive Reactance

freq

$X_L = 2\pi f L = (31\pi)(0.592) = 57.7\ \Omega$

$V_0 = I_0 Z \qquad (120\ V) = I_0(57.7\ \Omega)$

$V_{rms} = \frac{V_0}{\sqrt{2}} \qquad I_{rms} = \frac{I_0}{\sqrt{2}}$

Note: L is a constant for a given inductor.

What capacitor would have the same reactance at this frequency?

$$X_L = X_C$$

$$2\pi f L = \frac{1}{2\pi f C}$$

$$(31\pi)(0.592) = \frac{1}{(31\pi)C}$$

$$C = \frac{1}{(31\pi)^2(0.592)} = 1.78 \times 10^{-4}\ F$$

$$= 178 \times 10^{-6}\ F$$

$$= 178\ \mu F$$

To get a lower frequency, what do we do to the capacitance?

more C

$$f = \frac{1}{2\pi\sqrt{LC}}$$

To cover a range of frequencies, a range of capacitances is needed. The lowest capacitance goes with the highest frequency. And vice versa.

Generator voltage

Tuesday, October 29, 2019 1:12 PM

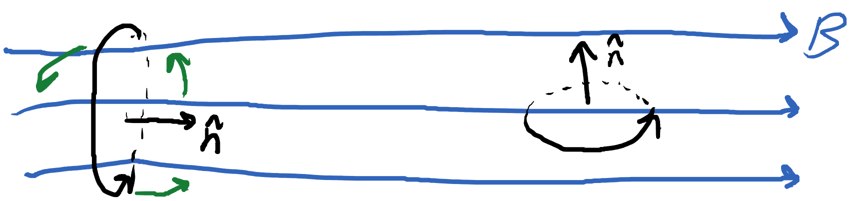
Rotating Coil: $\mathcal{E}_{max} = \underbrace{NBA}_{\text{max } \Phi_B} \omega$

rotational speed

$$\omega = 1670 \frac{rev}{min}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 174.9 \text{ rad/s}$$

Excel: $= A2 * 2 * \underbrace{PI()}_{\pi}/60$

$\Phi_B = NBA\cos\theta$



$-\frac{d\Phi_B}{dt} = NBA\sin\theta\ \omega$

max when $\theta = 90°$

$\theta = 0$

max Φ_B

$\frac{d\Phi_B}{dt} = 0$

minimum $|\mathcal{E}|$

$\theta = 90°$

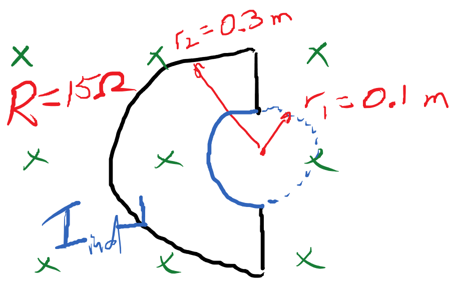
$\Phi_B = 0$

$\left|\frac{d\Phi_B}{dt}\right| = max$

$|\mathcal{E}| = max$

Lenz's Law with motional EMF

Tuesday, October 29, 2019 1:23 PM



$B_{in} = 0.07$ T

$\mathcal{E}_{max} = NBA\omega$

$= (0.5)(0.07)(\pi(0.1)^2)(377)$

$= 0.415$ V

$I_{max} = (0.415\,V)/(15\,\Omega) = 0.028$ A

$\omega = 3600 \frac{rev}{min} \frac{2\pi\, rad}{rev} \frac{1\, min}{60\, s}$

$= 377\ s^{-1}$

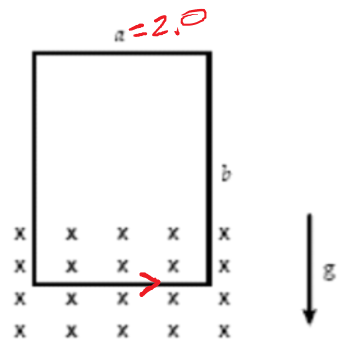
A small rotatable half-loop of wire is attached to a large fixed loop of wire. The small loop rotates at 3600 rpm. What is the peak magnitude and direction of the current as the loop rotates from the solid to the dashed location?

- Φ_B is increasing, B is "in."
- Lenz B_{ind} = "out", I_{ind} = CCW around whole loop

The half-loop is moving rightward. B is "in". So the force on + charges is toward the top of the page. The current will go this way then follow the solid wire CCW around the big loop.

14. A rectangular loop with mass 0.6 kg is 2.0 m wide and 3.0 m high. It is dropped so that the bottom leg of the loop is in a magnetic field $B_{in} = 6.0$ T, while the top leg is out of the magnetic field. If the resistance of the loop is 40 Ω, at what speed does it fall with zero acceleration?

v



$F_g = F_B$

$mg = I\ell B$

$(0.6)(9.8) = I(2.0)(6.0)$

$0.49\text{ A} = I$

$V = IR$

$vB\ell = IR$

$v(6.0)(2.0) = (0.49)(40\,\Omega)$

$v = 1.63$ m/s