Average: 59%
It's the Course Average, updated later today, that is your best estimate of your status. (60% Exams, 15% HW, 25% Lab)

Equivalent unit to the farad (F)?

\[ C = \frac{k \varepsilon_0 A}{\ell} \]
\[ Q = CV \]
\[ Q = RC \]
\[ \left[ C \right] = \left[ \text{F} \right] \left[ \text{V} \right] \]
\[ \left[ Q \right] = \left[ \text{C} \right] \]
\[ \left[ \frac{Q}{V} \right] = \left[ \text{C/V} \right] \]
\[ \left[ \frac{Q}{V} \right] = \left[ \text{F} \right] \]

Mass is the only quantity where the fundamental unit has a prefix.

Kilogram (kg)

\[ 25 \text{ mg} = 25 \times 10^{-3} \text{ g} \]
\[ = 25 \times 10^{-6} \text{ kg} \]

Cost of electricity

\[ \frac{1.25}{\$0.12/\text{kWh}} = 10.4 \text{ kWh} \]
\[ \frac{10400 \text{ Wh}}{50 \text{ W}} = 208 \text{ hours} \]

Two-battery Kirchhoff's Law circuit

\[ I_1 = 1 \text{ A} \]
\[ I_1 + I_2 = I_3 \]
\[ I_3 = 3 \text{ A} \]
\[ \varepsilon_2 = V_3 + V_2 \]
\[ 12 = V_3 + 4 \]
\[ 8 = V_3 \]

Combination resistor circuit
Capacitors in electrostatics and RC Circuits.

\[ C = 300 \mu F \quad V_0 = 15.0 \text{ V} \quad Q_0 = CV_0 = 4500 \mu \text{C} \]

\[ E_x = -\frac{dV}{dx} \]

\[ V = V_0 \quad V = 0 \]

\[ E = \frac{V}{d} = \frac{15 \text{ V}}{0.1 \times 10^{-3} \text{ m}} = 150000 \text{ V/m} \]

\[ V = V_0 e^{-t/\tau} \]

\[ 2.5 = 15 e^{-20/2} \]

\[ e^{-20/2} = 0.5 \]

\[ \frac{V_2}{V_1} = \frac{V_0 e^{-t_2/\tau}}{V_0 e^{-t_1/\tau}} = e^{-t_2/\tau} \]

\[ \frac{V_2}{V_1} = e^{-20/2} \]

If the voltage is cut in half in one 20 s interval, it must be cut in half in any 20 s interval.

\[ e^{-20/2} = 0.5 \]

\[ \frac{-20}{\ln(0.5)} = t = 29 \]
Magnetic fields are a lot like electric fields. Magnetic fields (\( \mathbf{B} \)) always form loops.

- No "sources" where B points away, or "sinks" where B points toward.

Magnetic fields are created by:
- Electric Currents
- Magnetic materials (dipoles)
- Fluctuating Electric Fields

Effects of magnetic fields:
- Forces on currents and moving charges
- Torques and forces on dipoles
- Generate electric fields and voltages

Magnetic geometries are always 3-D.

By itself, "Up" is ambiguous.

Ex: Magnetic field of a straight wire:
Side View:

Only points in plane are shown above.

3D Version:

(B)
Generally, $\vec{B}$ is caused by currents.

Biot-Savart Law

$\vec{B} = \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2}$

$\hat{r} = \frac{\vec{r}}{r}$

$\times = \text{Cross Product}$

$\mu_0 = 4\pi \times 10^{-7} \text{T m/A}$

Vector Cross Product:
- $\vec{C}$ is perpendicular to $\vec{A}$ and $\vec{B}$
- $\vec{C}$ has a magnitude of $AB \sin(\theta)$, where $\theta$ is the angle between $\vec{A}$ and $\vec{B}$.
- Pick direction of $\vec{C}$ via RHR

Ex: Infinite, straight current

$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{r} \times \hat{r}}{r^2}$

$\hat{r} = -x \hat{i} + y \hat{j}$

$r^2 = x^2 + y^2$

$r = \sqrt{x^2 + y^2}$

$\hat{r} = \frac{-x \hat{i} + y \hat{j}}{\sqrt{x^2 + y^2}}$

$\hat{\vec{C}} \times \hat{\vec{A}} = \hat{\vec{k}}$
Generally, we quote this formula as:

\[ \mathbf{B} = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{y \, dx}{(x^2 + y^2)^{3/2}} \hat{k} = \frac{\mu_0 I}{2\pi y} \hat{k} \]

Example: Magnetic field due to two currents:

\[ \mathbf{B}_{\text{wire}} = \frac{\mu_0 I}{2\pi r} \]

Ex: \( I_1 = I_2 = 1.5 \) A, with 4 cm between them.

\[ B_1 = \frac{\mu_0 (1.5)}{2\pi (0.02)} + \frac{\mu_0 (1.5)}{2\pi (0.02)} = 3 \times 10^{-5} \text{T} \]

Hint: type \( \mu_0 \) as \( (\pi \times 4e-7) \)

Magnetic Field of a circle of current, at the origin.

\[ \oint \frac{\mu_0 I \, d\theta}{2\pi r^2} = R d\theta \hat{\theta} \]

\[ \mathbf{F}_{\text{outward}} = -R \mathbf{r} \]

\[ \oint \mathbf{dA} = R^2 d\theta \hat{r} = R d\theta \hat{r} \]
\[ \vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{R \, d\Theta \, \hat{r}}{R^2} = \frac{\mu_0 I}{2R} \hat{r} \]

Usually quoted as:

\[ B_{\text{loop}} = \frac{\mu_0 I}{2R} \]

\[ B_{\text{wire}} = \frac{\mu_0 I}{2\pi R} \]
This is akin to Gauss's Law for electrostatics. For any imaginary looping path:

\[ \int \mathbf{B} \cdot d\mathbf{A} = \mu_0 I_{\text{enc}} = \int (B \, dl \, \cos \Theta) = \int B \, dl = B \int dl \]

When applied, we make the integral easy.
- Make sure \( B = \) constant along the path.
- Keep theta (of the dot product) constant along the path.

Ex: Magnetic field inside and outside a thick wire:

\[ B_1 \frac{2\pi r}{2\pi} = \mu_0 I \]
\[ B_1 = \frac{\mu_0 I}{2\pi \, r} \]

\[ B_2 \frac{2\pi r}{2\pi} = \mu_0 I \left( \frac{r}{R} \right)^2 \]
\[ B_2 = \frac{\mu_0 I r}{2\pi R^2} \]

When \( r = R \), \( B_1 = B_2 \)
Solenoid: \( \int \mathbf{B} \cdot d\mathbf{l} = B_0 L = \mu_0 I_{enc} \)

\[ I_{enc} = \frac{N}{\ell}(x)I \]

Inside: \( B = B_0 \)

Outside: \( B = 0 \)

\( B_0 \times = \mu_0 I \frac{N}{\ell} \)

going to zero
Magnetic fields cause magnetic forces on moving charges.

\[ \vec{F}_B = q \vec{v} \times \vec{B} \]

The cross product does a few things beyond \( F_B = qvB \):
- The velocity must have some component perpendicular to \( B \).
- The direction of the force is by the RHR for cross products.
  - \( F_B \) is perpendicular to \( v \)
  - \( F_B \) is perpendicular to \( B \)

Note: If the charge is negative, reverse the result after doing the RHR.

Since the magnetic force is always perpendicular to velocity:
- \( F_B \) cannot change the velocity magnitude, only its direction.
- \( F_B \) cannot transfer energy.

\[ W_B = \int \vec{F}_B \cdot d\vec{l} = \int \vec{F}_B \cdot \vec{v} \, dt \]

\[ = 0 \quad \text{because} \quad \vec{F}_B \perp \vec{v} \]

When the magnetic force is the only significant force acting on a particle:
- The perpendicular magnetic force is also the centripetal force.
The radius of the path depends on the mass of the particle. This allows building a mass spectrometer. There are multiple methods of feeding the particles into the mass spec.

Method 1: Linear accelerator (like Lab 6)

$$R = \frac{mV}{qB}$$

With this method, the speed also depends on mass.

$$R = \frac{mV}{qB} = \frac{m}{\sqrt{2q\Delta V/m}} = \frac{1}{B} \sqrt{\frac{2m\Delta V}{q}}$$

Now it's clear that $R$ is proportional to the square root of mass. The only problem is this "compresses" large masses into similar $R$.

It would be nice to give all particles the same velocity.

Method 2: Velocity Selector

- Start with particles of various velocities.
- Pick out the ones with the velocity you want.

When $v=E/B$, the particles feel zero net force.
- Too fast: particles bend left.
• Too slow: particles bend right.

When a velocity selector is fed into a mass spectrometer, we get the simple:

\[ R = \frac{mv}{qB} \]

\[ R \propto m \]
Direct application: Force exerted by wires on each other.

\[
\vec{F} = q \vec{v} \times \vec{B}
\]

\[
\vec{F} = I \vec{l} \times \vec{B}
\]

Length and direction of wire.

Unlike with electrostatics, here "opposites repel".

\[
F_B = \frac{\mu_0 I_1 I_2 l}{2\pi d}
\]

This served as the original definition of the ampere.
\[ \tau = 2 F_B d = 2 (IAB \frac{w}{2} \sin \Theta) \]

\[ \tau = NBAI \sin \Theta \]

\[ \tau_{\text{max}} = NBAI \]

\[ \Phi_B = \text{magnetic flux} \]
Faraday's Law describes induced EMF, but doesn't explain it. There are two separate causes that are both summarized by the