Return Exam2

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Average: 68%

#15-16, capacitor with changing scenario

$$C = \frac{K\varepsilon_0 A}{d} \qquad X_C = \frac{1}{2\pi f C} \qquad V_0 = I_0 X_C$$

Doubles

half const ? Doubles

half

freq halved, V_0 constant } C = const, I_0 = ?

#9 Solenoid

$$B = \frac{\mu_0 N I}{\ell}$$

$$I = \frac{B\ell}{\mu_0 N} = 249\ \text{A}$$

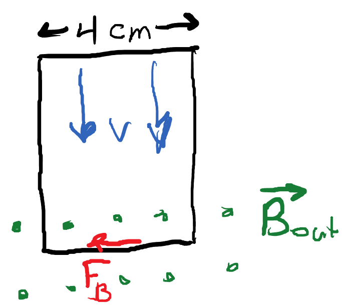
$B = 0.25$ T

$N = 1200$

$\ell = 1.5$ m

$\mu_0 = 4\pi \times 10^{-7}$

#10-11, Loop falling in field.



$\vec{F}_B = q\vec{v} \otimes \vec{B}$ on ⊕ charges drives current, CW

$F_B = I\ell B$ = Force resisting gravity

perpendicular

$$F_g = F_B$$

$$mg = I \ell B$$

0.001 kg 0.04 m 0.25 T

$I = 0.98$ A

Oscillations Intro

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Textbook: Chap 15, especially 15.1-15.4.

Simplest oscillator: Harmonic oscillator made of a mass and spring.

Example from Phys I:

k = spring constant

$F_s = -kx_s$

$U_s = \frac{1}{2}kx_s^2$

x_f = max compression

$$F_{net} = ma$$
$$F_s = m a_m$$
$$-kx_s = m\frac{d^2x_s}{dt}$$

$$(Energy)_i = (Energy)_f$$
$$K_i + U_{si} = K_f + U_{sf}$$
$$\frac{1}{2}mv_i^2 + 0 = 0 + \frac{1}{2}kx_f^2$$
$$\frac{m}{k}v_i^2 = x_f^2$$
$$\sqrt{\frac{m}{k}}\,v_i = x_f$$

Need x_s as a function of time.

- $t=0 \Rightarrow x_s = 0$
- $t=0 \Rightarrow \frac{dx_s}{dt} = v_i$

} Boundary Conditions

- 2^{nd} Derivative $\propto$ -value $\Rightarrow$ sines & cosines

$$x_s = x_0 \sin(\omega t)$$
$$v_s = x_0 \cos(\omega t)\,\omega = v_0\cos(\omega t)$$
$$v_0 = \omega x_0$$
$$a_s = x_0\omega(-\sin(\omega t)\,\omega) = a_0(-\sin(\omega t))$$
$$a_0 = \omega^2 x_0 = \omega v_0$$

$$-kx_s = m\frac{d^2x_s}{dt^2}$$
$$-kx_0\sin(\omega t) = m(-\omega^2 x_0 \sin(\omega t))$$
$$-k = -m\omega^2$$
$$\frac{k}{m} = \omega^2 \qquad \omega = \sqrt{\frac{k}{m}}$$

$$\omega = 2\pi f$$
$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Other boundary condition:

$t = 0 \Rightarrow v_s = v_i \qquad v_s = v_0 \cos(\omega t)$

$v_s = v_0$

$v_0 = \omega x_0 = v_i$

$x_0 = \frac{v_i}{\omega} = v_i \sqrt{\frac{m}{k}}$

Force analysis and energy analysis produce the same result.

If the specifics of position and velocity at various times are important, the actual oscillation function must be found.

If only the amplitude and max velocity are important, energy analysis works.

Alternative forms:

$$x = A \sin(\omega t) + B \cos(\omega t)$$

$$x = x_0 \sin(\omega t + \Phi)$$

Phase shift

$$x_0^2 = A^2 + B^2 \qquad \tan\Phi = B/A$$

Example:

$m = 0.25\ \text{kg} \qquad k = 50\ \text{N/m}$

$v_i = 15\ \text{cm/s} \qquad x_i = 10\ \text{cm}$

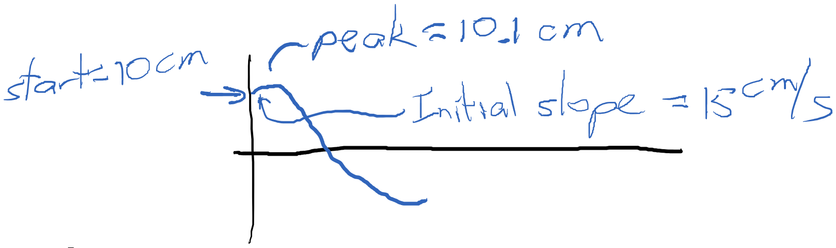
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{50}{0.25}} = 2.25\ \text{Hz}$$

$\omega = \sqrt{\frac{50}{0.25}} = 14.14$

cos part: $x_i = B$, $B = 0.10\ \text{m}$

sine part: $v_i = \omega A$, $0.15\ \text{m/s} = (14.14)A$, $0.0106 = A$

$$x_0 = \sqrt{A^2 + B^2} = \sqrt{0.0106^2 + 0.1^2} = 0.101\ \text{m} = 10.1\ \text{cm}$$



Initial position -> cosine component.
Initial velocity -> sine component.

Energy analysis: Energy is constant

Initial: $\frac{1}{2}(0.25\,kg)(0.15\,m/s)^2 + \frac{1}{2}(50\,N/m)(0.1\,m)^2$

Max potential: $\frac{1}{2}(50\,N/m)(0.101\,m)^2$

Amplitude

22. Waves

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A wave is an organized disturbance in a set of coupled oscillators.

- Coupled oscillators: Each exerts a force on its neighbors.
- Disturbance: Can be a pulse, but often an oscillation.

Note: A wave is not stuff. The oscillators generally stay in place.

General moving waveform: $f(x - vt)$

Oscillating waveform: $\sin\left(\frac{2\pi}{\lambda}x - 2\pi f t\right)$

Wave velocity: $\frac{2\pi x}{\lambda} - 2\pi f t = \frac{2\pi}{\lambda}(x - \lambda f t)$

$v = f\lambda$

Repeat in space: $\Delta\left(\frac{2\pi x}{\lambda}\right) = 2\pi$ $\Delta x = \lambda = \text{wavelength}$

Repeat in time: $\Delta(2\pi f t) = 2\pi$ $\Delta t = \frac{1}{f} = T = \text{period}$

$v = f\lambda$ $v = \frac{\lambda}{T}$

The wave speed depends on the type of wave and the environmental conditions.

Sound	Light	String	Water
$v \approx 340\ m/s$	$v = 3\times10^8\ m/s$	$v = \sqrt{F_T/\mu}$	$v = 1-10\ m/s$

warmer = faster

All waves involve oscillations of at least 2 variables.
At least one of them is a vector. (no scalar waves)

Two general types:

- Longitudinal: oscillations along the wave propagation direction
- Transverse: oscillations perpendicular to propagation direction.

Visuals: Dan Russell waves

$v = f\lambda$

Range of audible sounds: 20 Hz — 20 kHz

$f_1 = 20 \text{ Hz}$ $\lambda_1 = \frac{340 \text{ m/s}}{20 \text{ Hz}} = 17 \text{ m}$

Typical wind instrument ~1/4 lambda.

$f_2 = 20 \text{ kHz}$ $\lambda_2 = \frac{340 \text{ m/s}}{20 \text{ kHz}} = 17 \text{ mm} = 0.017 \text{ m}$

Doppler Effect

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When oscillations are propagated, generally the frequency is maintained.

Exception: Doppler effect due to moving source or observer.

- Wave gets compressed in front of a source, and stretched behind it.
- An observer moving toward a wave receives peaks more quickly than if they were stationary.

Together, these give a frequency shift between the source and observed freqs.

Classical: $$f_o = f_s\left(\frac{V + V_o}{V - V_s}\right)$$

V = wave speed
V_o = observer speed toward source
V_s = source speed toward observer

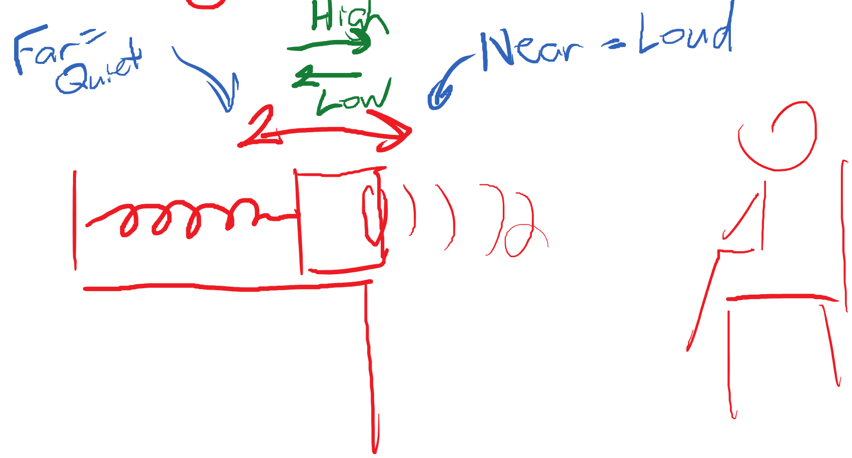
Relativistic: $$f_o = f_s\sqrt{\frac{V + V_{rel}}{V - V_{rel}}}$$

V_{rel} = relative speed toward each other

Low-Speed Approx: $$\frac{\Delta f}{f} = \frac{V_{rel}}{V_{wave}}$$

Δf = freq shift

Good for $V_{rel} < 15\%\ V_{wave}$



$$\text{extreme } x = x_o \pm \text{Amplitude}$$

$$\text{extreme } v = v_o = \omega(\text{Amplitude})$$

$\omega = 2\pi f$ of motion of speaker

extreme $V = V_0 = \omega(\text{Amplitude})$

ω = ω of motion of speaker

23. Standing Waves

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Standing waves are apparently-stationary oscillations formed when traveling oscillating waves are reflected back and forth on top of each other.

Oscillating traveling waves: $f(x,t) = A\sin\left(\frac{2\pi}{\lambda}x - 2\pi f t\right)$

Wavelength $= \lambda =$ repeat distance

Frequency $= f = 1/T$ $T =$ period $=$ repeat time

$v = f\lambda$

Wave superposition: When two wave functions overlap, the functions add.

Interference: When two sinewaves add, and they have the same frequency, their amplitudes can add or subtract depending on the relative phase.

- When peaks line up, that is said to be "in phase", which produces "constructive interference".
- When the peaks of one wave line up with the troughs of the other, the waves are "out of phase", and produces "destructive interference".

$$\left.\begin{aligned} f_1 &= \sin\left(\frac{2\pi}{\lambda}x - 2\pi f t + \phi_1\right) \\ f_2 &= \sin\left(\frac{2\pi}{\lambda}x - 2\pi f t + \phi_2\right) \end{aligned}\right\} \Delta\phi = \phi_2 - \phi_1$$

$\Delta\phi = 0, 2\pi, 4\pi, \ldots$ "in Phase"

$\Delta\phi = \pi, 3\pi, 5\pi, \ldots$ "out of phase"

When two waves traveling in opposite directions are superimposed, standing waves are formed.

$$y(x,t) = y_m \sin(\mathrm{kx} - \omega \mathrm{t}) + y_m \sin(\mathrm{kx} + \omega \mathrm{t}) = 2y_m \sin(\mathrm{kx}) \cos(\omega \mathrm{t})$$

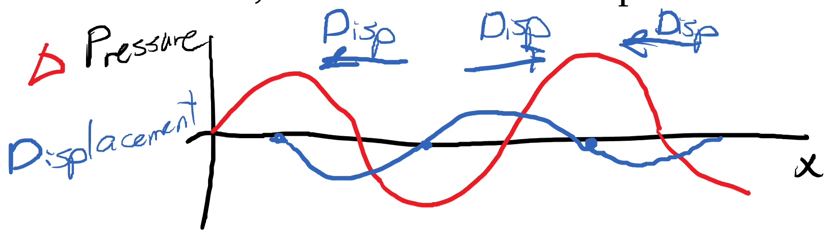
Instead of seeing two moving waves, we see one stationary oscillation, with its own nodes (zero points) and anti-nodes (peaks).

Reflection: A wave's energy can bounce off a surface. During reflection, a wave generates a reverse-direction copy of itself.

- String with fixed end: The end is forced to have zero displacement. The incident wave has non-zero displacement. In

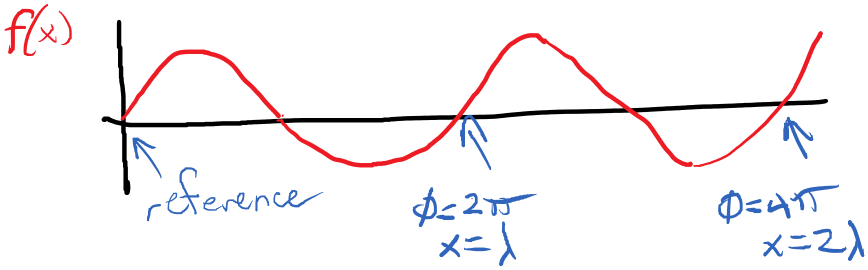
order to maintain the zero requirement at the end, an upside-down copy of the wave is generated.
- String with loose end: The end can have any displacement, so when the reflection is generated, it is right-side up.

Every oscillating wave has variables that are conjugates of each other. One variable or the other is forced to a node at a reflection.
- In sound waves, the variables are displacement and pressure.



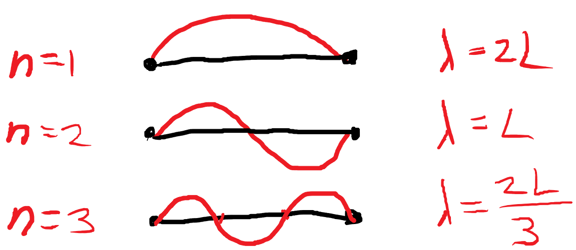
To get resonant standing waves, we surround a region with reflectors. This is called a "cavity". To get resonance, a wave must interfere constructively with itself.

Similar ends: After one round-trip (2*L*), a wave pulse gets back to where it started. If the sinewave looks identical at that point, it interferes constructively.



$2L = n\lambda$

$n = 1, 2, 3, \ldots$



$\lambda = \frac{2L}{n} = \frac{\lambda_1}{n}$

$v = f\lambda$

$f = \frac{v}{\lambda} = \frac{v}{\lambda_1} n = f_1 n$

The oscillating frequency of each standing wave is a multiple of the fundamental.

Ex: Let's say we want a string (clamped at both ends) to have a fundamental frequency of 220 Hz, and the string has a mass per unit length of 4.0 g/m and a length of 0.9 m. What is the required tension?

$\lambda_1 = 2L = 1.8\,m$

$v = f\lambda = (220\ Hz)(1.8\,m) = 396\ m/s$

$v = \sqrt{\frac{F_T}{\mu}} \Rightarrow F_T = v^2\mu = (396\ m/s)^2(0.004\ kg/m)$
$= 627\ kg\,m/s^2 = 627\ N$

What happens to the wave if we increase the tension by 1%?

$F_T = 633.27\ N$

$v = \sqrt{\frac{F_T}{\mu}} = 397.89\ m/s$

$\Delta v =$ 397.89 - 396 = 1.89

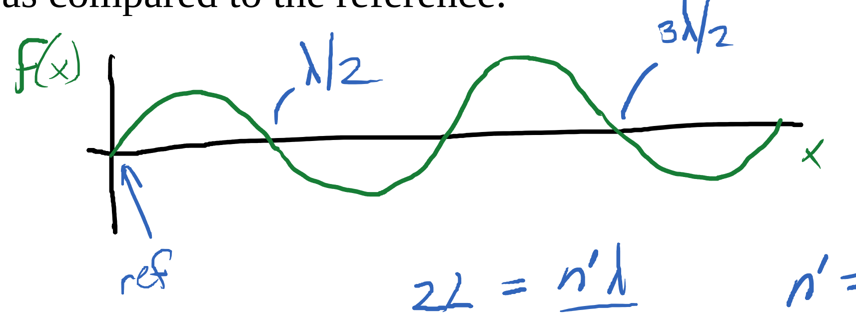
$\frac{\Delta v}{v} =$ 1.89 / 396 = 0.0048 $= 0.5\%$

$f = \frac{v}{\lambda} =$ 397.89 / 1.8 = 221.05

$\Delta f = 1\ Hz$

$\frac{\Delta f}{f} = 0.5\%$

If the two ends of the cavity produce *different* kinds of reflections, then the wave is upside-down after traveling down-and-back.
To find constructive interference, we look for parts of the wave that are inverted as compared to the reference.



$2L = \frac{n'\lambda}{2}$ $\quad n' = 1, 3, 5, \ldots$

$4L = n'\lambda$

Strings with loose ends are rare. More common are sound cavities with one end open and one end closed.

Graphs:
https://www.acs.psu.edu/drussell/Demos/StandingWaves/StandingWaves.html

Ex: If you want 220 Hz fundamental from a closed-open tube, how long should it be?

$$\lambda = \frac{v}{f} = \frac{340\ m/s}{220\ Hz} = 1.546\ m = 4L$$

$$0.386\ m = L$$

What is the spacing between harmonic frequencies?

$$\Delta f = f_7 - f_5 = 7f_1 - 5f_1 = 2f_1 = 440\ Hz$$

Diff Ends

$\lambda_1 = 4L$

$f_1 = \frac{v}{4L}$ $\quad f = n' f_1$

$\Delta f = 2f_1 = \frac{v}{2L}$

odd only

Same Ends

$\lambda_1 = 2L$

$f_1 = \frac{v}{2L}$ $\quad f = n f_1$

$\Delta f = f_1 = \frac{v}{2L}$

every integer

How can we use a variable that takes on every integer value to generate a pattern of only odd integers?

$n = 0, 1, 2, 3, \ldots$

$2n = 0, 2, 4, 6, \ldots$

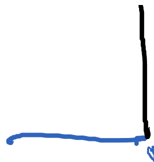
$(2n+1) = 1, 3, 5, \ldots$

Application: RF Antennas often use 1/4 wavelength.

free end $I = 0$

$f = 4\ MHz$

$L = 17.83\ m$



$L = 17.83$ m

feed point I = antinode

$\lambda = 4L = 71.32$ m (on the wire)

$v = f\lambda = 285.6 \times 10^6$ m/s $\quad c = 300 \times 10^6$ m/s

Electrical waves on the wire travel at almost c. $\quad \frac{c}{v} \sim 1.05$

Intensity

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$P = \frac{\Delta \text{Energy}}{\Delta t}$

Spread over Time

Energy (J)

Accumulate thru Time

$\Delta \text{Energy} = \int P\,dt = P_{avg} \Delta t$

Power $(W = J/s)$

$I = \frac{P}{A}$

Spread over Area

Expose Large Surface

$P = \iint I \, dA \cos\theta = I_{avg} A_{\perp}$

Cross-sectional Area

Intensity (Flux Density) (W/m^2)

Two basic shapes of waves:

- Plane waves: Generated from far-away source.
  Propagation direction is uniform (like z-direction).
  Wave fronts are planes perpendicular to the propagation.
  Intensity is uniform.
- Spherical waves: Generated by a point source.
  Propagation is radially outward from source.
  Wave fronts are spherical shells surrounding the source.
  Intensity decreases with distance.

$$I = \frac{P}{A} = \frac{P}{4\pi r^2} \qquad I \propto \frac{1}{r^2}$$

area of wave front.

Ex: A light source has an intensity of 100 W/m^2 at a distance of 10 m, how bright is it at 20 m?

$$\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{10}{20}\right)^2 = \frac{1}{4}$$

$$I_2 = \frac{1}{4} I_1 = 25\ W/m^2$$

Decibels

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Decibels are the unit of intensity "level". Level measures energy-like quantities on a logarithmic scale.

- All decibel levels measure ratios between two energy-like quantities.
- Log scales compress large values and stretch out small values.
- Zero decibels (0 dB) refers to "the same level", not zero intensity.
- Decibel values are always added and subtracted, which corresponds to multiplying or dividing the intensity by some factor.

$$\frac{I}{I_0} = 10^{\beta/10}$$

$\beta =$ level in dB

$$\beta = 10 \log_{10}(I/I_0)$$

β	I/I_0	
0 dB	$10^0 = 1$	
1 dB	$10^{0.1} = 1.26$	26% increase
→ 3 dB	$10^{0.3} = 2$	
5 dB	$10^{0.5} = \sqrt{10} \approx 3$	
7 dB	$5 = 10/2$	
→ 10 dB	$10^{1.0} = 10$	
20 dB	100	
-17 dB	1/50	

A siren has an intensity level of 110 dB at a distance of 4 m. What is the intensity level 100 m away?

$$\frac{I_2}{I_1} = 1/\left(\frac{r_2}{r_1}\right)^2 = \left(\frac{1}{25}\right)^2 = \frac{1}{625}$$

Guess: 625 = 2 * 3.1 * 10 * 10, so the level change is

I_1

Guess: 625 = 2 * 3.1 * 10 * 10, so the level change is
(3 dB + 5 dB + 10 dB + 10 dB) = 28 dB.

$$\log(625) = 2.796$$
$$\Delta\beta = -27.96 \text{ dB}$$
$$\beta_2 = \beta_1 + \Delta\beta = (110 \text{ dB}) - (28 \text{ dB}) = 82 \text{ dB}$$

What is a 110 dB sound anyway? $I_0 = 10^{-12}$ W/m^2 $\Rightarrow$ 0 dB

$I_1 = 10^{-1}$ W/m^2 $\leftarrow$ $10^{11.0}$ more $\leftarrow$ 110 dB

Exception: EEEN's often talk about voltages in terms of decibels.
Decibel values always refer to energy ratios.

$$P = V^2/R$$

$$\frac{V}{V_0} = 10 \qquad \frac{P}{P_0} = 100 \Rightarrow 20 \text{ dB increase}$$

A tenfold increase in voltage is actually a 20 dB increase.

Back to regular power measurements:

Signal strength in dBm: $P_0 = 1$ mW

$\beta = -38$ dBm

$$P = P_0\, 10^{\beta/10} = (1 \text{ mW})\, 10^{-3.8}$$
$$= 0.000158 \text{ mW}$$
$$= 158 \text{ nW}$$

Polarization

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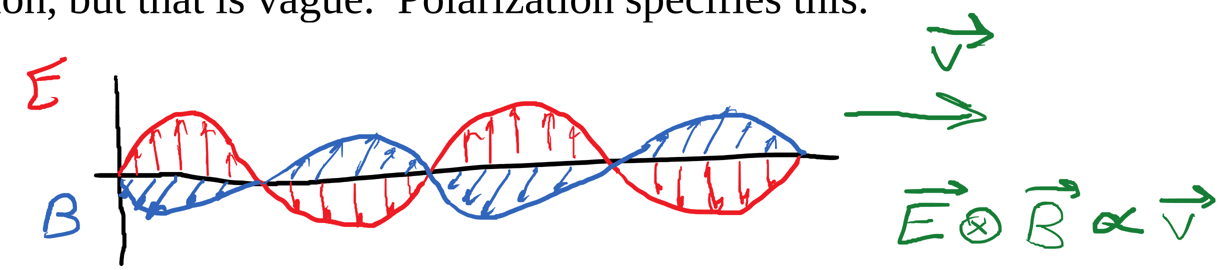
Transverse waves involve an oscillation perpendicular to the propagation, but that is vague. Polarization specifies this.



By convention, we use E to describe the polarization of light.

Most light is "unpolarized", meaning E changes direction randomly.
There are some light sources that produce particular polarizations:

- Glare off of a water surface tends to be horizontally polarized.
- A polarizer passes light of a particular polarization and filters out light of the perpendicular polarization. Ex: vertically polarized sunglasses.

Polarizer calculations: How much intensity gets through?

Unpolarized Light

$$I = \frac{1}{2} I_0$$

Polarized Light

$$I = I_0 \cos^2 \theta$$

Twist Angle

Unpolarized light with I0 = 100 W/m² hits a stack of 3 polarizers. The first is oriented at 0°, the second at 30°, and the third at 90°, each measured from vertical. What gets through?

Unpolarized

$I_0 = 100\ W/m^2$ | $0°$ | $I_1 = 50\ W/m^2$ | $30°$ | $I_2 = 37.5\ W/m^2$ | $90°$ | $I_3 = ? = 9.4\ W/m^2$

$$I_2 = I_1 \cos^2 \theta_{12} = 50\ W/m^2 \cos^2(30°) = 37.5\ W/m^2$$

$$I_3 = I_2 \cos^2 \theta_{23} = (37.5) \cos^2(60°) = 9.4\ W/m^2$$

What would happen if the 2nd polarizer was removed? (no light gets thru)

Ray Optics

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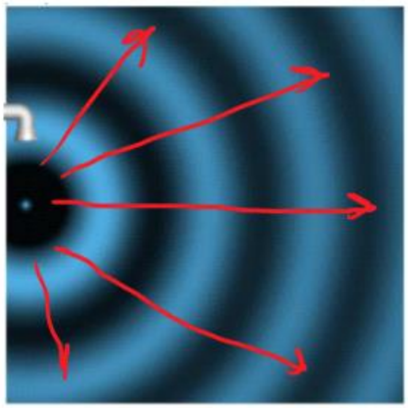
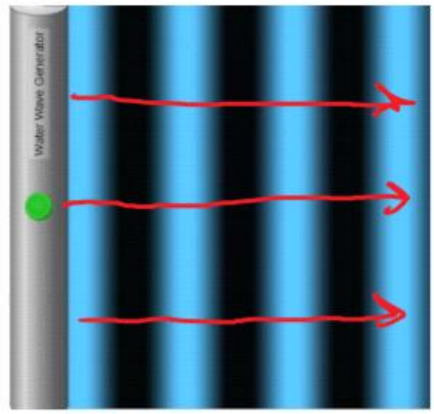


Spherical Waves
Diverging Rays



Plane Waves
Parallel Rays

Rays trace the path of energy as it flows from the source.
We can only focus on the above patterns of rays.

How can these patterns be formed?

- Small light emitters make diverging rays.
- Far light sources make parallel rays.
- Detailed objects with broad illumination make diverging rays.
- Plain objects with detailed illumination make diverging rays.
- Rays passing through a lens or mirror can be redirected.

Speed of light: $v = c = 3\times10^8\ m/s$

In a material: $v = c/n$ $n =$ index of refraction

Water: $n = 1.33$ $v = \frac{3\times10^8}{4/3} = 2.25\times10^8\ m/s$

Optics consists of controlling waves by manipulating the rays.

- Refraction: Bending of light because of a change in index of refraction.
- Reflection: Organized deflection of light from a surface.
- Absorption: Energy taken from the wave by a material.
- Scattering: Disorganized deflection of light from small particles.
- Diffraction: Light passing a barrier spreads into the "shadow" region.

Refraction and Reflection of a Ray

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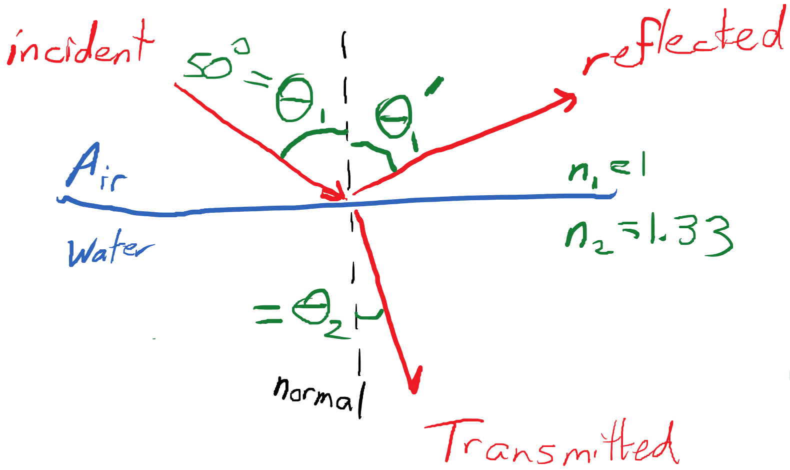


$$\theta_1' = \theta_1$$

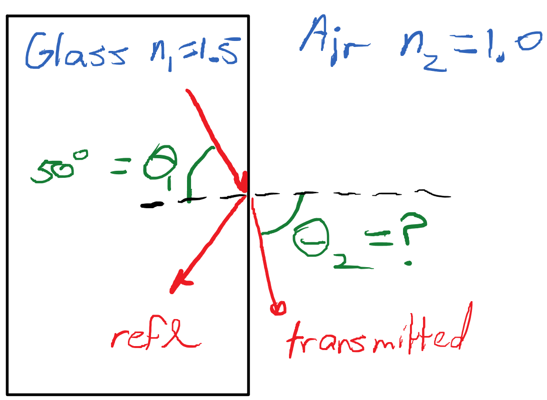
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$(1)\sin(50^\circ) = (1.33)\sin\theta_2$$

$$0.5760 = \sin\theta_2$$

$$35.2^\circ = \theta_2$$

What if the light "speeds up" at the surface?



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$(1.5)\sin(50^\circ) = (1)\sin\theta_2$$

$$1.149 = \sin\theta_2$$

no valid θ_2

no transmitted ray

Total Internal Reflection

Critical Angle: Set $\theta_2 = 90^\circ$

$$n_1 \sin\theta_c = n_2$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

Ex: glass/air

$$\sin\theta_c = \frac{1}{1.5} = 0.667$$

$$\theta_c = 41.8^\circ$$

Summary:

- When n increases, theta decreases. Light bends toward the normal when it slows.

Brewster's Angle

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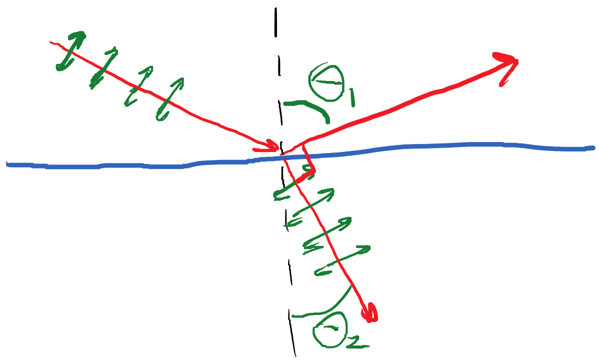
Polarization by reflection



If the transmitted and reflected rays are perpendicular, then the oscillations of the transmitted ray might not be able to form the reflected ray.

Ex: Light reflecting off a water surface can have the vertically polarized light not reflected. So the reflected light is mostly horizontally polarized. That's why we wear vertically polarized sunglasses.

$\Theta_1 + \Theta_2 = 90^\circ$ $\quad\quad \sin\Theta_2 = \cos\Theta_1$

$$n_1 \sin\Theta_1 = n_2 \sin\Theta_2$$
$$n_1 \sin\Theta_1 = n_2 \cos\Theta_1$$

$$\tan\Theta_1 = \frac{n_2}{n_1}$$ Brewster's Angle

Ex: $n_1 = 1$ $\quad n_2 = 1.33$ $\quad \Theta_B = \tan^{-1}\left(\frac{1.33}{1}\right) = 53^\circ$

Lenses

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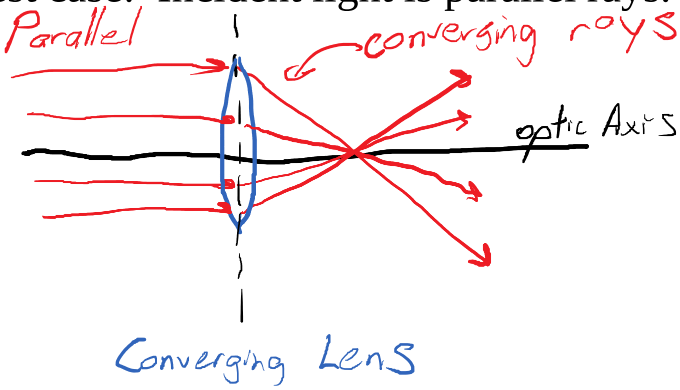
Lenses take advantage of refraction to reorganize a field of rays so they appear different to an observer.

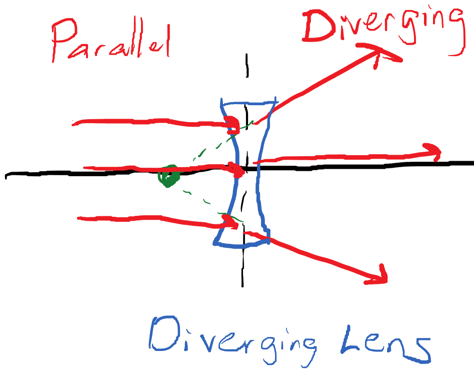
Easiest case: Incident light is parallel rays.



The point where the rays meet has two names:

- Focal point - where incoming parallel rays meet.
- Real image - where any rays leaving the lens actually meet.

Real images are typically viewed by placing a screen at the location of the real image.
Could also place a detector array at that point.



The diverging rays *look like* they are coming from a common point. That point is the image. Since the rays don't actually touch that point, it's a **virtual image**.

Virtual images can be viewed by looking into the lens.

Calculations:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

p = object location
q = image location
f = focal length

First diagram: $p = \infty$
(Converging Lens) $q = +5$ cm

$$\frac{1}{\infty} + \frac{1}{5\text{ cm}} = \frac{1}{f} \qquad f = +5\text{ cm}$$

Diverging lens: $p = \infty$

Diverging lens: $p = \infty$, $q = -5\text{ cm}$ $\quad \frac{1}{\infty} + \frac{1}{-5\text{ cm}} = \frac{1}{f}$ $\quad f = -5\text{ cm}$

What if the object is placed 15 cm from the f=+5 cm lens?

$p = 15\text{ cm}$

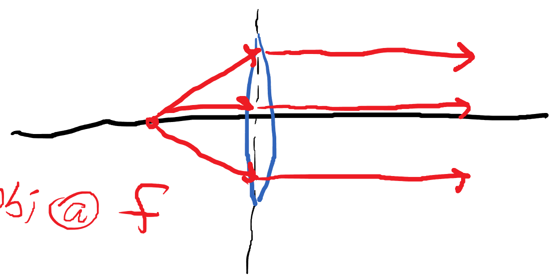
$f = 5\text{ cm}$

$$\frac{1}{15} + \frac{1}{q} = \frac{1}{5}$$

$$q = \left(\frac{1}{5} - \frac{1}{15}\right)^{-1} = \frac{15}{2} = 7.5\text{ cm}$$

Bringing the object closer made the real image further away.

Is there a place where the real image would go to infinity?



obj @ f

$p = ?$

$q = \infty$

$f = 5\text{ cm}$

$$\frac{1}{p} + \frac{1}{\infty} = \frac{1}{5}$$

$$p = 5\text{ cm}$$

What if we bring the object even closer?

$p = 2\text{ cm}$

$f = 5\text{ cm}$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{2} + \frac{1}{q} = \frac{1}{5}$$

$$\frac{1}{q} = -0.3 \qquad q = \frac{-10}{3}$$

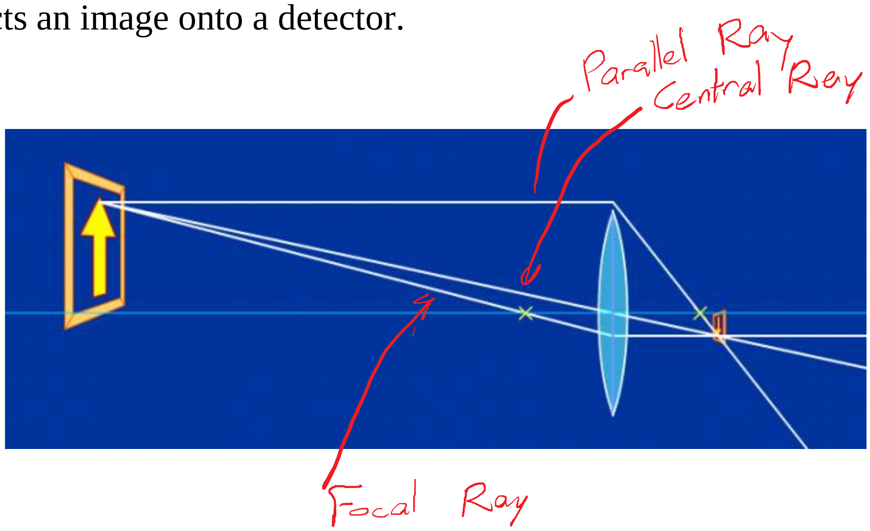
Vision

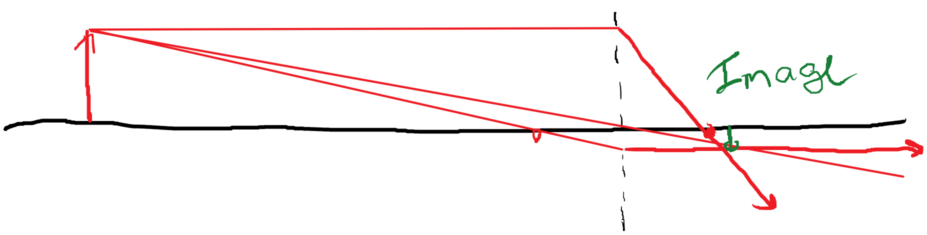
Thursday, November 21, 2019 12:24 PM

Obviously we can see objects. How does that work?
How and why can we see images?

- Virtual images
- Real images

Our eye is basically a biological camera. There is a converging lens which projects an image onto a detector.





For our eye, the detector is about 3 cm from the lens.

Nearest Objects: (d_{np})

$p = 25$ cm
$q = 3$ cm

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$f = \left(\frac{1}{25} + \frac{1}{3}\right)^{-1} = 2.68 \text{ cm}$$

Furthest Objects (d_{fp})

$p = \infty$
$q = 3$ cm

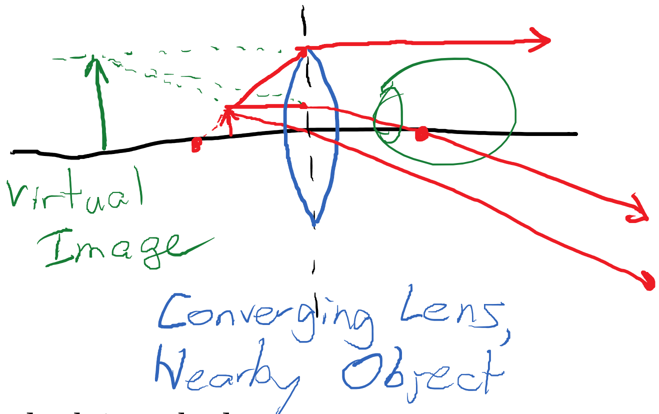
$$f = \left(\frac{1}{\infty} + \frac{1}{3}\right)^{-1} = 3 \text{ cm}$$

Typically, we use corrective lenses to adjust the range of focal lengths so we can see everything.

Seeing Virtual Images

Thursday, November 21, 2019 12:46 PM

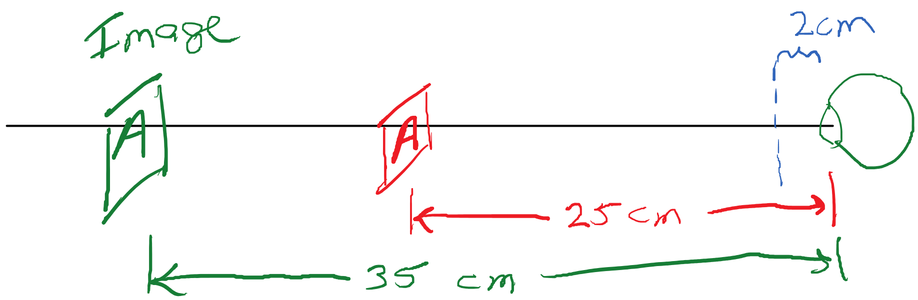
Diverging lenses generate virtual images.
Converging lenses can generate virtual images.



To view a virtual image, we look into the lens.
The rays that get to us look like they are coming from a common point - the virtual image.

When light goes through two lenses, the image from one lens becomes the object for the next.

Ex: I have a near point of 35 cm. If I want to see my phone at a distance of 25 cm, what kind of reading glasses do I need to wear 2 cm in front of my eyes?



Lens: $\left.\begin{array}{l} p = 23 \text{ cm} \\ q = -33 \text{ cm} \end{array}\right\} f = \left(\frac{1}{23} + \frac{1}{-33}\right)^{-1} = 75.9 \text{ cm}$

Lens power is the inverse of the focal length (in meters).

$$\text{Power} = \frac{1}{f} = \frac{1}{0.759} = 1.32 \text{ diopters}$$

A diverging lens does basically the same thing, but:

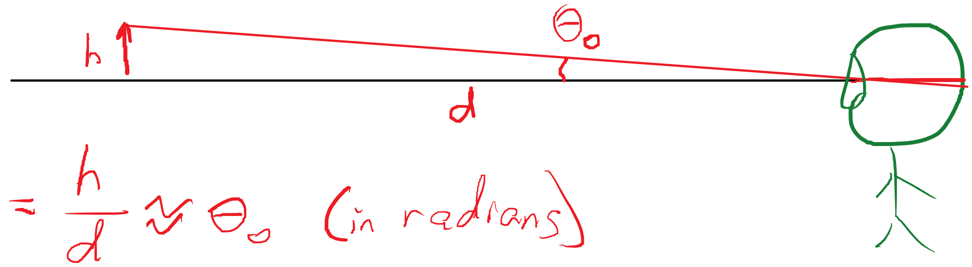
- The focal length is negative.
- The virtual image is closer (and smaller) than the image.

Magnifying Glass

Thursday, November 21, 2019 1:02 PM

The practical measure of a magnifying glass is what it does to the **angular size** of the image.
This is what affects the size of the image on the back of our eye.

h = object size (Small detail)



$$\tan\theta_0 = \frac{h}{d} \approx \theta_0 \quad \text{(in radians)}$$

When we can approach an object (or bring it near our face), the maximum possible subtended angle is:

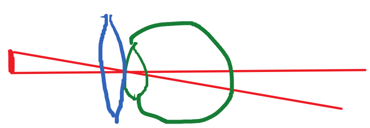
$$\theta_0 = \frac{h}{25\text{ cm}}$$

With a magnifying glass, we need to make a virtual image.
The most comfortable virtual image to look at is far away.
To view it most easily, we put our eye right up against the lens.

$$p \le f \qquad q = -\infty$$

$$\frac{1}{p} + \cancel{\frac{1}{q}}^{\,0} = \frac{1}{f}$$

$$p = f$$



$$\theta = \frac{h}{d} = \frac{h}{f}$$

The angular magnification is the ratio of these angles.

$$m_\theta = \frac{\theta}{\theta_0} = \frac{h/f}{h/25\text{ cm}} = \frac{25\text{ cm}}{f}$$

As it turns out, we can move the object a little closer and squeeze out another factor of magnification.

$$\max m_\theta = \frac{25\text{ cm}}{f} + 1$$

What is the lens power (in diopters) of a magnifier advertised as being a 6 times magnifier?

$$6 = \frac{25\text{ cm}}{f} + 1$$

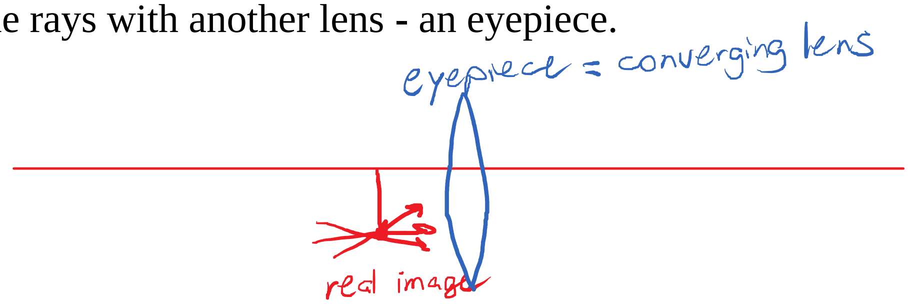
$$\text{Power} = \frac{1}{0.05} = 20 \text{ diopters} \longleftarrow f = 5\text{ cm} \longleftarrow 5 = \frac{25\text{ cm}}{f}$$

Viewing Real Images

Thursday, November 21, 2019 1:17 PM

A real image is formed by converging rays, but we can only see diverging rays. How can we view a real image?

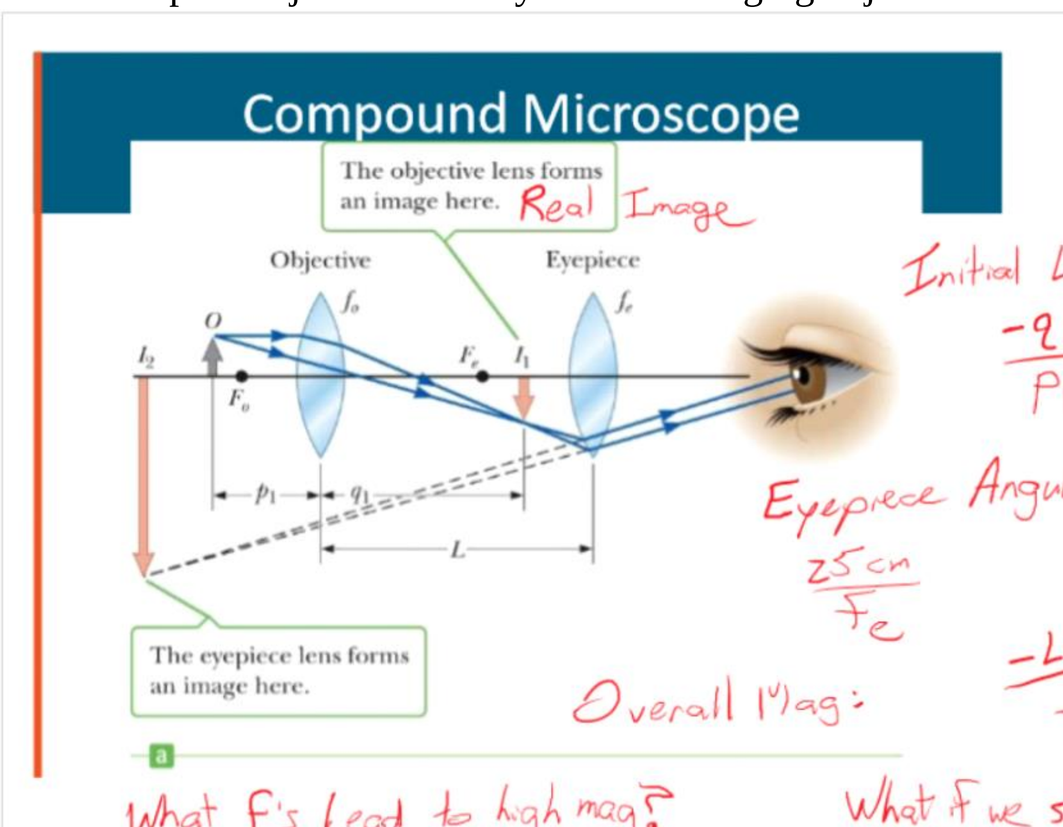
- Could move further from the lens. Awkward.
- Place a screen or detector at the image location.
- Intercept the rays with another lens - an eyepiece.



The eyepiece generates a virtual image that we can view through the eyepiece.

Most common devices:

- Microscope - Object is placed near (but outside f) a converging lens. This is the objective lens.
- Telescope - Object is far away from converging objective lens.



Initial Linear Mag.

$$\frac{-q_1}{p_1} \approx \frac{-L}{f_o}$$

Eyepiece Angular Mag

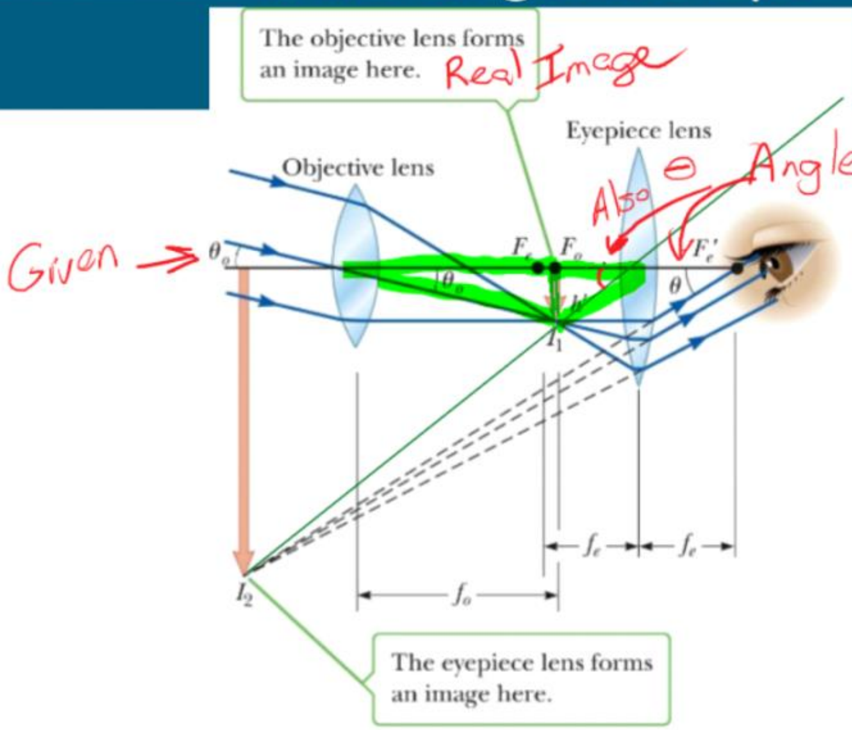
$$\frac{25\text{ cm}}{f_e}$$

Overall Mag: $\frac{-L(25\text{ cm})}{f_o f_e}$

What f's lead to high mag?

What if we swap the lenses

Refracting Telescope



$$\theta = \frac{h'}{f_e} \qquad \theta_o = \frac{-h'}{f_o}$$

$$\frac{\theta}{\theta_o} = \frac{-h'/f_e}{h'/f_o} = \frac{-f_o}{f_e}$$

Interference

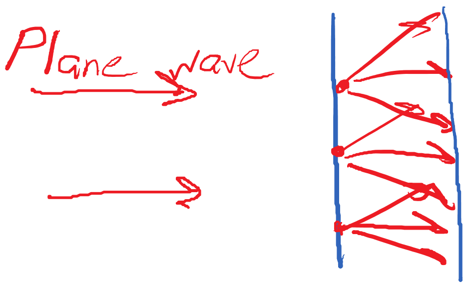
Tuesday, November 26, 2019 12:15 PM

Waves are global patterns that emerge from local equations.

- Each point is only affected by its neighbors.
- A disturbance in one point radiates outward in all directions.
- Every oscillating point radiates in all directions.
- That's too much, so we can reduce it down to some outward bound waves.

Huygens's Principle

- Only worry about the points along a wave front.
- These points radiate in all "forward" directions.



Diffraction: If we block most of the wave, the remaining wave front will radiate outward from the hole in the blockage. This allows waves to propagate around corners and spread out when they go through a hole.

$$f = A \sin\left(2\pi f t - \frac{2\pi x}{\lambda} + \phi\right)$$

x = distance from source

A = amplitude that may vary slowly

Interference is the result of combining two waves.
The total amplitude depends on the relative phase difference.

$$\Delta\phi = \left(2\pi f t - \frac{2\pi x_1}{\lambda} + \phi_1\right) - \left(2\pi f t - \frac{2\pi x_2}{\lambda} + \phi_2\right)$$

$$\Delta\phi = \frac{2\pi \Delta x}{\lambda} + \Delta\phi_i$$

Overall Phase Diff

Path Length Diff

Explicit Phase Diff

Overall Phase Diff

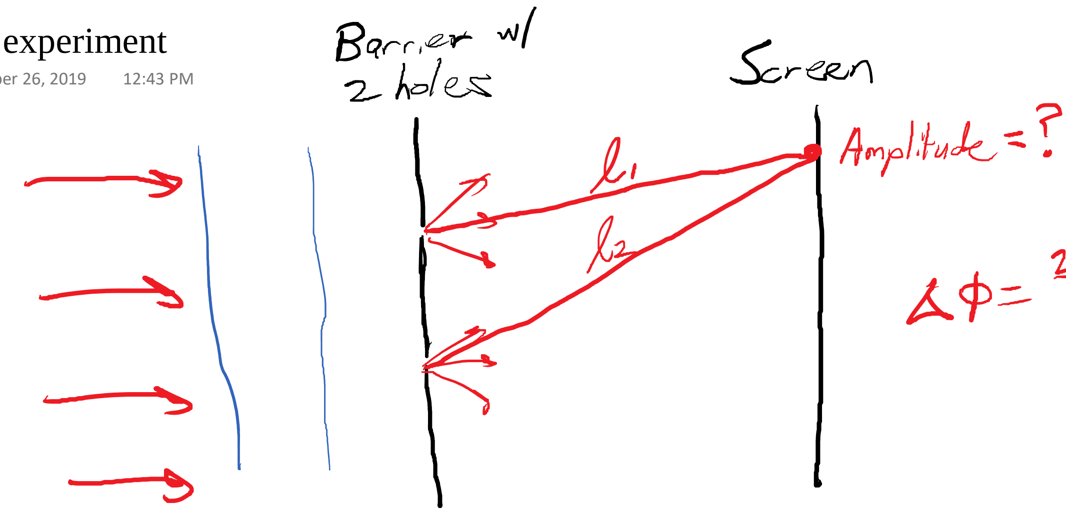
Path Length Diff

Phase Diff

$\Delta\phi = 0, 2\pi, 4\pi, \ldots$ Constructive

Two-slit experiment

Tuesday, November 26, 2019 12:43 PM



$$\Delta\phi = \frac{2\pi\Delta\ell}{\lambda}$$

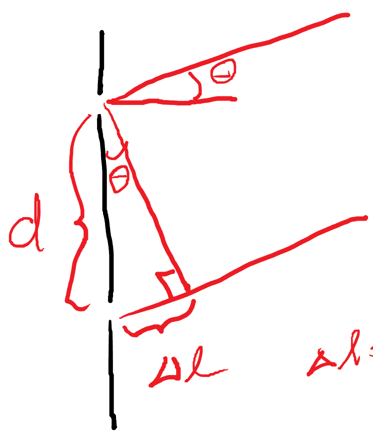
Constructive: $\Delta\phi = m2\pi = \frac{2\pi\Delta\ell}{\lambda}$

$$m\lambda = \Delta\ell$$

If the path length difference ($\Delta\ell$) is an integer number of wavelengths, we get constructive interference.

If the screen is "very far" away, we can approximate the two rays as being parallel to each other.

Zoomed in

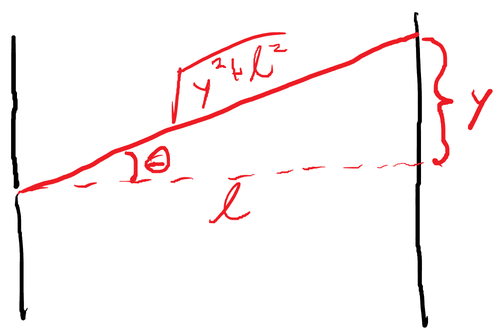


$\Delta\ell = d\sin\theta$

$\Delta\ell = m\lambda$

$m\lambda = d\sin\theta$

Zoomed Out



$\tan\theta = \frac{y}{\ell}$

$\sin\theta = \frac{y}{\sqrt{y^2+\ell^2}}$

If the offset *y* causes *m* to be an integer, there is a bright spot.

Ex: A diffraction grating with a density of 250 lines/mm has green light with a wavelength of 530 nm shined through it. What is the first angle of diffraction?

$d = 250$ lines/mm

diffraction?

$\rho = 250$ lines/mm

$d = \frac{1\text{ mm}}{250} = 0.004\text{ mm} = 4\ \mu\text{m} = 4000\text{ nm}$

$(1)(530\text{ nm}) = (4000\text{ nm})\sin\theta_1$

$\theta_1 = 7.6^\circ$

We can list out the angle of each spot:

m	θ_m
0	0
±1	±7.6°
⋮	
±7	±68°
±8	n/a

m=8?

$(8)(530\text{ nm}) = (4000\text{ nm})\sin\theta_8$

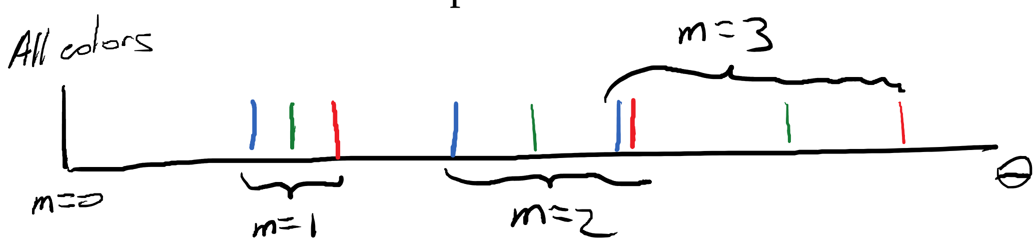
$\frac{4240}{4000} = \sin\theta_8$

In this case, there are only 15 spots.

Diffraction gratings are used to separate light into its constituent wavelengths. Only the first or second "orders" are actually well separated.

	m=1	m=2	m=3
$\lambda_{red} = 650$ nm	$m\lambda = 650$	1300	1950
$\lambda_{blue} = 410$ nm	$m\lambda = 410$	820	1230

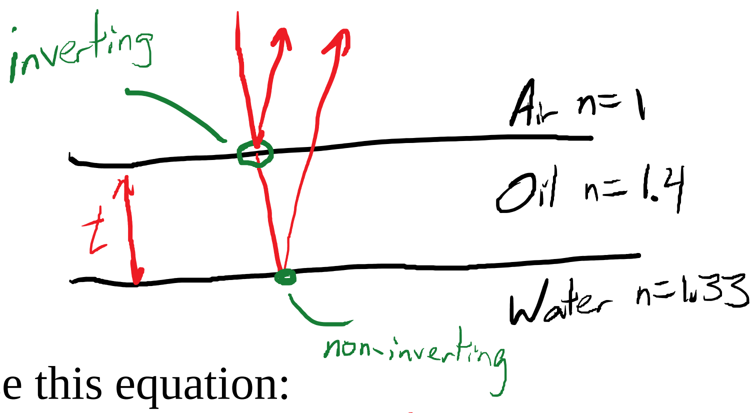
The m=3 angle for blue light is actually less than the m=2 angle for red light. The 2nd and 3rd rainbows will overlap.



Thin Film Interference

Tuesday, November 26, 2019 1:10 PM

Examples: Sheen of oil on water. Smear of grease on glass.



Phase Diff between rays

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta\ell + \Delta\phi_E$$

To use this equation:

- The path length diff ($\Delta\ell$) is twice the thickness.
- The wavelength must be measured in the film.

$\Delta\ell = 2t$

$\lambda = \lambda_0/n$

$v = f\lambda$

- The explicit phase diff ($\Delta\phi_E$) depends on the types of reflection.
  - If the index of refraction increases, the reflection is inverted.
  - If the index of refraction decreases, the reflection is non-inverted.

$$\Delta\phi_E = \begin{cases} 0 & \text{same type of reflection} \\ \pi & \text{different types of reflection} \end{cases}$$

How does this all go together?

$$m2\pi = \Delta\phi = \frac{2\pi}{\lambda}\Delta\ell \overset{?}{+} \pi$$

$$\left(m2\pi = \frac{2\pi}{\lambda}\Delta\ell \overset{?}{+} \pi\right)\frac{\lambda}{2\pi}$$

$$m\lambda = \Delta\ell \overset{?}{+} \frac{\lambda}{2}$$

$$\left(m \overset{?}{-} \frac{1}{2}\right)\lambda = \Delta\ell$$

yes, $-\frac{1}{2}$

For a film of oil on water, what is the minimum oil thickness that will reflect blue light

$\lambda = 410\text{ nm}$

For a film of oil on water, what is the minimum oil thickness that will reflect blue light constructively?

$\lambda_o = 410\ nm$

$\lambda = \frac{410\ nm}{1.4} = 293\ nm \qquad n_{film} = 1.4$

$\left(1 - \frac{1}{2}\right) 293\ nm = \Delta l$

$146\ nm = \Delta l = 2t$

$73\ nm = t$

What is the minimum thickness to see red light?

$\lambda_o = 650\ nm$

$\lambda = 464\ nm$

$\left(1 - \frac{1}{2}\right) 464 = \Delta l$

$232\ nm = \Delta l = 2t$

$116\ nm = t$

Color Theory

Tuesday, November 26, 2019 1:27 PM

Light ranges from 400 nm to 750 nm

Violet

Blue

Green

Yellow

Red

Review for Final

Tuesday, December 3, 2019 12:26 PM

Final Info:

- Thu 12/12, 11am, CI-109 (i.e. here)
- Bring pencil(s), calculator with fresh batteries

Office hours: 11-4 every day except Fri 12/6 and Mon 12/9

Last minute HW extensions: Use WebAssign Extension Request. See Syllabus for policy. Final deadline is day of Final.

- Electrostatics
- DC Circuits

25-30%

- Magnetism
- AC Circuits

25-30%

- Oscillations and Waves
- Optics

40-50%

Level and Decibels

Tuesday, December 3, 2019 12:38 PM

- Decibels (dB) always measure a ratio of an energy-like quantity (energy, power, or intensity) as compared to some reference.
- As the energy is multiplied or divided, the decibels are added or subtracted.

dB Level

$$\beta = 10 \log_{10} I/I_0 \qquad I = I_0 10^{\beta/10}$$

Example: Signal strengths in dBm: $I \to P$ $\quad P_0 = 1\,mW$

Sun: $P = 3.8 \times 10^{26}\ W$

$$\beta = 10 \log_{10}\left(\frac{\boxed{3.8} \times 10^{26}\ W}{10^{-3}\ W}\right) = 10(29.\boxed{58})$$

$$\beta = 295.8\ dBm$$

Compare to a 4G phone signal of -115 dBm.

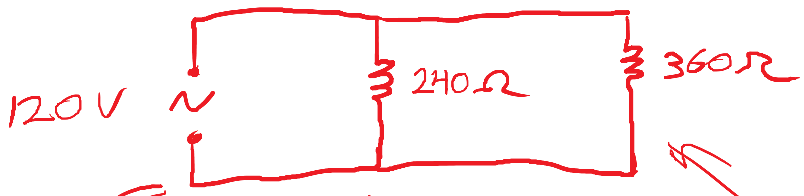
Remember: 0 dB is not zero signal, it's a ratio of 1.

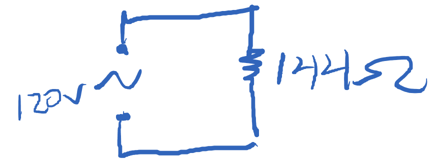
Series/Parallel Circuits

Tuesday, December 3, 2019 12:48 PM

Series: $I = I_1 = I_2 = \cdots$ $V_{\text{tot}} = V_1 + V_2 + \cdots$ $R_{\text{eq}} = R_1 + R_2 + \cdots$

Parallel: $I_{\text{tot}} = I_1 + I_2 + \cdots$ $V = V_1 = V_2 = \cdots$ $R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots\right)^{-1}$





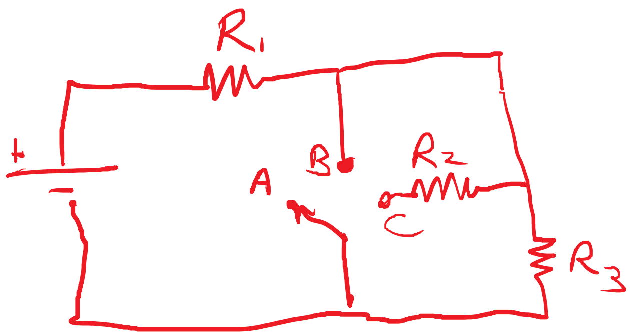
$I = I_1 + I_2$
$= 0.5 + 0.333$
$= 0.833 \text{ A}$

$V = IR$
$(120 \text{ V}) = I(240\,\Omega)$
$0.5 \text{ A} = I$

$120 \text{ V} = I(360\,\Omega)$
$0.333 \text{ A} = I$

$R_{eq} = \left(\frac{1}{240} + \frac{1}{360}\right)^{-1} = 144\,\Omega$

$120 \text{ V} = I(144\,\Omega)$
$0.833 = I$



A: $R_{eq} = R_1 + R_3$

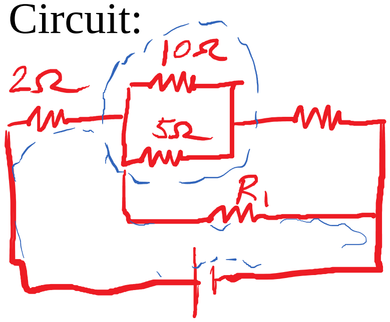
B: $R_{eq} = R_1$

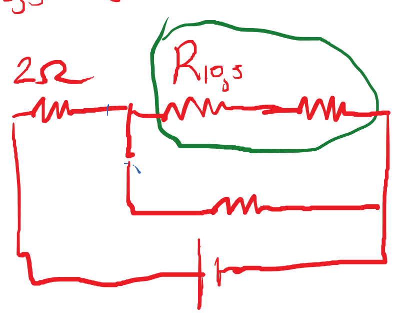
C: $R_{eq} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$

B: $0.0\,\Omega \parallel R_3$ $R = \left(\frac{1}{0.0} + \frac{1}{R_3}\right)^{-1} = \left(\infty + \frac{1}{R_3}\right)^{-1} = 0.0$

$R_{10,5} = \left(\frac{1}{10} + \frac{1}{5}\right)^{-1} = 3.33\,\Omega$

Combination Circuit:



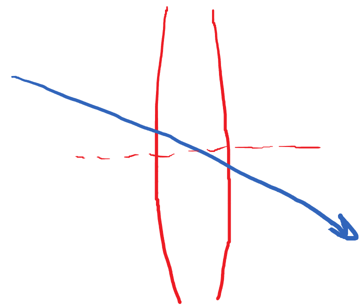


$\mathcal{E} = V_{2\Omega} + V_1$

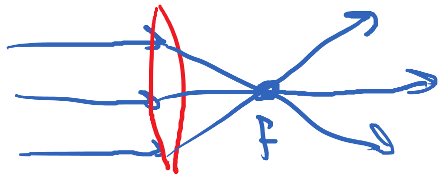
Optics Ray Diagrams

Tuesday, December 3, 2019 1:10 PM

- Central Ray: goes straight thru the middle of the lens

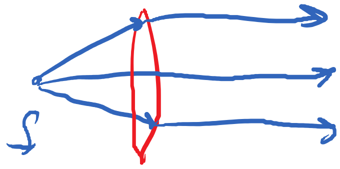


- Parallel Ray: Focuses toward (or away from) axis



$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

- Focal Ray: Mirror image of parallel ray



$$\frac{h'}{h} = \frac{-q}{p}$$

If we see an upright image through a converging lens, is it magnified or reduced?

h' is (+)

q is (−)

f is (+) (converging)

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{p} + \frac{1}{-|q|} = \frac{1}{f}$$

$$\frac{1}{p} = \frac{1}{f} + \frac{1}{|q|}$$

$\frac{1}{p}$ is bigger than $\frac{1}{|q|}$

p is smaller than $|q|$

$$\frac{h'}{h} = \frac{-q}{p}$$

$$\frac{h'}{h} = \frac{|q|}{p}$$

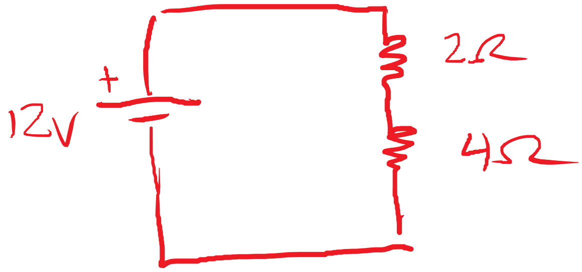
$|q|$ ← Bigger

p ← smaller

Conclusion: Image is magnified (not reduced)

Measuring Circuit values

Tuesday, December 3, 2019 1:28 PM



Votmeter across 2Ω:

- Series $I_2 = I_4$

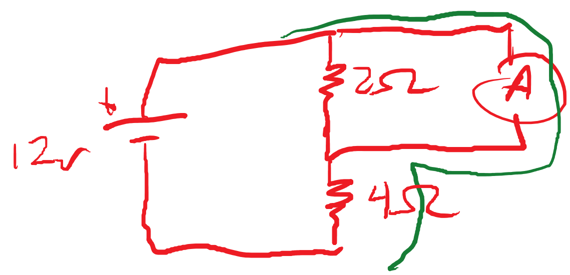
$$\frac{V_2}{R_2} = \frac{V_4}{R_4}$$

- $V_2 = 4\text{ V}$

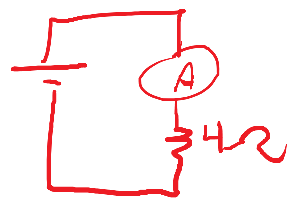
Ammeter in series with battery:

$$I = {}^{12V}/_{6\Omega} = 2A$$

Ammeter across 2 ohm resistor:

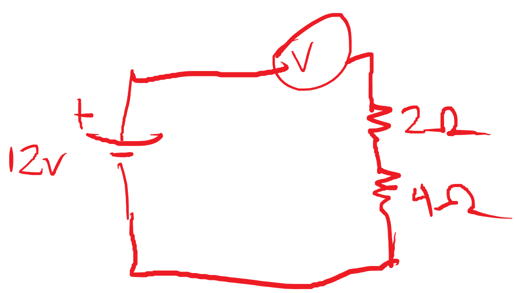


- Ammeter has $R_A = 0$



$I = 3A$

Voltmeter in series with whole circuit



- Voltmeter has $I = 0$

$V_2 = 0$

$V_4 = 0$

$V_m = 12V$