linear Functions and Curve Fitting

Motivation: When laboratory rats are exposed to asbestos fibers, some of them develop lung tumors. The following table lists the results of several such experiments by different scientists:

<table>
<thead>
<tr>
<th>Asbestos Exposure (fibers/ml)</th>
<th>50</th>
<th>400</th>
<th>500</th>
<th>900</th>
<th>1100</th>
<th>1600</th>
<th>1800</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>% that Develop Lung Tumors</td>
<td>2</td>
<td>6</td>
<td>5</td>
<td>10</td>
<td>26</td>
<td>42</td>
<td>37</td>
<td>28</td>
</tr>
</tbody>
</table>

Use your skills in Algebra and graphing to quantify the relationship between asbestos exposure and % of lung tumors development.

To answer the above problem and other life problems, you need some skills in Algebra. Let us start by linear functions.

A linear function is any function of the form \( y = f(x) = ax + b \) where, \( a, b \) are real numbers and \( x, y \) any two variables.

Examples: \( y = f(x) = -2x + 1 \) or \( y = g(x) = x - 2 \), etc.

Ex0 For \( y = f(x) = -2x + 6 \), find the \( x \)-intercepts, \( y \)-intercepts then sketch its graph.

Solution:

let \( x = 0 \), to get \( y = -2(0) + 6 = 6 \) \( \Rightarrow (0,6) \)

\( y = 0 \) to get \( 0 = -2x + 6 \Rightarrow x = 3 \) \( i.e. (3,0) \)
Ex(2) Given the points P(1,6) and Q(3,2).
   a) Find the slope of the line \( \overline{PQ} \).
      \[
      \text{slope} = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4}{2} = -2.
      \]
   b) Find the equation of the line joining \( P \& Q \) in
      (i) slope-point form \( y - y_1 = m(x - x_1) \)
      (ii) slope-intercept form \( y = mx + b \)
      (iii) general form \( Ax + By + C = 0 \)

      (i) \( y - 6 = -2(x - 1) \)
      (ii) \( y = -2x + 8 \)
      (iii) \( 2x + y - 8 = 0 \)

Ex(3) Given \( 2x + 4y = 8 \)
   (i) Solve for \( y \).
      \( y = -\frac{1}{2}x + 4 \)
   (ii) Find slope and intercepts \( m = -\frac{1}{2}, (4,0), (0,2) \)
   (iii) Graph \( 2x + 4y = 8 \).

Remarks:
1. Parallel lines have the same slopes.
2. Perpendicular lines slopes are negative reciprocals.
3. Vertical lines have the form \( x = a \)
4. Horizontal lines have form \( y = b \)
5. Slopes = Average rate of change.
Ex4 Find the equation of the line that is (i) parallel to the line on Ex6 and passes through (-1,2), (ii) perpendicular to the line in Ex3 and passes through (2,-1).

Ex5 Use the given table to find and interpret the average rate of change.

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>20</td>
<td>16</td>
<td>12</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

Average rate of change
\[ \frac{\Delta y}{\Delta x} = -4,000 \text{ \$/year} \]

Ex6 Find the linear equation that expresses the per capita expenditure E in terms of the year y and the per capita expenditure in the year 1996.

The linear equation is \[ E = 205.90y - 15810.00 \].

(Round the coefficient to two decimal places.)

In the year 1996, the formula predicts the per capita expenditure will be \[ \$\text{________} \].

(Round to the nearest whole number.)

<table>
<thead>
<tr>
<th>Year (y)</th>
<th>Per Capita Expenditure (E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>2694</td>
</tr>
<tr>
<td>91</td>
<td>2962</td>
</tr>
<tr>
<td>92</td>
<td>3128</td>
</tr>
<tr>
<td>93</td>
<td>3351</td>
</tr>
<tr>
<td>94</td>
<td>3529</td>
</tr>
</tbody>
</table>