**Quadratic Functions and Models**

The graph of a quadratic function \( y = f(x) = ax^2 + bx + c \), where \( a, b, c \) are real numbers and \( x, y \) are variables, is a parabola.

\[
\text{Domain: All real numbers } (-\infty, \infty) \\
\text{Range: from vertex up (or down) on y-axis.}
\]

![Diagram of a parabola with vertex and axis of symmetry]

\[
f(x) = ax^2 + bx + c
\]

**Factored Form**

\[
f(x) = a(x - r)(x - s)
\]

**Vertex Form**

\[
f(x) = a(x - h)^2 + k
\]

where \( h = \frac{-b}{2a} \)

\( k = f(h) \)

**Example**

Given an equation \( y = f(x) = (x - 5)^2 - 4 \)

a) Give the coordinate of the vertex and axis of symmetry.

b) Give the domain and range then graph it.

c) Find the x-intercepts and y-intercepts.

d) Write the equation on

   i) factored form \( f(x) = (x - 3)(x - 7) \)

   ii) general form \( f(x) = x^2 - 10x + 21 \)
Ex(2) Graph the quadratic function \( f(x) = -3(x-2)^2 + 1 \) then given the vertex, axis, domain, and range.

- Vertex (2,1), axis: \( x = 2 \), D: \( (-\infty, \infty) \), R: \( (-\infty, 1] \).

Ex(6) (Modeling)

Carbon monoxide (CO) combines with hemoglobin of the blood to form carboxy hemoglobin (COHb) which reduces transport of oxygen to tissues. Smokers routinely have a 4% to 6% COHb level in their blood, which can cause symptoms such as blood flow alterations, visual impairment, and poor vigilance ability. The quadratic model

\[ T(x) = 0.0078x^2 - 1.528x + 75.89 \]

approximates the exposure time in hours necessary to reach this 4% to 6% level, where \( 50 \leq x \leq 100 \) is the amount of carbon monoxide present in air in parts per million. (Source: Indoor Air Quality Environmental Info. Handbook, US Dept. of Energy 1985).

a) How long would it take for a non-smoking person to start feeling the above symptoms if a room full of smokers is capable of producing 50 ppm of carbon monoxide.

b) Find the carbon monoxide concentration necessary for a person to reach the 4% to 6% COHb level in 3 hours. Round to the nearest tenth.
**Curve Fitting**

The table (on the right) gives carbon dioxide (\( \text{CO}_2 \)) emissions from all sources on the US and Canada, in million of tons.

<table>
<thead>
<tr>
<th>Year</th>
<th>Millions of Tons of ( \text{CO}_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>1633</td>
</tr>
<tr>
<td>1999</td>
<td>1664</td>
</tr>
<tr>
<td>2000</td>
<td>1725</td>
</tr>
<tr>
<td>2001</td>
<td>1710</td>
</tr>
<tr>
<td>2002</td>
<td>1733</td>
</tr>
</tbody>
</table>

a) Sketch a scatter plot of the given data.

b) Use a graphing calculator to find a quadratic model that approximates the emission for these years. Use \( x \) as the number of years since 1998 and the emission in tons of \( \text{CO}_2 \).

c) Find the year that the model in (b) predicts the emissions reached 1700 million tons.

d) According to the model in (b), what would be the emissions in 2003 and 2011. Do you see any problem with these estimates.