Exercise 1: The distribution is a standard normal because the readings are normally distributed with μ = 0 and \( \sigma = 1 \).

We need to know \( P(Z \leq 1.27) \).

\[ P(Z \leq 1.27) = 0.8980 \]

b) \( P(Z > -1.23) \)

Table A-2 always give area to the left of \( Z \) is 0.1093.

\[ P(Z > -1.23) = 1 - 0.1093 = 0.8907 \]

c) \( P(-2 \leq Z \leq 1.50) \)

\[ P(-2 \leq Z \leq 1.50) = 0.9104 \]

\( Z \) when \( Z = -2 \), \( P = 0.0228 \)

\( Z = 1.5 \), \( P = 0.9332 \)

So \( P(-2 \leq Z \leq 1.50) = 0.9332 - 0.0228 = 0.9104 \)

d) \( Z \) is between 1.64 and 1.65

Search \( P = 0.95 \) (0.9495 and 0.9505)

So \( Z = 1.645 \)
Exercise 2

a) $P(-1.96 < t < 1.96) = 0.95$

b) $P(t < 1.645) = 0.95$

c) $P(t < -2.575 \text{ or } t > 2.575) = 0.0100 = 0.9950$

d) $P(t < 1.96 \text{ or } t > 1.96) = 0.05$
Section 6.3 - Exercises

Exercise 1:

Step 1: \( \mu = 69 \text{ in} \) \( \sigma = 2.8 \text{ in} \).

\[
\begin{array}{c}
\mu = 69 \text{ in} \\
X = 80 \\
\sigma = 2.8 \\
Z = 0 \\
Z = 3.93
\end{array}
\]

Step 2: \( Z = \frac{X - \mu}{\sigma} = \frac{80 - 69}{2.8} = 3.93 \).  

Step 3: From table A-2 and \( Z = 3.93 \)

"3.90" and up: cumulative area = .9999

Interpretation: The proportion of men who can fit through the standard doorway height of 80 in. is .9999 or .9999%.

Very few men will not be able to fit through the doorway.

Exercise 2: \( \mu = 3420 \) \( \sigma = 495 \)

\[
\begin{align*}
Z_1 &= \frac{X - \mu}{\sigma} = \frac{2450 - 3420}{495} = -1.96 \\
Z_2 &= \frac{X_2 - \mu}{\sigma} = \frac{4390 - 3420}{495} = 1.96
\end{align*}
\]

Using table A-2 we find that

\( Z = 1.96 \) corresponds to an area of 0.9750

\( Z = -1.96 \) corresponds to an area of 0.0250. Shaded area

\( .9750 - .0250 = .9500 \)
Interpretation: Expressing the result as a percentage we conclude that 95% of the babies do not require special treatment because they have birth weights between 2450g and 4390g. It follows that 5% do require it. Not too high for typical hospitals.

Exercise 3:

\[
\begin{align*}
\text{Prob} &= \text{area} = .9500, \text{ so} \\
Z &= 1.645 \\
\mu &= 69 \quad \text{and} \quad \sigma = 2.8
\end{align*}
\]

Now with \( Z = 1.645 \)

\[
1.645 = \frac{x - 69}{2.8}
\]

\[
x = 73.606 \text{ in}
\]

The solution of \( x = 73.6 \text{ in} \) is reasonable because it is greater than the mean of 69.0 in.

Interpretation: A doorway height of 73.6 in \( \text{or} 6\text{ft}1.6\text{in} \) would allow 95% of men to fit without bending or bumping their head. So 5% of men would not fit through a doorway. Because not practical.

Exercise 4:

\[
\begin{align*}
\text{Prob} &= \text{area} = .9900, \text{ so} \\
Z &= 2.33 \\
\mu &= 3420 \\
\sigma &= 49
\end{align*}
\]

When \( p = 0.03 \) \( Z = -1.88 \)

When \( p = 0.01 \) we are looking at \( p = 0.99 \), so \( Z = 2.33 \)

because it is the cumulative area from the left.
left most value of $x$:
\[ x_1 = \mu + (2\alpha) = 34.20 + (-1.88 \cdot 4.95) = 24.89.4 \]

right most value of $x$:
\[ x_2 = \mu + (2\beta) = 34.20 + (2.33 \cdot 4.95) = 45.73.85 \]

$x_1 = 24.89.4$ g is reasonable because it is less than the mean of 34.20 g.

Also $x_2 = 45.73.85$ is reasonable because it is above the mean of 34.20 g.
Unbiased Estimators

The sample mean and sample proportions are unbiased estimators. That is, they target the population parameter. These statistics are better in estimating the population parameter if they do not target the population parameter, they are biased estimators.

Section 6.4 - Exercises

Exercise 1: Sample = 2, 3, 10
   With n = 2, Total = 9

<table>
<thead>
<tr>
<th>Sample</th>
<th>Sample mean</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2, 2</td>
<td>2</td>
<td>1/9</td>
</tr>
<tr>
<td>2, 3</td>
<td>2.5</td>
<td>1/9</td>
</tr>
<tr>
<td>2, 10</td>
<td>6</td>
<td>1/9</td>
</tr>
<tr>
<td>3, 8</td>
<td>2.5</td>
<td>1/9</td>
</tr>
<tr>
<td>3, 3</td>
<td>3</td>
<td>1/9</td>
</tr>
<tr>
<td>3, 10</td>
<td>6.5</td>
<td>1/9</td>
</tr>
<tr>
<td>10, 2</td>
<td>6</td>
<td>1/9</td>
</tr>
<tr>
<td>10, 3</td>
<td>6.5</td>
<td>1/9</td>
</tr>
<tr>
<td>10, 10</td>
<td>10</td>
<td>1/9</td>
</tr>
</tbody>
</table>

b) \[ \mu = \frac{1}{n} \sum_{i=1}^{n} x_i \cdot P(x_i) = \]
\[ = 2 \left( \frac{1}{9} \right) + 2.5 \left( \frac{1}{9} \right) + 6 \left( \frac{1}{9} \right) + 2.5 \left( \frac{1}{9} \right) + 3 \left( \frac{1}{9} \right) + 6.5 \left( \frac{1}{9} \right) \]
\[ + 6 \left( \frac{1}{9} \right) + 6.5 \left( \frac{1}{9} \right) + 10 \left( \frac{1}{9} \right) = \frac{45}{9} \approx 5 \]
c) Yes, the sample mean is equal to the mean of the population. These mea are always equal because the mean is an unbiased estimator.

\[ \bar{X} = \frac{2 + 3 + 10}{3} = \frac{15}{3} = 5 = \mu. \]

Exerciced: M, A, B, C (female)

g) Samples | Proportion of females | Sample | Prop | Sample | Prop
---|---|---|---|---|---
M-A & 0.5 & A-M & 0.5 & B-M & 0.5
M-B & 0.5 & A-B & 1 & B-A & 1
M-C & 0.5 & A-C & 1 & B-C & 1
M-M & 0 & A-A & 1 & B-B & 1

\[ \text{Proportion of females} \]

\[ \begin{array}{c}
\text{Sample} \\
C-M \\
C-A \\
C-B \\
C-C
\end{array} \quad \begin{array}{c}
\text{Prop} \\
0.5 \\
1 \\
1 \\
1
\end{array} \]

\[ \text{Probability} \quad \begin{array}{c}
1/16 \\
6/16 \\
9/16
\end{array} \]

b) \[ M = \sum (x_i \cdot P(x_i)) \]
\[ = 0 \left( \frac{1}{16} \right) + 0.5 \left( \frac{6}{16} \right) + 1 \left( \frac{9}{16} \right) \]
\[ = 0 + \frac{3}{16} + \frac{9}{16} = \frac{12}{16} = \frac{3}{4} = \underline{0.75} \]

c) Yes, the sample mean is equal to the population proportion of females. These values are always equal, because proportion is an unbiased estimator.

\[ \bar{X} = \frac{3}{4} = 0.75 = \mu. \]
Exercise 6.5 - Exercises

a) Approach: Use the methods in Section 6.3 because we are dealing with an individual value from a normally distributed population.

\[ \mu = 172 \quad \sigma = 29 \quad x = 180 \]

\[ z = \frac{x - \mu}{\sigma} = \frac{180 - 172}{29} = \frac{8}{29} = 0.28 \]

Use Table A-2 and use \( z = 0.28 \) to find the cumulative area to the left of 180 lb.

\[ P(z < 0.28) = 0.6103 \]

So the prob. of a randomly selected man weighing more than 180 lbs is 0.3897.

b) Approach: Use the central limit theorem because we are dealing with the mean for a sample of 20 men, not an individual man. Even though \( n \neq 30 \), we use a normal distribution because the original population of men has a normal distribution.

\[ \mu_\bar{x} = \mu = 172 \]

\[ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{29}{\sqrt{20}} \approx 6.4845 \]
\[
Z = \frac{X - \mu}{\sigma_X} = \frac{180 - 172}{\frac{6.4845}{6.4845}} = 1.23
\]

From Table A-2 we find that \( Z = 1.23 \) corresponds to a cumulative left area of \( .8907 \).

So the shaded area is \( 1 - .8907 = .1093 \).

Interpretation: There is a .3897 probability that an individual man will weigh more than 180 lb and there is .1093 probability that 50 men will have a mean weight of more than 180 lb, which is fairly too high.

Exercise 2: \( \mu = 1518, \sigma = 325 \)

a) \( X_1 = 1550, X_2 = 1575 \)

\[
Z_1 = \frac{1550 - 1518}{325} = \frac{32}{325} = 0.09
\]

\[
Z_2 = \frac{1575 - 1518}{325} = \frac{57}{325} = 0.17
\]

\( Z_1 = .5359, Z_2 = .5675 \)

Then the area \( Z_2 - Z_1 = .0316 \).

The probability of a randomly selected SAT score that is between 1550 and 1575 is 0.0316.

b) \( \mu_X = \mu = 1518, n = 25 \)

\[
\sigma_X = \frac{\sigma}{\sqrt{n}} = \frac{325}{\sqrt{25}} = \frac{325}{5} = 65
\]

\[
X - \mu = \frac{65}{6.4845} = 1.01
\]
\[ z_1 = \frac{\bar{x}_1 - \mu}{\sigma/\sqrt{n}} = \frac{1550 - 1578}{65} = \frac{32}{65} = 0.49 = 0.49 \]

\[ z_2 = \frac{\bar{x}_2 - \mu}{\sigma/\sqrt{n}} = \frac{1575 - 1518}{65} = \frac{57}{65} = 0.88 \]

\[ z_2 - z_1 = 0.88 - 0.49 = 0.39 \]

The probability that 25 SAT scores are randomly selected if they have a mean between 1550 and 1575 is 0.1227.

\( c \) If the original population is normally distributed, then the distribution of sample means is normally distributed for any sample size.
Section 7.2 - Exercises

Exercise 1:
a) \( z = .99 \Rightarrow 1 - .99 = .01 = \frac{z}{2} \)
\( 2\frac{z}{2} = 2.575 \)

b) \( z = .995 \Rightarrow 1 - .995 = 0.005 = \frac{z}{2} \)
\( 2\frac{z}{2} = 2.81 \)

c) \( z = 0.02 \Rightarrow \frac{z}{2} = .01 \)
\( 2\frac{z}{2} = 2.33 \)

Exercise 2

a) \( 0.200 < p < 0.500 \)
\( \hat{p} = \frac{500 + 200}{2} = .35 \)
\( \hat{p} \pm E = .350 \pm .150 \)

b) \( (0.437, 0.529) \)
\( \hat{p} = \frac{0.437 + 0.529}{2} = .483 \)
\( E = \frac{0.529 - .437}{2} = .046 \)
\( \hat{p} \pm E = .483 \pm .046 \)

Exercise 3

a) \( n = 500 \ x = 220 \ 99\% \) conf. int. \( 1.99 = 0.01 = \frac{z}{2} \)
\( 2\frac{z}{2} = 2.005 = 2.575 \) \( \hat{p} = \frac{220}{500} = .44 \) \( \hat{q} = .56 \)
\( E = \frac{z}{2} \sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.575 \sqrt{\frac{.44\.56}{500}} = 0.0572 \)
Exercise 4

a) \( n = 200 \), \( x = 40 \), 95% confidence

\[ z = 1 - .95 = 0.05 \]

\[ z_{\alpha/2} = z_{0.025} = 1.96 \]

\[ \hat{p} = \frac{40}{200} = 0.20 \]

\[ \hat{q} = 0.80 \]

\[ E = 1.96 \sqrt{\frac{0.20 \times 0.80}{200}} = 0.05 \text{ SE} \]

\[ \hat{p} - E < p < \hat{p} + E \]

\[ 0.15 < p < 0.255 \]

b) \( n = 5200 \), \( x = 4821 \), 99% confidence

\[ z = 1 - .99 = 0.01 \]

\[ z_{\alpha/2} = z_{0.005} = 2.575 \]

\[ \hat{p} = \frac{4821}{5200} = 0.93 \]

\[ \hat{q} = 0.07 \]

\[ E = 2.575 \sqrt{\frac{0.93 \times 0.07}{5200}} = 0.009 \]

\[ \hat{p} - E < p < \hat{p} + E \]

\[ 0.93 - 0.009 < p < 0.93 + 0.009 \]

\[ 0.921 < p < 0.939 \]

Exercise 5

a) \( E = 0.005 \), 99% CI

\[ z = 1 - .99 = 0.01 \]

\[ z_{\alpha/2} = z_{0.005} = 2.575 \]

\[ \hat{p}, \hat{q} \text{ unknown} \]

\[ n = \frac{\left(2z_{\alpha/2}\right)^2 \hat{p} \hat{q}}{E^2} = \frac{(2.575)^2 (0.25)}{(0.005)^2} = 66,307 \]
b) \( k = 3 \% = 0.03 \), \( z = 95\% \)
\[ d = 1 - 0.95 = 0.05 \]
\[ z_{z} = 2 \times 0.025 = 2.86 \]
\[ \hat{p} = 0.87, \hat{q} = 0.13 \]
\[ n = \frac{(2z_{z})^2 \hat{p} \hat{q}}{E^2} = \frac{(2 \times 2.86)^2 (0.87)(0.13)}{(0.03)^2} = 483 \]

**Exercise 6:** \( n = 1228 \), \( x = 856 \)

a) \[ \hat{p} = \frac{856}{1228} = 0.697 \]
b) \( CI: 99\% \) \( 1 - 0.99 = \alpha = 0.01 \)
\[ z_{z} = 2 \times 0.005 = 2.575 \]
\[ \hat{p} = 0.697, \hat{q} = 0.303 \]
\[ E = \frac{2z_{z} \sqrt{\hat{p} \hat{q}}}{n} = 2.575 \sqrt{\frac{(0.697)(0.303)}{1228}} = 0.034 \]
\[ \hat{p} - E < \hat{p} < \hat{p} + E \]
\[ 0.697 - 0.034 < \hat{p} < 0.697 + 0.034 \]
\[ 0.663 < \hat{p} < 0.731 \]

C) Yes. The population proportion does appear to be a value that is greater than 0.5.

**Exercise 7:** \( n = 1002 \)

a) \( CI = 99\% \) \( \alpha = 1 - 0.99 = 0.01 \)
\[ p = 0.61 \]
\[ E = 2.575 \sqrt{\frac{0.70 \times 0.30}{1002}} = 0.037 \]
\[ 0.70 - 0.037 < \hat{p} < 0.70 + 0.037 \]
\[ 0.663 < \hat{p} < 0.737 \]

b) No, because 0.61 is not included in the confidence interval.
Finding the point estimate and confidence interval.

Point estimate of $\mu$:

$$\bar{x} = \frac{(\text{upper conf. int}) + (\text{lower conf. int})}{2}$$

Margin of Error:

$$E = \frac{\text{upper conf. int} - \text{lower conf. int}}{2}$$

Section 7.4 - Exercises

Exercise 1

a) $t_{22} = 2.074$ (since $n = 23 \Rightarrow \text{dof} = 22$, $\alpha = 0.05$)

b) $t_{20} = 2.575$ (since $d = 0.01 \Rightarrow t_{20} = 0.005$)

Neither normal nor $t$ distribution applies.

t_{20} = 2.023 (since $n = 40 \Rightarrow \text{dof} = 39$, $\alpha = 0.05$)

Exercise 2

C.I. = 95%, $d = 0.05$, $df = 19$, $E = 1.645$ (since $d = 0.1 \Rightarrow t_{20} = 0.05$).

$$E = 1.645 \quad n = 20 \quad \bar{x} = 900 \quad s = 569$$

$$\text{dof} = 19$$

a) $E = 2.093 \div \sqrt{\frac{569}{20}} = 26.6$

b) $900 - 26.6 \leq \mu \leq 900 + 26.6$

$873.4 \leq \mu \leq 926.6$
Exercise 3: $\bar{x} = 3103 \quad s = 696 \quad d.f. = 185$

a) $\mu = \bar{x} = 3103 \quad n = 186$ (use 200)

b) $\alpha = 0.05 \quad t_{\frac{\alpha}{2}} = 1.972$

$$E = \frac{t_{\frac{\alpha}{2}} \cdot s}{\sqrt{n}} = 1.972 \cdot \frac{692}{\sqrt{186}} = 100.64$$

Confidence interval

$$\bar{x} - E < \mu < \bar{x} + E$$

$$3103 - 100.64 < \mu < 3103 + 100.64$$

$$3002.36 < \mu < 3204.64$$

\[c\] The mean weight of babies born to mothers who used cocaine appear to be substantially less than the mean weight of babies born to mothers who did not use cocaine.

\[c\] Cocaine use appears to be associated with lower birth weights.
Exercise 1: Critical z values
a) $z = 0.01$ - Two tailed: $\frac{z}{2} = 0.005$

$b.575$ $2.005 = 2.05$

b) $z = 0.02$ - Right tailed. $2.02 = 2.05$

c) $z = 0.005$ - $H_1: p > 0.20$

Left tailed test
$2.005 = -2.575$

Exercise 2: $p = 0.25$

$P = \frac{304}{1122} = 0.28$

$z = \frac{25 - 0.25}{\sqrt{(0.25)(0.75)}} = 2.82$

Exercise 3: $z = 0.05$

$a)$ Left tailed, $z = -1.25$

$b)$ Right tailed, $z = 2.50$

$P = 0.1056$

$P$-value $\Rightarrow z = 0.05$

Fail to reject $H_0$. $P = 0.9938$

$P$-value $= 0.0062$

Since $P < 0.05$, Reject $H_0$. $P = \frac{1}{0.9938}$

Since $P < 0.05$, Reject $H_0$. $P = 0.0062$

$H_1: p \neq 0.07$

$z = -2.75$ (Two tailed test)

$P$-value $= 2(0.0030) = 0.0060$

Since $P < 0.05$, Reject $H_0$. $P = 0.0030$
Exercises

4) There is not sufficient evidence to support the claim that the percentage of blue M&M is greater than 5%.

5) There is sufficient evidence to support the rejection claim that the percentage of on-time U.S. airline flights is less than 75%.

Exercises

5) a) Type I error: Reject the claim that the percentage of non-smokers exposed to secondhand smoke is equal to 41%, when that percentage is actually 41%.

Type II error: Fail to reject the claim that the percentage of non-smokers exposed to secondhand smoke is equal to 41%, when that percentage is actually different from 41%.

b) Same.

Type I = Reject when it is actually true
Type II = Fail to reject when it is actually different
Section 8.3 - Exercise

Step 1: Original claim  \( p = 43\% = 0.43 \)

Step 2: Opposite of original  \( p \neq 0.43 \)

Step 3: Because \( p \neq 0.43 \) does not contain equality it becomes \( H_1 \):

\( H_0: p = 0.43 \)  Two tailed test.
\( H_1: p \neq 0.43 \)

Step 4: \( \alpha = 0.01 \)

Step 5: Because it is a test about proportion \( p \) sample \( \hat{p} \) is relevant to this test.

Step 6: \( Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{\frac{308}{611} - 0.43}{0.074} = 3.65 \)

* P-value method:
Two tailed test: \( Z (\text{area}) = 2 \times 0.0002 \)
For values of \( Z = 3.50 \) or higher: area = 0.0001

\( p\)-value = 0.0002 \( < \) 0.01  Reject \( H_0 \).

* Critical value: \( \frac{Z}{\alpha} = 0.005 \)  \( 2.575 = \pm 2.575 \)

3.65 (test statistic)

\( -2.575 \quad 2.575 \)

Step 7: Since test statistic falls within the critical region

Reject \( H_0 \).

Step 8: There is sufficient evidence to warrant rejection of the claim that the percentage who believe that they voted for the winning candidate is equal to 43%.
**Section 6.5**

**Step 1:** Original claim: \( \mu < 5.4 \)

**Step 2:** Opposite of original claim: \( \mu > 5.4 \)

**Step 3:**
- Ho: \( \mu = 5.4 \)  
- Hi: \( \mu < 5.4 \)  
  "Left-tail test"

**Step 4:** \( \alpha = 0.01 \)  

**Step 5:** The claim is about population mean \( \mu \), the sample mean \( \bar{x} = 5.23 \), pop \( \sigma = 0.54 \), \( n = 50 \). C.L.T. indicates that the distribution is normal.

**Step 6:** Test statistic

\[
Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{5.23 - 5.4}{\frac{0.54}{\sqrt{50}}} = -2.23
\]

**Step 7:**
- \( P \)-value: left-tailed test using \( Z = -2.23 \), area = 0.0132
- Critical value: \( Z = 2.01 \)

Since \( P \)-value > 0.01, Fail to reject Ho

Since test statistic is not in the critical area, Fail to reject Ho

**Step 8:** There is not sufficient evidence to support the claim that the sample is from a population with a mean less than 5.4.