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OPTIMUM SYNTHESIS OF SLIDER-ROCKER PLANAR MECHANISM FOR TWO PRESCRIBED POSITIONS

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ABSTRACT

The planar slider-rocker mechanism is synthesized for two exact positions for the condition of maximum motion transmission efficiency. Particular advantageous configurations are identified, which render this mechanism suitable for generation of large swinging amplitudes of the output member. Also a new method of representing graphically objective functions of more than two variables is proposed.

INTRODUCTION

Many of the pneumatic and hydraulic actuators used in practice utilize the slider-rocker mechanism (RTRR), arranged such that the input member is the translating element (Fig. 1).

The synthesis of such a mechanism for generating an imposed amplitude of the rocker for a given stroke of the input member (the pneumatic or hydraulic cylinder) can be carried out very easy graphically (Hunt 1978). However, obtaining a good motion transmitting efficiency is not guaranteed, and a trial and error must be carried out, until the transmission angle γ remains in-between desired limits, usually greater than 45° and less than 135°. If a self-return of the output member is ensured due to weight or other active forces, a transmission angle in-between 30° and 150° can be considered acceptable.

In the present paper the slider-rocker mechanism is investigated for the generation of large amplitude displacements of the output member, while ensuring maximum motion transmission efficiency.

PROBLEM FORMULATION

Given a TRRR mechanism, a $\Delta\varphi_{max}$ displacement of the output member between the extreme positions φ_1 and φ_0 is imposed to be generated with a given maximum stroke of the piston $\Delta x_{Cmax}=x_{C0}-x_{C1}$. For the sake of generality, a unit value of the input member stroke will be considered ($\Delta x_{Cmax}=1$), the final dimensions of the elements being obtained by scaling the entire mechanism, until the actual piston stroke of the employed cylinder is obtained (usually chosen from pneumatic or hydraulic components catalogs).

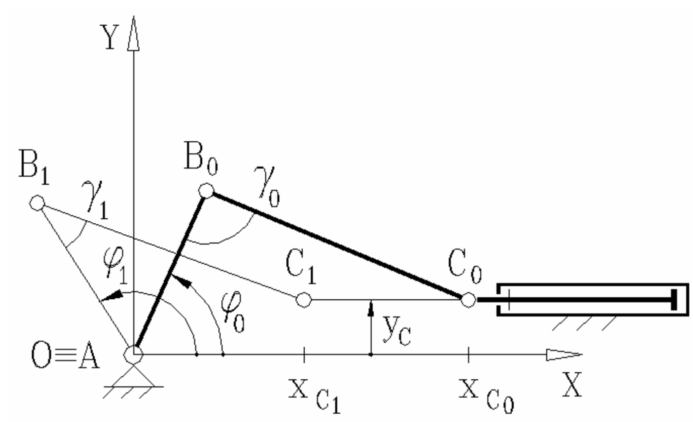

Fig. 1 Slider-rocker mechanism shown in two positions (x_{C0}, φ_0) and (x_{C1}, φ_1).

With the notations in Fig. 1, for the two imposed extreme positions (x_{C0},φ_0) and (x_{C1},φ_1), the equations of constraint of the mechanism are:

$$\begin{aligned}(x_{B0}-x_{C0})^2+(y_{B0}-y_{C0})^2 &= BC^2\\ (x_{B1}-x_{C1})^2+(y_{B1}-y_{C1})^2 &= BC^2\end{aligned} \quad (1)$$

where

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$$x_{B0} = AB \cdot \cos\varphi_0 \qquad y_{B0} = AB \cdot \sin\varphi_0 .$$
$$x_{B1} = AB \cdot \cos\varphi_1 \qquad y_{B1} = AB \cdot \sin\varphi_1 \tag{2}$$

Substituting (2) into (1), and for $y_C=y_{C0}=y_{C1}$ gives:

$$x_{C0}^2 - 2AB \cdot x_{C0} \cos\varphi_0 - 2AB \cdot y_C \sin\varphi_0 + + AB^2 + y_C^2 = BC^2 \tag{3}$$

$$x_{C1}^2 - 2AB \cdot x_{C1} \cos\varphi_1 - 2AB \cdot y_C \sin\varphi_1 + + AB^2 + y_C^2 = BC^2 \tag{4}$$

By subtracting these equations, and for $x_{C1}=x_{C0}-1$ results the normalized length of the rocker:

$$AB = \frac{0.5 \cdot [x_{C0}^2 - (x_{C0}-1)^2]}{x_{C0}\cos\varphi_0 - (x_{C0}-1)\cos\varphi_1 + y_C(\sin\varphi_0 - \sin\varphi_1)} \tag{5}$$

in which $\varphi_1=\Delta\varphi_{max}+\varphi_0$. Choosing φ_0, x_{C0} and y_C as design variables, the normalized lengths AB and BC can be determined based on equation (5) and any of the equations (3) or (4), so that the desired output member amplitude $\Delta\varphi_{max}$ is obtained.

As can be seen, the problem has a triple infinity of solutions, and it is therefore possible to choose the values of φ_0, x_{C0} and y_C such that other conditions upon link length ratios, ground-joint disposition or transmission angle can be fulfilled.

In this paper, only the last of the mentioned conditions will be consider, i.e. to have a minimum variation of the angle γ from the ideal value of 90° over the working range of the mechanism. For a current piston displacement x_C, the transmission angle can be calculated with the equation:

$$\cos\gamma(x_C) = \frac{AB^2 + BC^2 - x_C^2 + y_C^2}{2AB \cdot BC}, \tag{6}$$

derived by applying the cosine law in the ABC triangle and in the right triangle of hypotenuses AC obtained by drawing the perpendicular from C over the OX axis. Apart from the extreme positions, the transmission angle can experience a limit value for $x_C=0$, and therefore this third position must be taken into consideration if $x_{C1}<0$ and $x_{C0}>0$.

An objective function of variables φ_0, x_{C0} and y_C has been defined with the following structure:

$$\begin{aligned}
&\textit{function} \;\; F(\varphi_0, x_{C0}, y_C) \\
&\textit{determine} \;\; AB \;\; \textit{from eq.}\,(5) \\
&\textit{determine} \;\; BC \;\; \textit{from eq.}\,(3)\; \textit{or}\; (4) \\
&c_0 = |\cos\gamma(x_{C0})| \;\; \textit{from eq.}\,(6) \\
&c_1 = |\cos\gamma(x_{C1})| \;\; \textit{from eq.}\,(6) \\
&\textit{IF} \;\; x_{C0} \cdot x_{C1} < 0 \;\; \textit{THEN} \;\; c_e = |\cos\gamma(0)| \\
&\textit{ELSE} \;\; c_e = 0 \\
&F = \max\{c_0, c_1, c_e\}
\end{aligned} \tag{7}$$

where the output member stroke $\Delta\varphi_{max}$ is considered given.

After several numerical experiments, it proved necessary to check that in the extreme positions, the vector loop ABCA keep the same orientation (i.e. the cross products $\mathbf{AB_0}\times\mathbf{B_0C_0}$ and $\mathbf{AB_1}\times\mathbf{B_1C_1}$ have the same sign). In case this requirement is not fulfilled, the two extreme positions can not be attained without

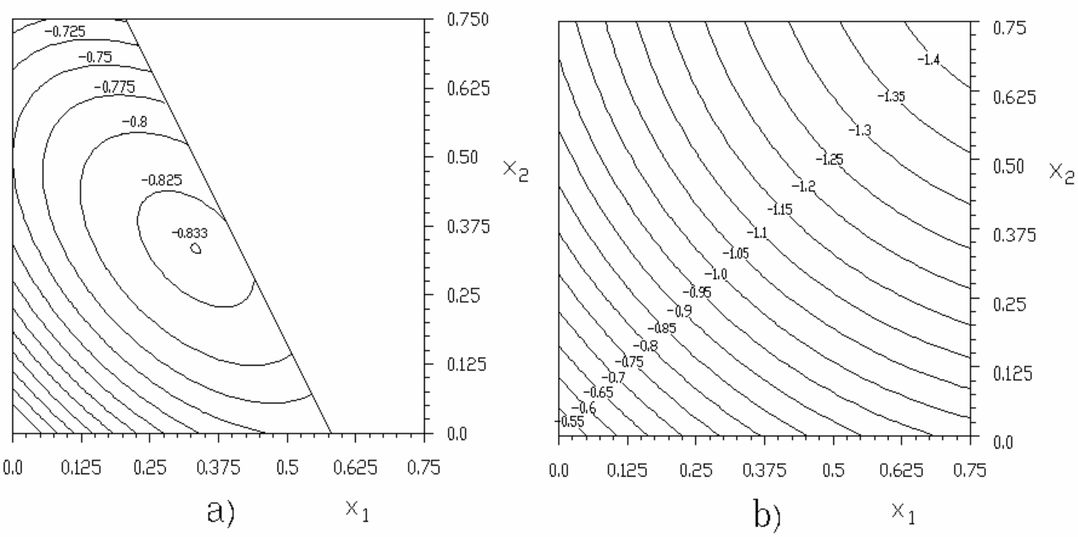

Fig. 2 Contour line plots of the test objective function F1 partially minimized with respect to x_3, (a) with restrictions, and (b) without restrictions.

breaking the joints, and the value of the objective function will be penalized.

After a systematic inspection of the design space through graphical representations of the objective function (7) for various values for the output-member stroke $\Delta\varphi_{max}$, some interesting properties have been identified. The method used for visualizing more-than-two variables objective function is believed to be new and will be described in detail in the following paragraph.

METHOD OF VISUALIZING MULTIVARIABLE OBJECTIVE FUNCTIONS

Single valued functions of two variables can be visualized in various ways, basically as projected level curves (Fig. 2) or as three-dimension surfaces (Fig. 3). For single-valued functions of 3 variables, it is customary to scan two of the dimensions through level curves or 3D representations, generated for different values of the third variable. In this way a feeling of the monotonicity, convexity, or the existence of multiple minima of the function can be obtained. For example Kota and Chiou (1993) inspected the design space of a three variable objective function used for the synthesis of a symmetric four bar path generator with no less that 15 such representations. It is obvious that the number of representations required for inspecting the design space of objective functions with more than 3 variables is prohibitively large, and the method becomes ineffective.

The proposed method of visualizing objective-functions of more than two variables $F(x_1,x_2,..x_n)$, is to scan at a constant step the domain of two of the variables (for example x_1 and x_2), while performing a minimization in respect to the remaining variables $x_3..x_n$.

Partial minimization of multivariable objective function is not a new concept. For example Papalambros and Wilde (1988) have routinely used partial minimization of functions in respect to only one of the variables in solving optimal design problems.

Following Liu and Angeles (1992), the sets x_{1i}, x_{2i} can be named *state variables*, and together with the values $\min F(x_3..x_n)_i$ of the partial minima can be used in generating appropriate level-curve diagrams or 3D surfaces. Such representations will give a good feeling of the properties of the objective function, mainly if several combinations of the state variables will be used for generating the graphical representations. To show the effectiveness of the method, the cases of two test objective functions taken from Hansen et al. (1989) will be further considered. The first test objective function was:

$$\begin{aligned}
&F1(x_1,x_2,x_3) = (x_1^2 + x_2^2 + x_3^2)/2 - \\
&-(x_1 + x_2 + x_3) \\
&\textit{subjected to:} \\
&x_1 + x_2 + x_3 - 1 \le 0 \\
&4x_1 + 2x_2 - 7/3 \le 0 \\
&x_1 \ge 0; \quad x_2 \ge 0 \quad \textit{and} \quad x_3 \ge 0;
\end{aligned} \tag{8}$$

Its minimum value is -5/6 and occurs in the point (1/3,1/3,1/3) for which the constraint $x_1+x_2+x_3-1\le 0$. From the graphical representation in Fig. 2-a, it is apparent that the minimum is located inside of the feasible domain, which is not true. This is revealed by the graphical representation of the partial minima with respect to x_3 of the same function, but without restrictions (Fig. 2-b), which shows a constant decrease of its value with the increase of x_1 and x_2. For generating the partial minima diagrams, Brent's algorithm has been used (Brent 1972) and ramp-type penalty functions that are very easy to program.

The second test objective function considered was:

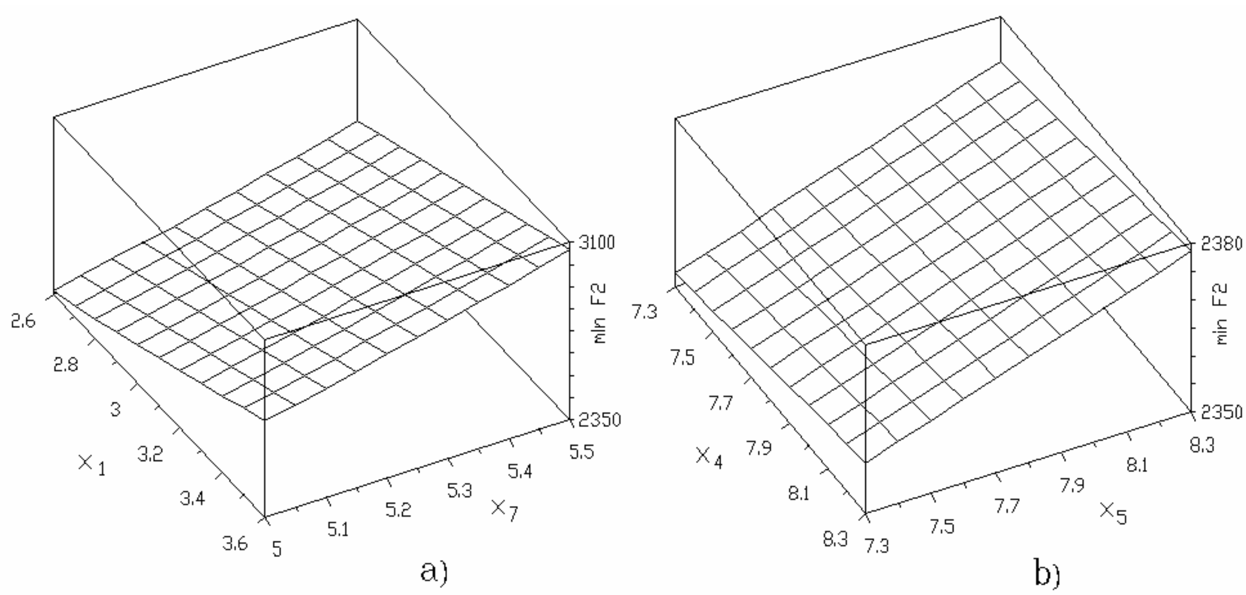

Fig. 3 Surface plots of the test objective function F2, partially minimized with respect (a) to x_2,x_3,x_4,x_5,x_6 and (b) to x_1,x_2,x_3,x_6,x_7.

$$
\begin{aligned}
&F2(x_1, x_2, ... x_7) = 0.7854 \cdot x_1 \cdot x_2^3(3.3333 \cdot x_3^2 + \\
&+14.9334 \cdot x_3 - 43.0934) - 1.508 \cdot x_1(x_6^2 + x_7^2) + \\
&+0.7477(x_6^3 + x_7^3) + 0.7854(x_4 \cdot x_6^2 + x_5 \cdot x_7^2) \\
&\textit{subjected to:} \qquad\qquad\qquad\qquad\qquad\qquad (9) \\
&2.6 \le x_1 \le 3.6 \\
&0.7 \le x_2 \le 0.8 \qquad 17 \le x_3 \le 28 \\
&7.3 \le x_4 \le 8.3 \qquad 7.3 \le x_5 \le 8.3 \\
&2.9 \le x_6 \le 3.9 \quad \textit{and} \quad 5 \le x_7 \le 5.5
\end{aligned}
$$

Its minimum is 2352.448 and occurs in the point (2.6, 0.7, 17, 7.3, 7.3, 2.9, 5), which is confirmed by the two graphs in Fig. 3. In this case, the partial minima of the function required to generate the 3D diagrams were calculated using Simplex algorithm due to Nelder and Mead (Press et al. 1989) and the same ramp-type penalty functions.

The generation of the successive partial minima used for sampling the function surface is CPU intensive, but with the advent of nowadays very high-speed personal computers, this is not a serious problem. The procedure is very well suitable for being implemented on a parallel computer. The CPU time can be further reduced by using the previous value found for one point on the diagram as an initial guess for the next search.

NUMERICAL RESULTS

The above-described procedure was applied to visualize the objective function (7). For an imposed value of the output member $\Delta\varphi_{max}$=120°, the level curve diagram of the partial minima of *arcos* $F(\varphi_0, x_{C0}, y_C)$ in respect with φ_0 is shown in Fig. 4. In Fig. 5 a detail representation of the minimum region of the same function is given. One can notice that the design space is symmetric relative to the vertical x_{C0}=0.5, and also that two global extrema occur, one for x_{C0}=0.3690 and y_C=0.4476, and the second one for x_{C0}=0.6310 and the same y_C=0.4476. The initial angle of the crank are φ_0=69.503° and φ_0=-9.502° respectively.

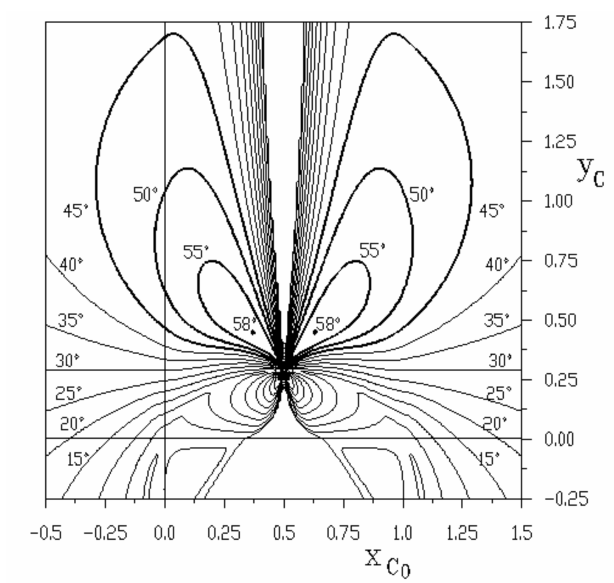

Fig. 4 Contour plots of the objective function F, partially minimized with respect to φ_0 and for $\Delta\varphi_{max}$=120°. The level of the contour-lines were edited so as to generate the function *arccos*(F), for which the maximum values are sought.

The graphical representations in Fig. 5 of the corresponding optimum mechanisms, shows that one is the mirror image of the other, both having the same member lengths i.e. AB=0.3471 and BC=0.5093. Consequently, the same transmission function $\varphi(x_C)$ and variation of the transmission angle $\gamma(x_C)$ is obtained for both mechanisms, but reverted over Δx_C axis (where $\Delta x_C = x_{C0} - x_C$ is the relative displacement of the piston from its initial position x_{C0}). The maximum departure of the transmission angle γ from the 90° value is in both cases ±31.5°, an unexpectedly small value for an output member swinging travel of 120°. In contrast, the transmission angle of the oscillating-cylinder actuating mechanism of RTRR structure ensures a departure of ±60° of the transmission angle from the ideal value of 90° for the same amplitude of the rocker (Simionescu 1999).

Even a 180° swinging amplitude of the rocker can be generated while still maintaining an acceptable variation of the transmission angle. By minimizing the same objective function $F(\varphi_0, x_{C0}, y_C)$ for $\Delta\varphi_{max}$=180° the following optimum dimensions were obtained: x_{C0}=0.75, (or x_{C0}=0.25), y_C=0.28868 AB=0.3307 and BC=0.5518. The maximum departure of the transmission angle from the optimum value of 90° is in this case ±57.32°.

It is to be noticed that the mechanisms described experience positions in which they locked completely if motion is suppose to be transmitted in reverse (from the rocker to the actuating cylinder). Bagci (1987) gave analytical relations for the synthesis of the slider-rocker mechanisms for two imposed positions, of which one is a locking position, but without any concern about the motion transmission efficiency.

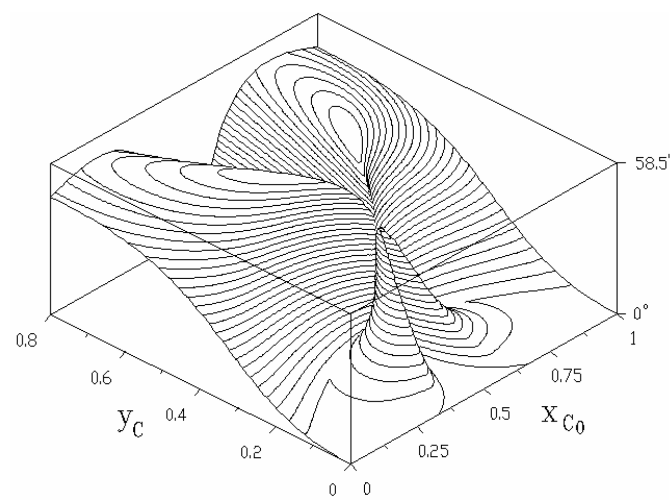

Fig. 5 Detail representation of the minimum area in Fig. 4

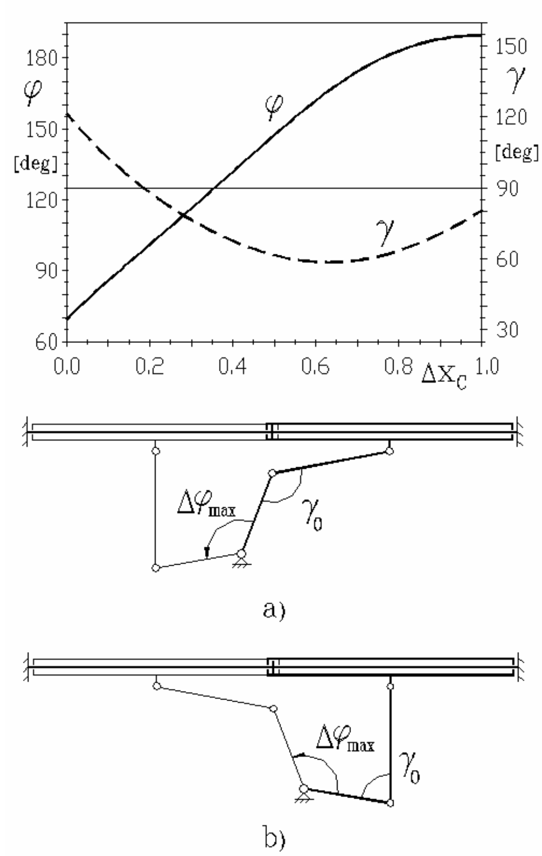

Fig. 5 The mechanisms corresponding to the global minima of the objective function $F(\varphi_0, x_{C0}, y_C)$, for $\Delta\varphi_{max}=120°$. The diagram corresponds to the mechanism (a).

CONCLUSIONS

The above results prove that the slider-rocker mechanism has good capabilities of converting a translating input into a rotary output, while simultaneously ensuring good transmission efficiency of the motion.

If the mechanism is supposed to be disposed into a confined space, a diagram like the one in Fig. 4 can indicate the maximum transmission angles that can occur for different locations of the actuating cylinder axis (y_C) and initial position of the pivot joint C (x_{C0}).

In a future paper design charts will be provided to help the designer in quickly choosing the appropriate dimensions of the mechanism members and the disposition of the ground joints, for an imposed swinging amplitude $\Delta\varphi$ of the rocker.

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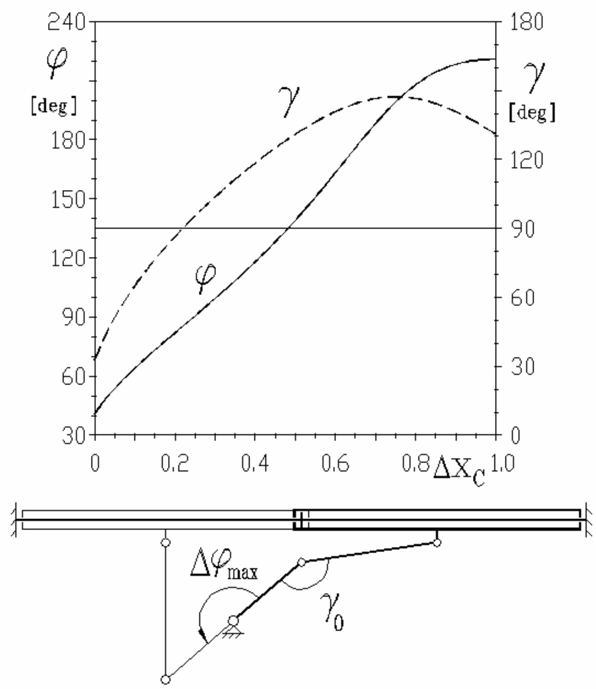

Fig. 7 One of the two optimum mechanism that generates an amplitude $\Delta\varphi_{max}=180°$ of the rocker, and the corresponding diagrams $\varphi(\Delta x_C)$ and $\gamma(\Delta x_C)$.

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