OPTIMIZATION OF THE MOLD ORIENTATION ON AN INVESTMENT CASTING CENTRIFUGE

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ABSTRACT

An optimum orientation of the mold inside the spinning drum of a casting centrifuge is sought, at which the resulting g-force has a maximum overall value inside the part's volume. An optimization problem is defined and solved with the aid of a hybrid algorithm that combines an evolutionary algorithm named 3*p* in series with Nelder and Mead's simplex algorithm. The 3*p* algorithm, employs an elitist, rank-based selection and introduces a new concept, that of *consanguination-avoidance crossover*.

INTRODUCTION

The Solidification Design Center of Auburn University (http://metalcasting.auburn.edu) was established by NASA in 1997, and together with its affiliates is working to develop advanced manufacturing products through improved understanding and control of gravitational and related effects in industrial processes. The R&D conducted in the field of gravitational effects upon metallurgical processes includes investment, centrifugal and semisolid casting as well as microgravity and high-*g* directional solidification studies.

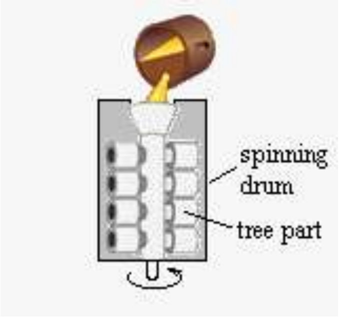

Fig. 1 Centrifugal-investment casting process (the parts are shown as short cylinders forming a tree).

Centrifugal-investment casting is a new process, where the tree-shaped mold as used in the classical investment casting process (Davis, 1998) is spun relative to a vertical axis on a centrifuge while pouring the molted metal inside (Fig. 1). The high *g*-forces thus created provide a better filling of the cavities with molted metals and, after solidification, an improved grain structure by separating the nonmetallic impurities in the tree shaft, close to the axis of rotation of the centrifuge.

Fig. 2 The Digibot II laser digitizing system.

In order to maximize the output per casting, a compact arrangement must be chosen to the parts inside the centrifuge drum. Secondly, if the resultant *g*-force is constantly large throughout the part's volume, the metal structure will be more homogenous after solidification. Such a goal can be achieved by locating the cavities corresponding to the parts as close as possible to the circumference of the centrifuge drum, and avoiding any thin protrusions to be oriented towards the axis of rotation of the centrifuge. The problem of optimum orientation of a single part only will be considered in this paper and solved for the actual case of a complex-shape part.

PROBLEM FORMULATION

For a given diameter of the centrifuge drum spinning at a constant RPM, an optimum disposition of the mold requires finding the position and orientation for which the distances between the infinitesimal volumes of material that will form the parts and the centrifuge's axis of rotation is on the average maximum.

One issue in defining an appropriate objective function to be maximized for achieving this goal was the availability of a suitable 3D representation of the part to be cast. The solution considered was to digitize the outside surface of an existing part using a 3D laser scanner available at Solidification Design Center of Auburn University (Fig. 2). The output file used from the laser-digitizing system is a DXF encoding of a number of equally spaced polylines, the vertices of which are acquired using a laser beam in a process described on http://www.digibotics.com/3adaptive.htm. These polylines can be viewed as a collection of level-curves equally spaced over the vertical axis as shown in Fig. 3.

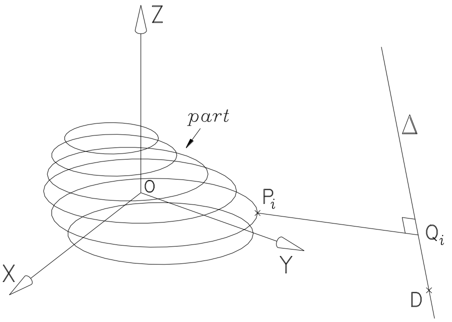

Fig. 4 Schematic used for defining the objective function (3). P_iQ_i is the distance segment between a current vertex P_i and the centrifuge's

Although this is not a "volume representation" of the part, the vertices of the polylines provide a useful numerical representation to be further used in formulating an optimization problem. Conversely, because the vertex concentration is higher in the areas of the part having increased detailed contours, the respective zones will favorably gain importance inside the objective function.

In order to avoid applying repeated translation and rotation transformations to a large number of points, an inverse-motion approach was adopted i.e. the part was considered fixed while the position and orientation of the axis of rotation (noted Δ in Fig. 4) was changed during the optimization process. The axis of the centrifuge has the equation:

$$(\mathrm{x}-\mathrm{x_D})/l = (\mathrm{y}-\mathrm{y_D})/m = (\mathrm{z}-\mathrm{z_D})/n \qquad (1)$$

and is repositioned by modifying the x_D and y_D coordinates of the through point D (Fig. 4), while its orientation can be changed by modifying two of its direction cosines, say *l*, *m*. The z_D coordinate of the through point is maintained at a constant value (possibly at z_D=0), while the third direction cosine *n* can be calculated using the equation:

$$l^2 + m^2 + n^2 = 1 \qquad (2)$$

The vertices read from the DXF file (summing a total of *np* points) were used in defining the following objective function:

$$\mathrm{F}(\mathrm{x_D}, \mathrm{y_D}, l, m) = \sum_{i=1}^{np} \mathrm{dist}\,(\mathrm{P}_i, \Delta) \qquad (3)$$

subjected to the constraints:

$$\begin{aligned} &\mathrm{dist}(\mathrm{P}_i, \Delta) \le \mathrm{R} \\ &l^2 + m^2 \le 1 \\ &0 \le l \le 1 \\ &0 \le m \le 1 \end{aligned} \qquad (4)$$

The first of these inequalities restricts the location of the part inside the centrifuge drum of radius R, while the remaining three constraints ensure that the direction cosines *l*, *m* and *n* satisfy equation (2).

In the above relations dist(P_i,D) is the distance between a current vertex $P_i(x_i,y_i,z_i)$, as read from the DXF file, and the axis of rotation of the centrifuge Δ, and is given by the formula:

$$\mathrm{dist}(\mathrm{P}_i, \Delta) = \sqrt{\frac{(m\,d\mathrm{x} - l\,d\mathrm{y})^2 + (n\,d\mathrm{x} - l\,dz)^2 + (n\,d\mathrm{y} - m\,dz)^2}{l^2 + m^2 + n^2}} \qquad (5)$$

where dx, dy and dz are:

$$d\mathrm{x} = \mathrm{x}_i - \mathrm{x_D}\,, \quad d\mathrm{y} = \mathrm{y}_i - \mathrm{y_D} \;\text{ and }\; dz = \mathrm{z}_i - \mathrm{z_D} \qquad (6)$$

THE SEARCHING SUBROUTINE

An evolutionary algorithm named 3*p* in series with a standard Simplex algorithm (Nelder and Mead 1965) has been used to maximize the objective function (3). The constraints (4) were introduced in the optimization problem as ramp-type penalty functions (Michalewicz 1996) that are very easy to program.

The 3*p* algorithm uses vectors of real numbers as individuals and has the following structure:

1. randomly generate an initial population of *p* members where *p* is a given number satisfying the inequality $3p>n^*$ (n^* is the number of variables of the objective function);
2. select the most fit individual (the dominant male) from among the total population of 3*p* members;
3. sort from among the remaining 3*p*-1 members of the population the most distant to the dominant male *p*-1 individuals i.e. the Euclidean norm between the 2nd ranked individual and the dominant male will be the largest; the Euclidean norm between the 3rd ranked individual and the dominant male will be the second largest etc., up to the *p*th rank (the remaining 2*p* individuals will remain unsorted);
4. crossover the dominant male with the individuals ranked between 2 and *p*-1 and replace the individuals ranking between *p*+1 and 2*p*;
5. replace by random generation the individuals ranked between 2*p*+1 and 3*p* (the mutation step);
6. go to step 2 until for *q* successive iterations (*q* a given number); the dominant male remains unchanged (the stop test should be done after completing step 3);

It can be seen that the 3p algorithm employs an elitist, rank-based selection (Bäck at al 1997, Poloni and Pediroda 1998, Yao 1999). This type of selection forces the best member of the population (the dominant male) to recombine with the most distant *p* individuals in the total population which avoids the search to settle in the vicinity of the dominant male. This inverse-distance ranking is equivalent to a *consanguination-avoidance* encountered in nature (like in African lions or modern humans).

The crossover operator used was:

$$x_{p+j,i} = \alpha_i \cdot x_{1,i} + \beta_i \cdot x_{j,i} \quad (i = 1..n, \;\; j = 1..p) \tag{7}$$

where $x_{\square,\square}$ represents an entry in the $3p\times n^*$ matrix storing the members of the population and α_i and β_i are uniform random numbers between 0 and 1. This type of crossover in turn facilitates a wider exploration of the design space of the objective function.

After the 3*p* algorithm reaches its stop condition, the search is continued by a Nelder-Mead simplex subroutine. The purpose of applying a simplex search subsequent to the evolutionary algorithm was to refine the optimum solution. The simplex step is less time consuming and was inspired by the similarities that exist between this particular zero-order searching algorithm and evolutionary algorithms in general.

Nelder and Mead’s simplex algorithm has received constant attention ever since it was published due to its robustness and the capability of coping well with nonsmooth, nondifferentiable objective functions (Humphrey and Wilson 2000, Parkinson and Hutchinson 1969, Trabia 2000). The vertices of the simplex in Nelder-Mead algorithm can be considered individuals of a population that experience successive directional crossovers and selection operations. If a mutation operator is introduced (like for example to replace every iteration some

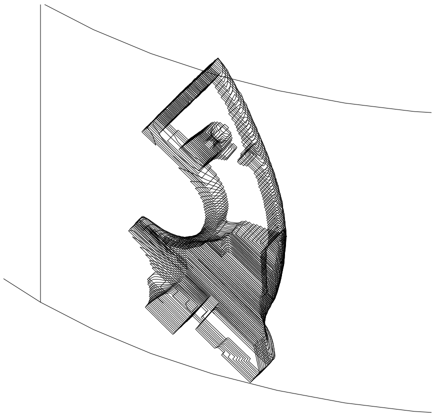

Fig. 5: Isometric view of the part in its optimum position inside the drum

of the vertices of the simplex with other vertices randomly generated), it can be converted into an evolutionary algorithm.

The hybrid algorithm used by the authors for maximizing the objective function (3) employs a classic variant of simplex algorithm, as it was originally described in Nelder and Mead (1965). After the 3*p* algorithm reaches its stop condition, the dominant male together with other n^* individuals are transferred to Nelder-Mead subroutine. These n^*+1 individuals will form the initial simplex and the search will continue until the convergence criteria or other predefined stop condition is achieved.

Regarding the transfer of data to the simplex algorithm, it was found more advantageous to exit the 3*p* subroutine after passing through step 3 (not immediately after step 2) and consider for the vertices of the initial simplex the first n^*+1 ranked members of the last population. In this way the exploratory contribution of the Nelder-Mead algorithm is increased to some extent.

NUMERICAL RESULTS

The actual case of a complex-shape part was considered as a numerical example. Every time the objective function is evaluated the (x_i, y_i, z_i) coordinates of 15224 points (corresponding to 138 polyline levels) have to be read from a DXF file. This file sums a total of 185 513 lines of ASCII symbols, which must be read sequentially making the evaluation of the objective function notably time consuming.

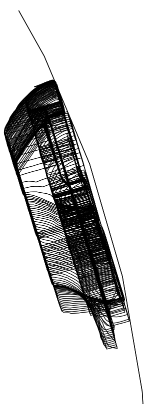

Fig. 6: Top view of the part in its optimum position inside the drum

After 8436 function evaluations, of which 7258 were consumed in the 3*p* algorithm (for a stop parameter q=75), the following results were obtained corresponding to an inner radius of the centrifuge drum R=6”: F_{max}=88492.016 (objective function value), x_D=0.1447”, y_D=-5.7521”, z_D=1.72597”, l=0.577919, m=0.001415, n=0.816093. The parameters positioning the axis of the centrifuge are expressed relative to the reference system attached to the part (the same used by the laser digitizing system).

By changing the reference frame, so that the axis of rotation of the centrifuge becomes the z axis, the configuration shown in Fig. 5 has been finally obtained. Fig. 6 is a top view of the same image, showing, as expected, that the part is aligned with the inner surface of the drum.

CONCLUSIONS AND FURTHER STUDIES

An optimum orientation of the mold inside the spinning drum of a casting centrifuge was sought in order to ensure a better filling of the cavities of the mold and improve the metal grain structure after solidification. A hybrid evolutionary-simplex algorithm was used in solving a proper maximization problem. The evolutionary algorithm named 3*p* employs an elitist, rank-based selection and introduces a new concept, that of *consanguination-avoidance crossover*. Thorough investigation of the convergence and robustness properties of this hybrid algorithm, and of the 3*p* algorithm alone will be the subject of future investigations the authors are considering to pursue.

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