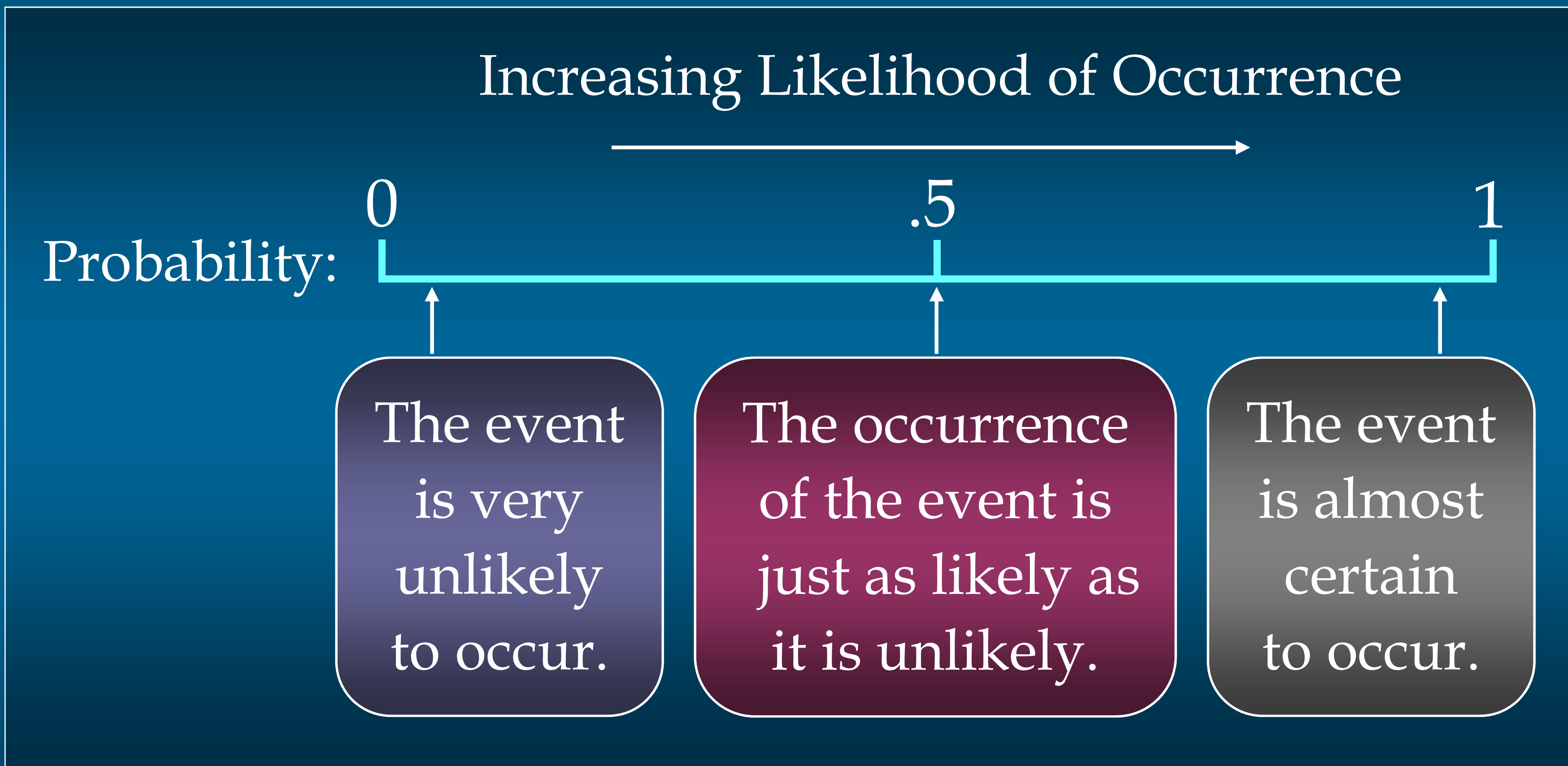
Chapter 4
Introduction to Probability

- Experiments, Counting Rules, and Assigning Probabilities
- Events and Their Probability
- Some Basic Relationships of Probability
- Conditional Probability

Probability as a Numerical Measure of the Likelihood of Occurrence



An Experiment and Its Sample Space

- An experiment is any process that generates well-defined outcomes.
- The sample space for an experiment is the set of all experimental outcomes.
- An experimental outcome is also called a sample point.

Example: Bradley Investments

Bradley has invested in two stocks, Markley Oil and Collins Mining. Bradley has determined that the possible outcomes of these investments three months from now are as follows.

Investment Gain or Loss in 3 Months (in $000)

Markley Oil	Collins Mining
10	8
5	–2
0	
–20	

A Counting Rule for Multiple-Step Experiments

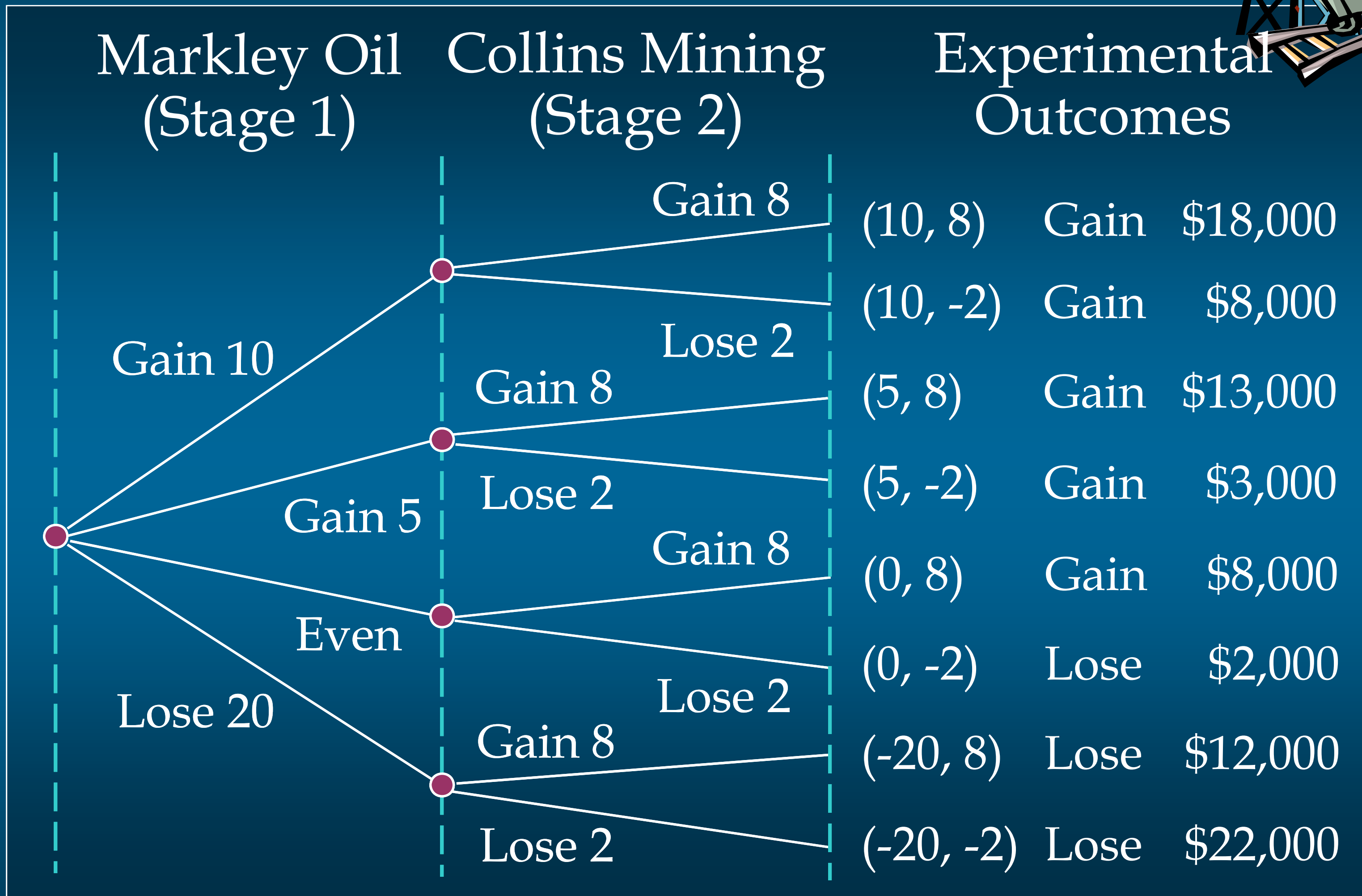
- If an experiment consists of a sequence of k steps in which there are n_1 possible results for the first step, n_2 possible results for the second step, and so on, then the total number of experimental outcomes is given by $(n_1)(n_2) \dots (n_k)$.
- A helpful graphical representation of a multiple-step experiment is a <u>tree diagram</u>.

A Counting Rule for Multiple-Step Experiments

Bradley Investments can be viewed as a two-step experiment. It involves two stocks, each with a set of experimental outcomes.

Markley Oil:	$n_1 = 4$
Collins Mining:	$n_2 = 2$
Total Number of Experimental Outcomes:	$n_1 n_2 = (4)(2) = 8$

Tree Diagram



Counting Rule for Combinations

A second useful counting rule enables us to count the number of experimental outcomes when n objects are to be selected from a set of N objects.

- Number of Combinations of N Objects Taken n at a Time

$$C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!}$$

where:

$$N! = N(N-1)(N-2)\ldots(2)(1)$$

$$n! = n(n-1)(n-2)\ldots(2)(1)$$

$$0! = 1$$

Counting Rule for Permutations

A third useful counting rule enables us to count the number of experimental outcomes when n objects are to be selected from a set of N objects, where the order of selection is important.

- Number of Permutations of N Objects Taken n at a Time

$$P_n^N = n!\binom{N}{n} = \frac{N!}{(N-n)!}$$

where: $N! = N(N-1)(N-2)\ldots(2)(1)$

$n! = n(n-1)(n-2)\ldots(2)(1)$

$0! = 1$

Assigning Probabilities

- **Classical Method**

  Assigning probabilities based on the assumption of equally likely outcomes

- **Relative Frequency Method**

  Assigning probabilities based on experimentation or historical data

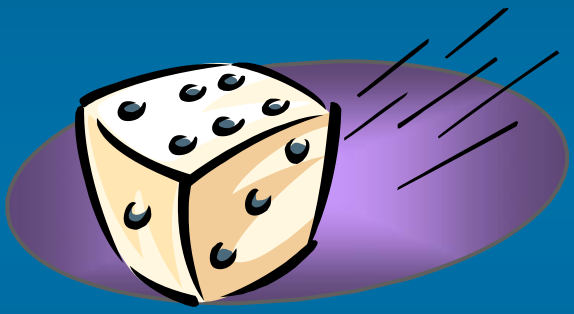
- **Subjective Method**

  Assigning probabilities based on judgment

Classical Method

If an experiment has n possible outcomes, this method would assign a probability of $1/n$ to each outcome.

Example



- Experiment: Rolling a die
- Sample Space: $S = \{1, 2, 3, 4, 5, 6\}$
- Probabilities: Each sample point has a 1/6 chance of occurring

Relative Frequency Method

- Example: Lucas Tool Rental
- Lucas Tool Rental would like to assign probabilities to the number of car polishers it rents each day. Office records show the following frequencies of daily rentals for the last 40 days.

Number of Polishers Rented	Number of Days
0	4
1	6
2	18
3	10
4	2

Relative Frequency Method

Each probability assignment is given by dividing the frequency (number of days) by the total frequency (total number of days).

Number of Polishers Rented	Number of Days	Probability
0	4	.10
1	6	.15
2	18	.45
3	10	.25
4	2	.05
	40	1.00

4/40

Subjective Method

Applying the subjective method, an analyst made the following probability assignments.

Exper. Outcome	Net Gain or Loss	Probability
(10, 8)	\$18,000 Gain	.20
(10, –2)	\$8,000 Gain	.08
(5, 8)	\$13,000 Gain	.16
(5, –2)	\$3,000 Gain	.26
(0, 8)	\$8,000 Gain	.10
(0, –2)	\$2,000 Loss	.12
(–20, 8)	\$12,000 Loss	.02
(–20, –2)	\$22,000 Loss	.06

Events and Their Probabilities

- An event is a collection of sample points.
- The probability of any event is equal to the sum of the probabilities of the sample points in the event.
- If we can identify all the sample points of an experiment and assign a probability to each, we can compute the probability of an event.

Events and Their Probabilities

- Event M = Markley Oil Profitable
- $M = \{(10, 8), (10, -2), (5, 8), (5, -2)\}$
- $P(M) = P(10, 8) + P(10, -2) + P(5, 8) + P(5, -2)$
- $= .20 + .08 + .16 + .26$

$= .70$

Events and Their Probabilities

- Event C = Collins Mining Profitable
- $C = \{(10, 8), (5, 8), (0, 8), (-20, 8)\}$
- $P(C) = P(10, 8) + P(5, 8) + P(0, 8) + P(-20, 8)$
- $= .20 + .16 + .10 + .02$

  $= .48$

Some Basic Relationships of Probability

There are some basic probability relationships that can be used to compute the probability of an event without knowledge of all the sample point probabilities.

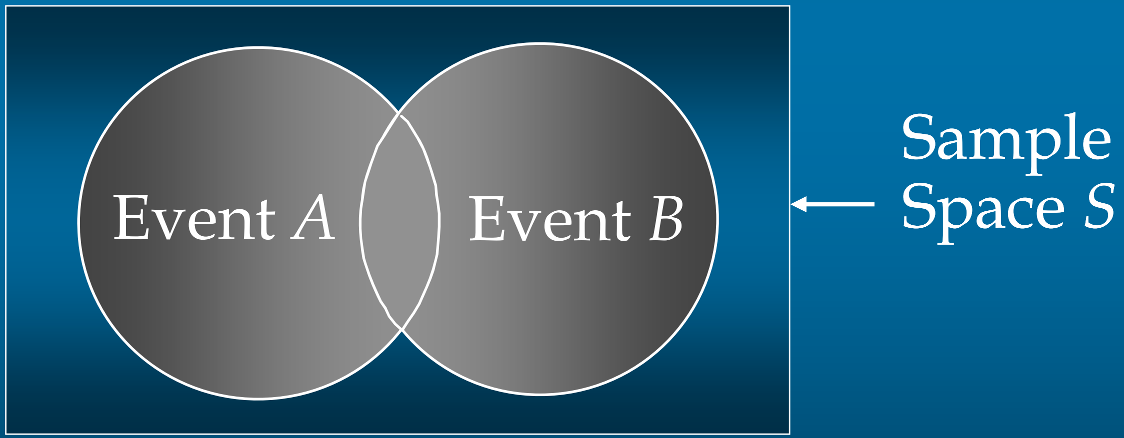
- Complement of an Event
- Union of Two Events
- Intersection of Two Events
- Mutually Exclusive Events

Complement of an Event

- The complement of event *A* is defined to be the event consisting of all sample points that are not in *A*.
- The complement of *A* is denoted by A^c.

Union of Two Events

- The <u>union</u> of events A and B is the event containing all sample points that are in A or B or both.
- The union of events A and B is denoted by $A \cup B$.

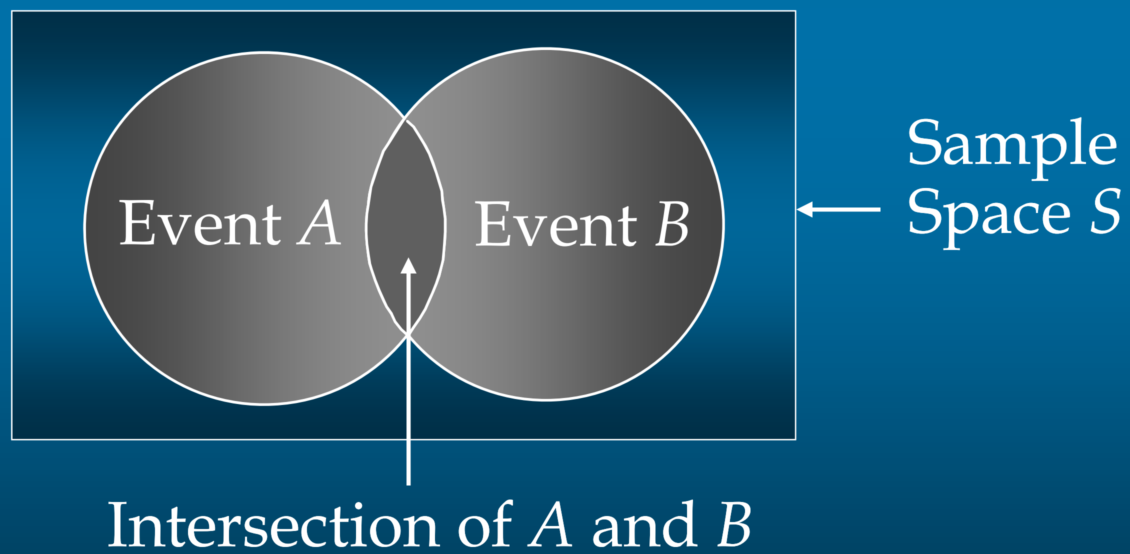


Union of Two Events

- Event M = Markley Oil Profitable
  Event C = Collins Mining Profitable
- $M \cup C$ = Markley Oil Profitable
  <u>or</u> Collins Mining Profitable
- $M \cup C = \{(10, 8), (10, -2), (5, 8), (5, -2), (0, 8), (-20, 8)\}$
- $P(M \cup C) = P(10, 8) + P(10, -2) + P(5, 8) + P(5, -2)$
  $+ P(0, 8) + P(-20, 8)$
- $= .20 + .08 + .16 + .26 + .10 + .02$
  $= .82$

Intersection of Two Events

- The <u>intersection</u> of events A and B is the set of all sample points that are in both A and B.
- The intersection of events A and B is denoted by $A \cap B$.



Intersection of Two Events

- Event M = Markley Oil Profitable

  Event C = Collins Mining Profitable

- $M \cap C$ = Markley Oil Profitable
  and Collins Mining Profitable

- $M \cap C = \{(10, 8), (5, 8)\}$

- $P(M \cap C) = P(10, 8) + P(5, 8)$

- $= .20 + .16$

  $= .36$

Addition Law

- The addition law provides a way to compute the probability of event A, or B, or both A and B occurring.

- The law is written as:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Addition Law

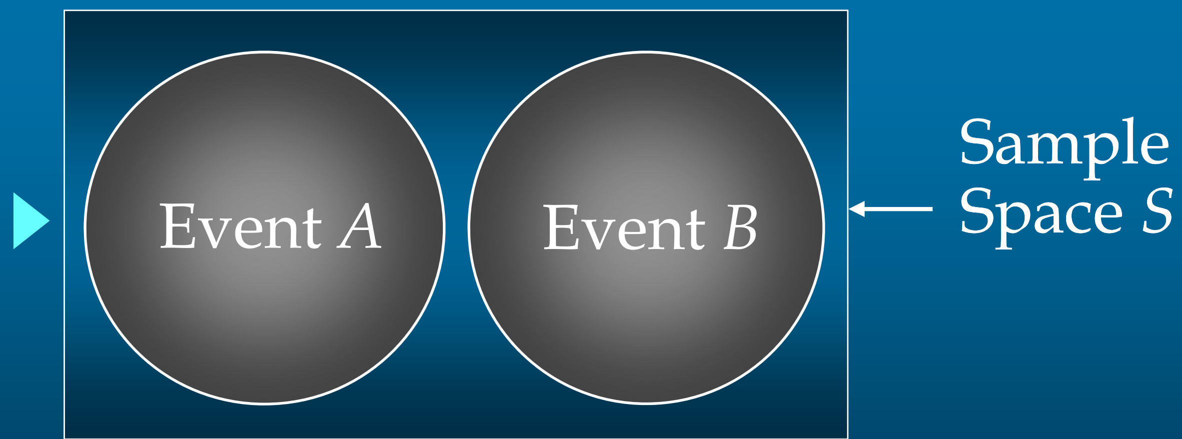
- Event M = Markley Oil Profitable
  Event C = Collins Mining Profitable
- $M \cup C$ = Markley Oil Profitable
  <u>or</u> Collins Mining Profitable
- We know: $P(M) = .70$, $P(C) = .48$, $P(M \cap C) = .36$
- Thus: $P(M \cup C) = P(M) + P(C) - P(M \cap C)$
- $= .70 + .48 - .36$

  $= .82$

(This result is the same as that obtained earlier using the definition of the probability of an event.)

Mutually Exclusive Events

- Two events are said to be <u>mutually exclusive</u> if the events have no sample points in common.
- Two events are mutually exclusive if, when one event occurs, the other cannot occur.



Mutually Exclusive Events

- If events A and B are mutually exclusive, $P(A \cap B) = 0$.
- The addition law for mutually exclusive events is:

$$P(A \cup B) = P(A) + P(B)$$

there's no need to include "$- P(A \cap B)$"

Conditional Probability

- The probability of an event given that another event has occurred is called a conditional probability.
- The conditional probability of A given B is denoted by $P(A \mid B)$.
- A conditional probability is computed as follows :

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Conditional Probability

- Event M = Markley Oil Profitable
  Event C = Collins Mining Profitable
- $P(C \mid M)$ = Collins Mining Profitable <u>given</u> Markley Oil Profitable
- We know: $P(M \cap C) = .36$, $P(M) = .70$
- Thus: $P(C \mid M) = \dfrac{P(C \cap M)}{P(M)} = \dfrac{.36}{.70} = .5143$

Multiplication Law

- The multiplication law provides a way to compute the probability of the intersection of two events.
- The law is written as:

$$P(A \cap B) = P(B)P(A \mid B)$$

Multiplication Law

- Event M = Markley Oil Profitable
  Event C = Collins Mining Profitable
- $M \cap C$ = Markley Oil Profitable
  and Collins Mining Profitable
- We know: $P(M) = .70,\ P(C\,|\,M) = .5143$
- Thus: $P(M \cap C) = P(M)P(M\,|\,C)$
- $= (.70)(.5143)$

  $= .36$

(This result is the same as that obtained earlier using the definition of the probability of an event.)

Independent Events

- If the probability of event A is not changed by the existence of event B, we would say that events A and B are <u>independent</u>.
- Two events A and B are independent if:

$$\boxed{P(A \mid B) = P(A)} \quad \text{or} \quad \boxed{P(B \mid A) = P(B)}$$

Multiplication Law for Independent Events

- The multiplication law also can be used as a test to see if two events are independent.
- The law is written as:

$$P(A \cap B) = P(A)P(B)$$

Multiplication Law for Independent Events

- Event M = Markley Oil Profitable
  Event C = Collins Mining Profitable
- Are events M and C independent?
- Does $P(M \cap C) = P(M)P(C)$?
- We know: $P(M \cap C) = .36$, $P(M) = .70$, $P(C) = .48$
- But: $P(M)P(C) = (.70)(.48) = .34$, not .36
- Hence: M and C are not independent.