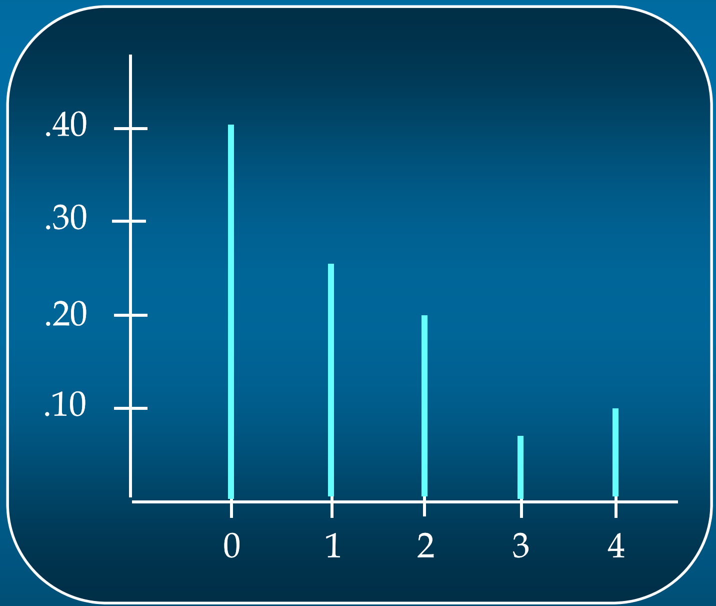
Chapter 5
Discrete Probability Distributions

- Random Variables
- Discrete Probability Distributions
- Expected Value and Variance



Random Variables

- A random variable is a numerical description of the outcome of an experiment.
- A discrete random variable may assume either a finite number of values or an infinite sequence of values.
- A continuous random variable may assume any numerical value in an interval or collection of intervals.

Example: JSL Appliances

- Discrete random variable with a finite number of values

Let x = number of TVs sold at the store in one day, where x can take on 5 values (0, 1, 2, 3, 4)

Example: JSL Appliances

- Discrete random variable with an infinite sequence of values

Let x = number of customers arriving in one day,
where x can take on the values 0, 1, 2, . . .

We can count the customers arriving, but there is no finite upper limit on the number that might arrive.

Random Variables

Question	Random Variable x	Type
Family size	x = Number of dependents reported on tax return	Discrete
Distance from home to store	x = Distance in miles from home to the store site	Continuous
Own dog or cat	x = 1 if own no pet; = 2 if own dog(s) only; = 3 if own cat(s) only; = 4 if own dog(s) and cat(s)	Discrete

Discrete Probability Distributions

- The probability distribution for a random variable describes how probabilities are distributed over the values of the random variable.
- We can describe a discrete probability distribution with a table, graph, or equation.

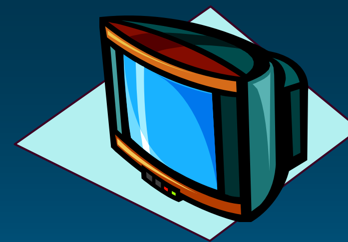
Discrete Probability Distributions

- The probability distribution is defined by a probability function, denoted by $f(x)$, which provides the probability for each value of the random variable.
- The required conditions for a discrete probability function are:

$$f(x) \geq 0$$

$$\Sigma f(x) = 1$$

Discrete Probability Distributions



- Using past data on TV sales, …
- a tabular representation of the probability distribution for TV sales was developed.

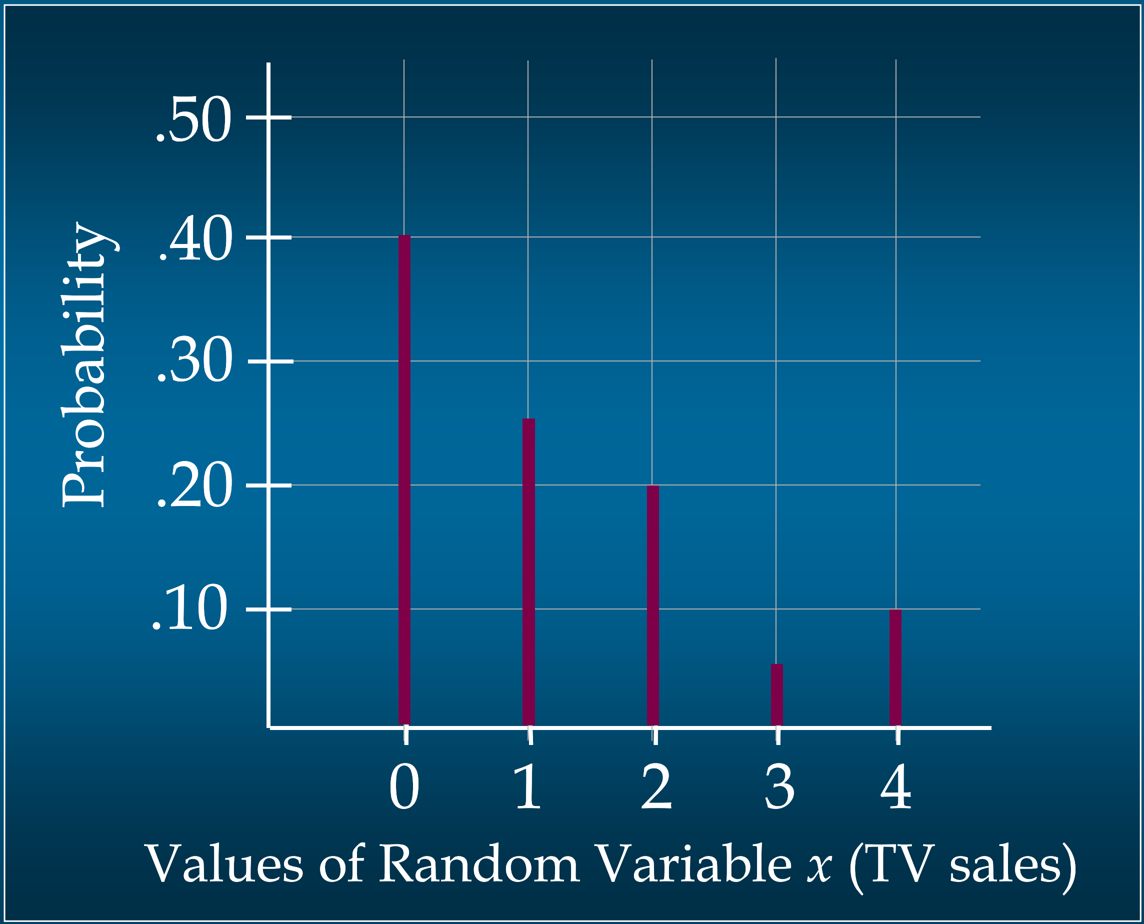
Units Sold	Number of Days
0	80
1	50
2	40
3	10
4	20
	200

x	$f(x)$
0	.40
1	.25
2	.20
3	.05
4	.10
	1.00

80/200

Discrete Probability Distributions

- Graphical Representation of Probability Distribution



Discrete Uniform Probability Distribution

- The discrete uniform probability distribution is the simplest example of a discrete probability distribution given by a formula.

- The discrete uniform probability function is

$$f(x) = 1/n$$

the values of the random variable are equally likely

where:

n = the number of values the random variable may assume

Expected Value and Variance

- The expected value, or mean, of a random variable is a measure of its central location.

$$E(x) = \mu = \Sigma x f(x)$$

- The variance summarizes the variability in the values of a random variable.

$$\text{Var}(x) = \sigma^2 = \Sigma(x - \mu)^2 f(x)$$

- The standard deviation, σ, is defined as the positive square root of the variance.

Expected Value and Variance

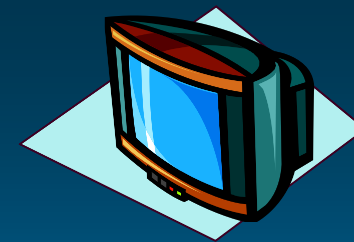
- Expected Value

x	$f(x)$	$xf(x)$
0	.40	.00
1	.25	.25
2	.20	.40
3	.05	.15
4	.10	.40

$E(x) = 1.20$

expected number of TVs sold in a day

Expected Value and Variance



- Variance and Standard Deviation

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x	$x - \mu$	$(x - \mu)^2$	$f(x)$	$(x - \mu)^2 f(x)$
0	-1.2	1.44	.40	.576
1	-0.2	0.04	.25	.010
2	0.8	0.64	.20	.128
3	1.8	3.24	.05	.162
4	2.8	7.84	.10	.784

Variance of daily sales = σ^2 = 1.660

TVs squared

Standard deviation of daily sales = 1.2884 TVs

Binomial Distribution

Four Properties of a Binomial Experiment

1. The experiment consists of a sequence of n identical trials.
2. Two outcomes, success and failure, are possible on each trial.
3. The probability of a success, denoted by p, does not change from trial to trial.

   stationarity assumption

4. The trials are independent.

Binomial Distribution

- Our interest is in the number of successes occurring in the n trials.
- We let x denote the number of successes occurring in the n trials.

Binomial Distribution

- Binomial Probability Function

$$f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)}$$

where:

$f(x)$ = the probability of x successes in n trials

n = the number of trials

p = the probability of success on any one trial

Binomial Distribution

- Binomial Probability Function

$$f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)}$$

$$\frac{n!}{x!(n-x)!}$$

Number of experimental outcomes providing exactly *x* successes in *n* trials

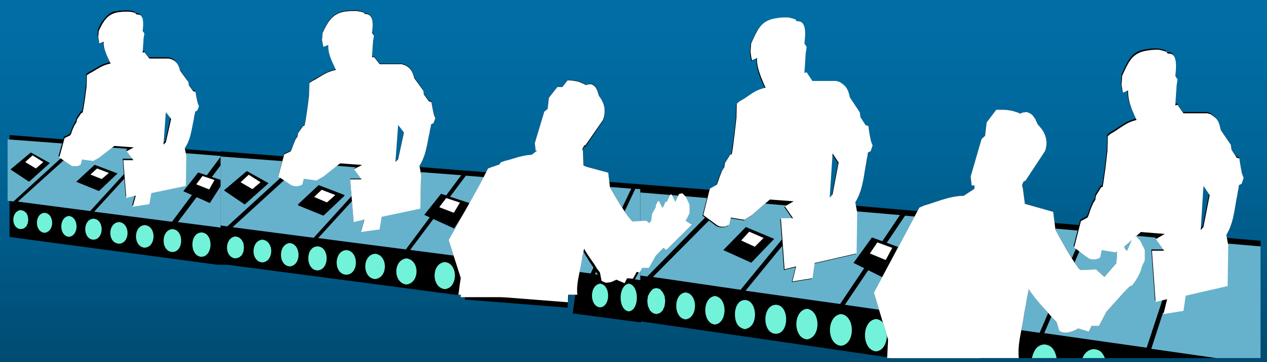
$$p^x (1-p)^{(n-x)}$$

Probability of a particular sequence of trial outcomes with x successes in *n* trials

Binomial Distribution

- Example: Evans Electronics

  Evans is concerned about a low retention rate for employees. In recent years, management has seen a turnover of 10% of the hourly employees annually. Thus, for any hourly employee chosen at random, management estimates a probability of 0.1 that the person will not be with the company next year.



Binomial Distribution

- Using the Binomial Probability Function

Choosing 3 hourly employees at random, what is the probability that 1 of them will leave the company this year?

Let: $p = .10$, $n = 3$, $x = 1$

$$f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)}$$

$$f(1) = \frac{3!}{1!(3-1)!}(0.1)^1(0.9)^2 = 3(.1)(.81) = .243$$

Binomial Distribution

- Using Tables of Binomial Probabilities

		p									
n	***x***	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
3	0	.8574	.7290	.6141	.5120	.4219	.3430	.2746	.2160	.1664	.1250
	1	.1354	.2430	.3251	.3840	.4219	.4410	.4436	.4320	.4084	.3750
	2	.0071	.0270	.0574	.0960	.1406	.1890	.2389	.2880	.3341	.3750
	3	.0001	.0010	.0034	.0080	.0156	.0270	.0429	.0640	.0911	.1250

Binomial Distribution

- Expected Value

$$E(x) = \mu = np$$

- Variance

$$\mathrm{Var}(x) = \sigma^2 = np(1 - p)$$

- Standard Deviation

$$\sigma = \sqrt{np(1 - p)}$$

Binomial Distribution

- Expected Value

$$E(x) = \mu = 3(.1) = .3 \text{ employees out of } 3$$

- Variance

$$\text{Var}(x) = \sigma^2 = 3(.1)(.9) = .27$$

- Standard Deviation

$$\sigma = \sqrt{3(.1)(.9)} = .52 \text{ employees}$$