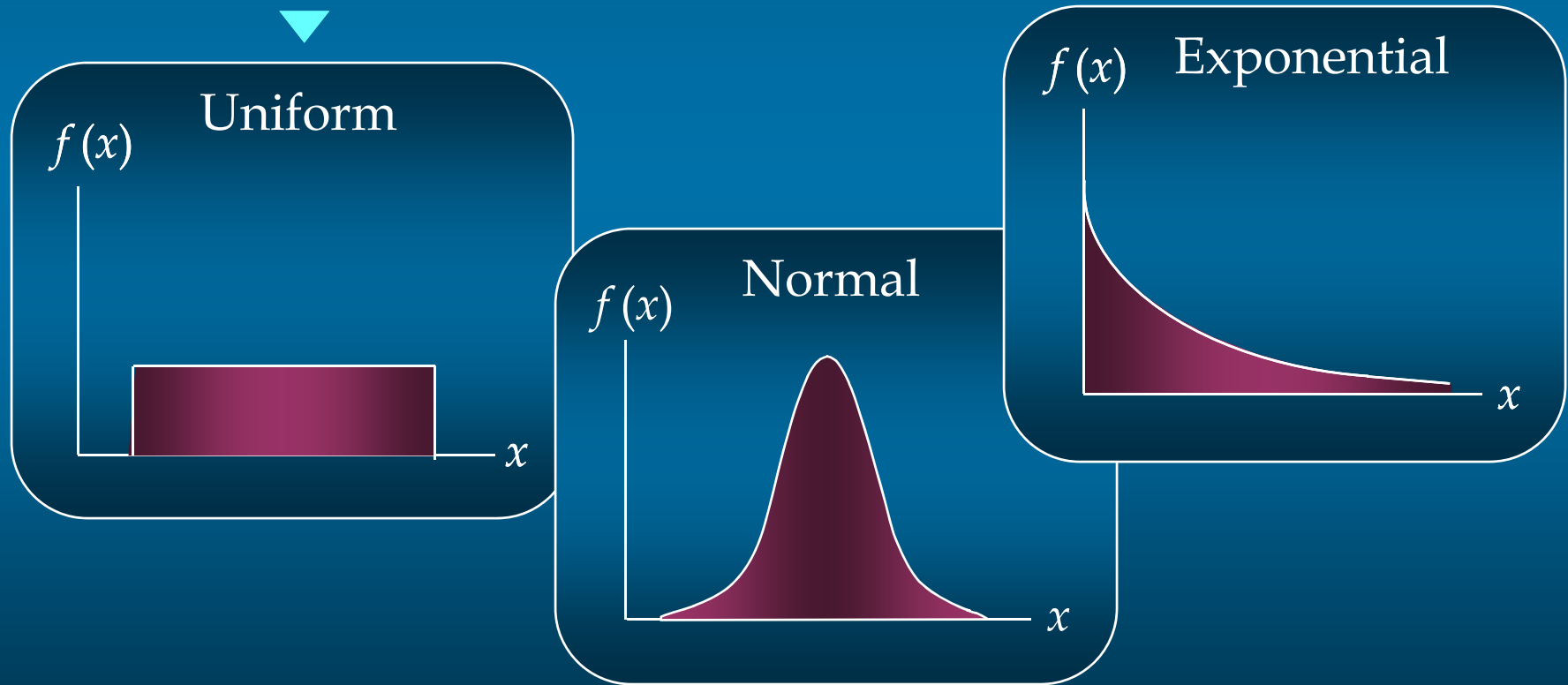


# Chapter 6

## Continuous Probability Distributions

- q Uniform Probability Distribution
- q Normal Probability Distribution
- q Exponential Probability Distribution

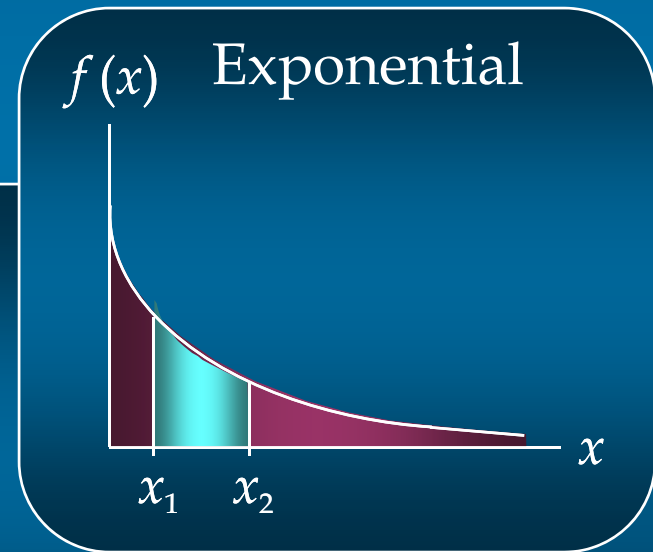
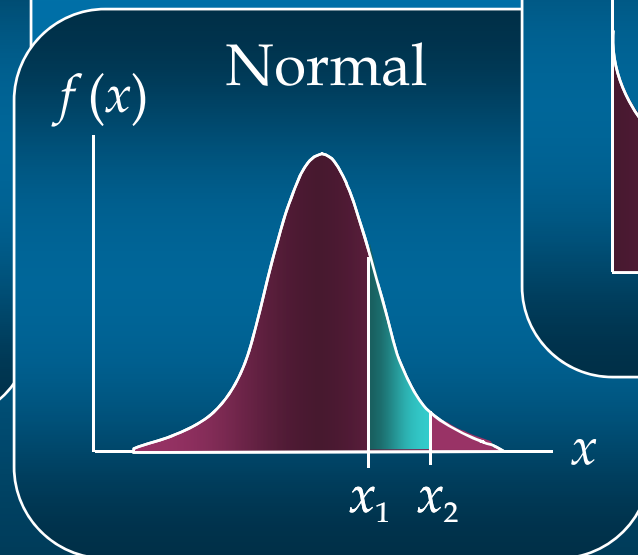
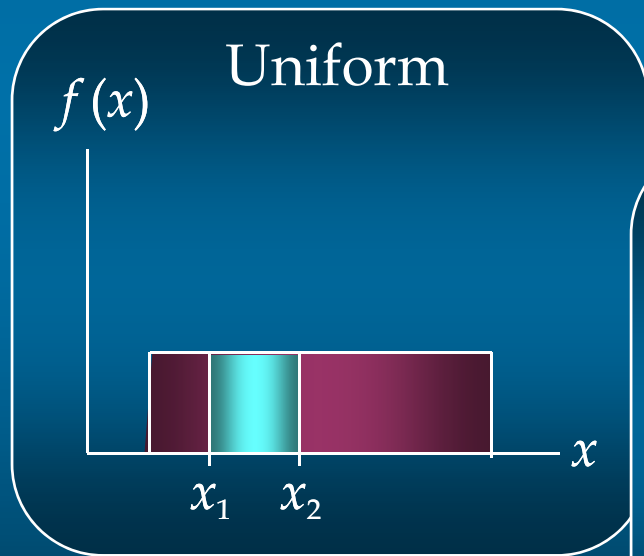


# Continuous Probability Distributions

- ▶  $q$  A continuous random variable can assume any value in an interval on the real line or in a collection of intervals.
- ▶  $q$  It is not possible to talk about the probability of the random variable assuming a particular value.
- ▶  $q$  Instead, we talk about the probability of the random variable assuming a value within a given interval.

# Continuous Probability Distributions

- The probability of the random variable assuming a value within some given interval from  $x_1$  to  $x_2$  is defined to be the area under the graph of the probability density function between  $x_1$  and  $x_2$ .



# Normal Probability Distribution

- q The normal probability distribution is the most important distribution for describing a continuous random variable.
- q It is widely used in statistical inference.

# Normal Probability Distribution

- q It has been used in a wide variety of applications:

Heights  
of people



Scientific  
measurements



# Normal Probability Distribution

- q It has been used in a wide variety of applications:

Test  
scores



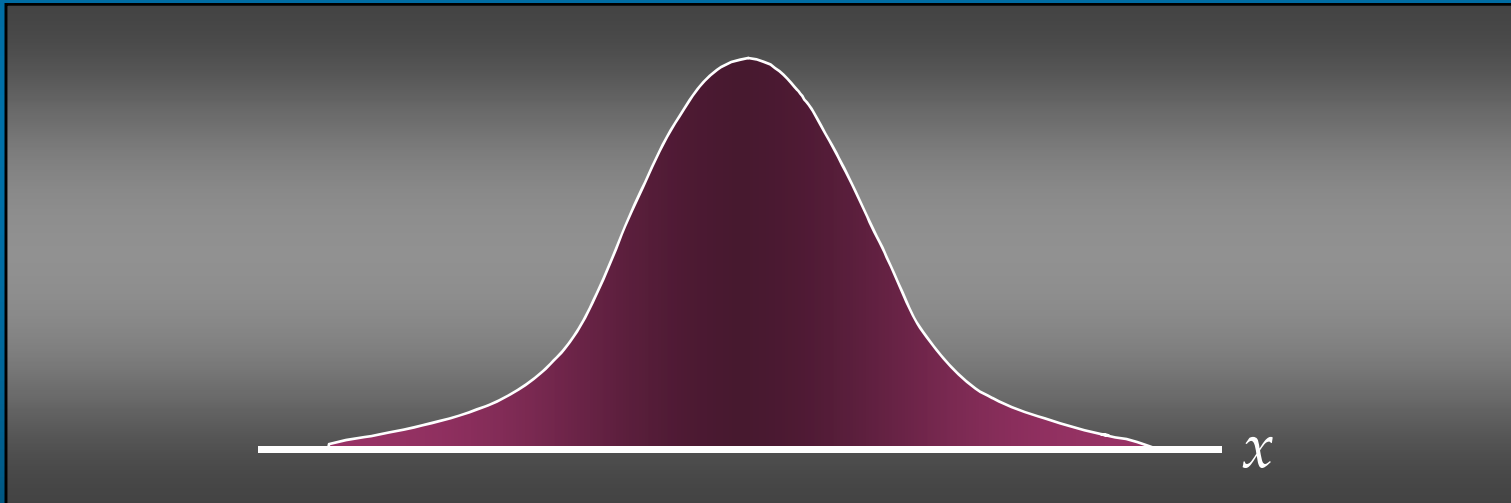
Amounts  
of rainfall



# Normal Probability Distribution

## q Characteristics

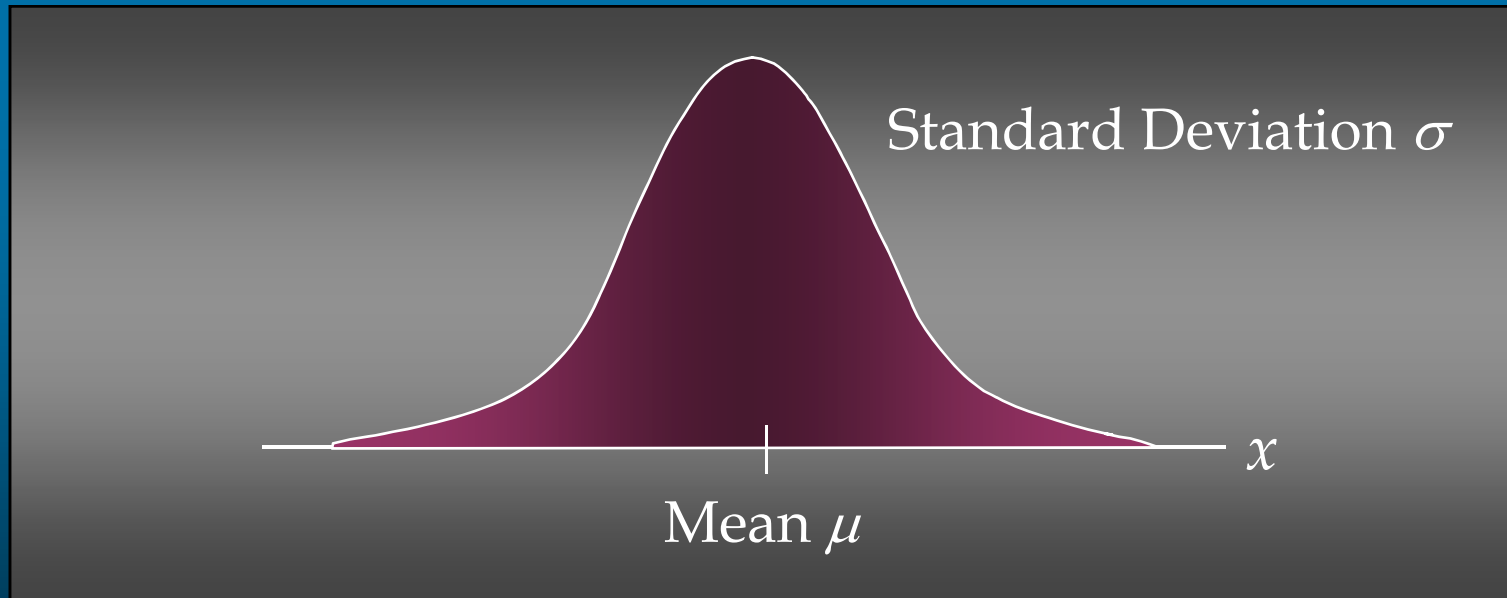
- ▶ The distribution is symmetric; its skewness measure is zero.



# Normal Probability Distribution

## q Characteristics

- ▶ The entire family of normal probability distributions is defined by its mean  $\mu$  and its standard deviation  $\sigma$ .

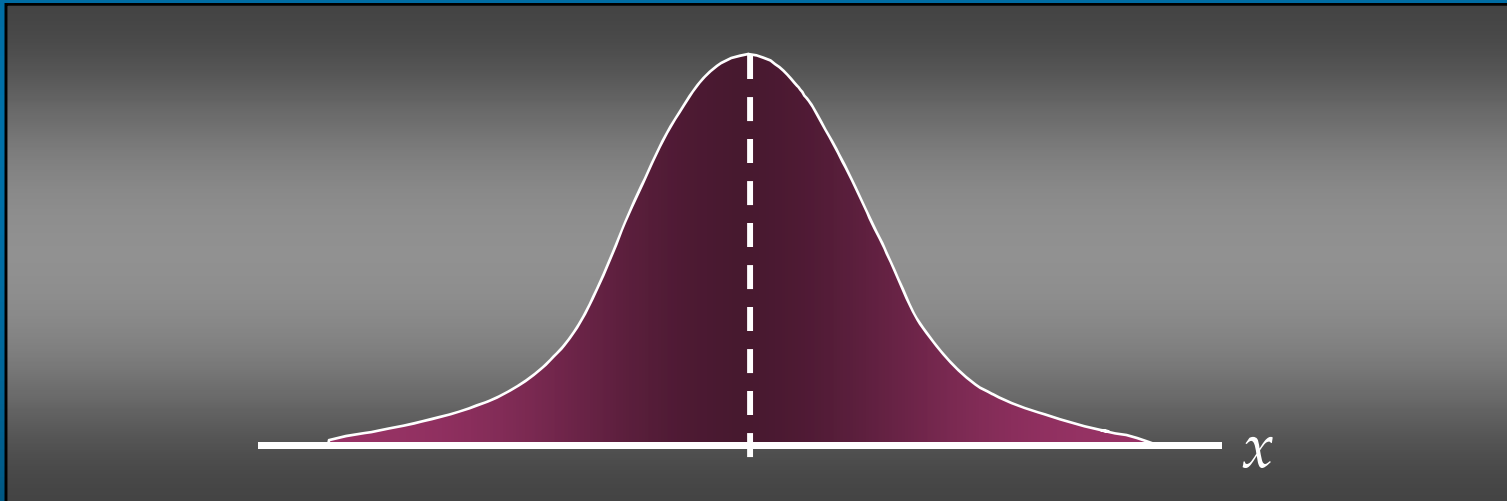




# Normal Probability Distribution

## q Characteristics

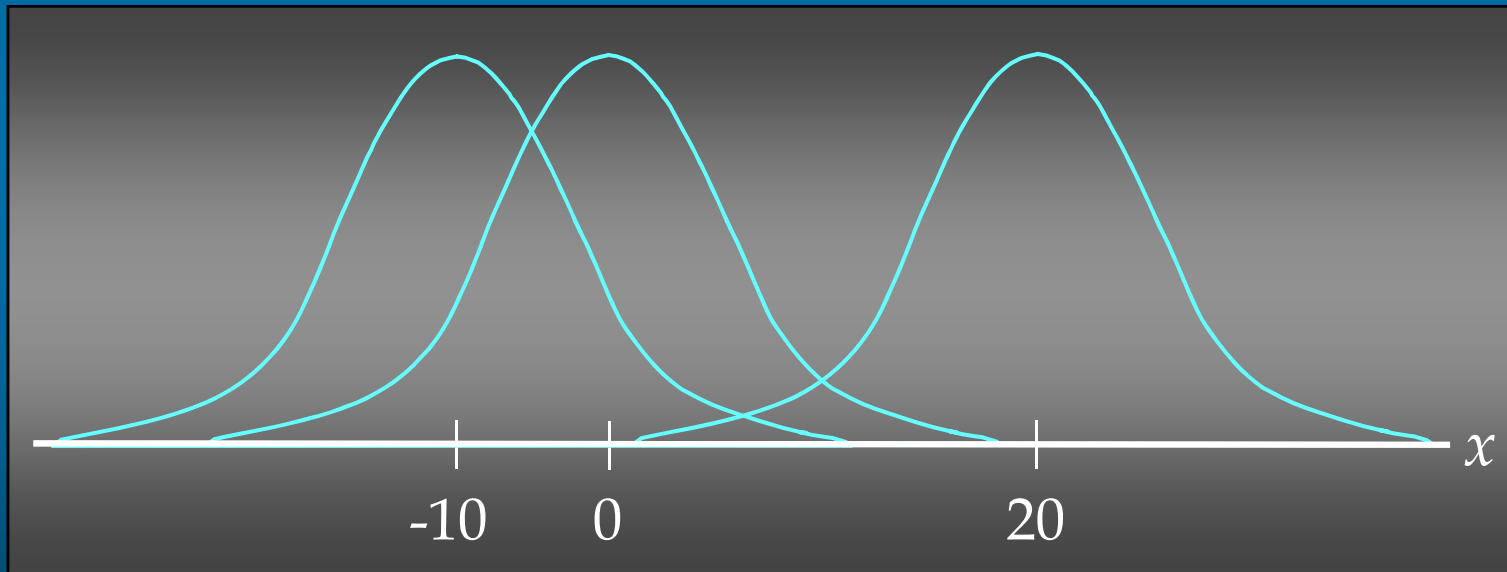
- ▶ The highest point on the normal curve is at the mean, which is also the median and mode.



# Normal Probability Distribution

## □ Characteristics

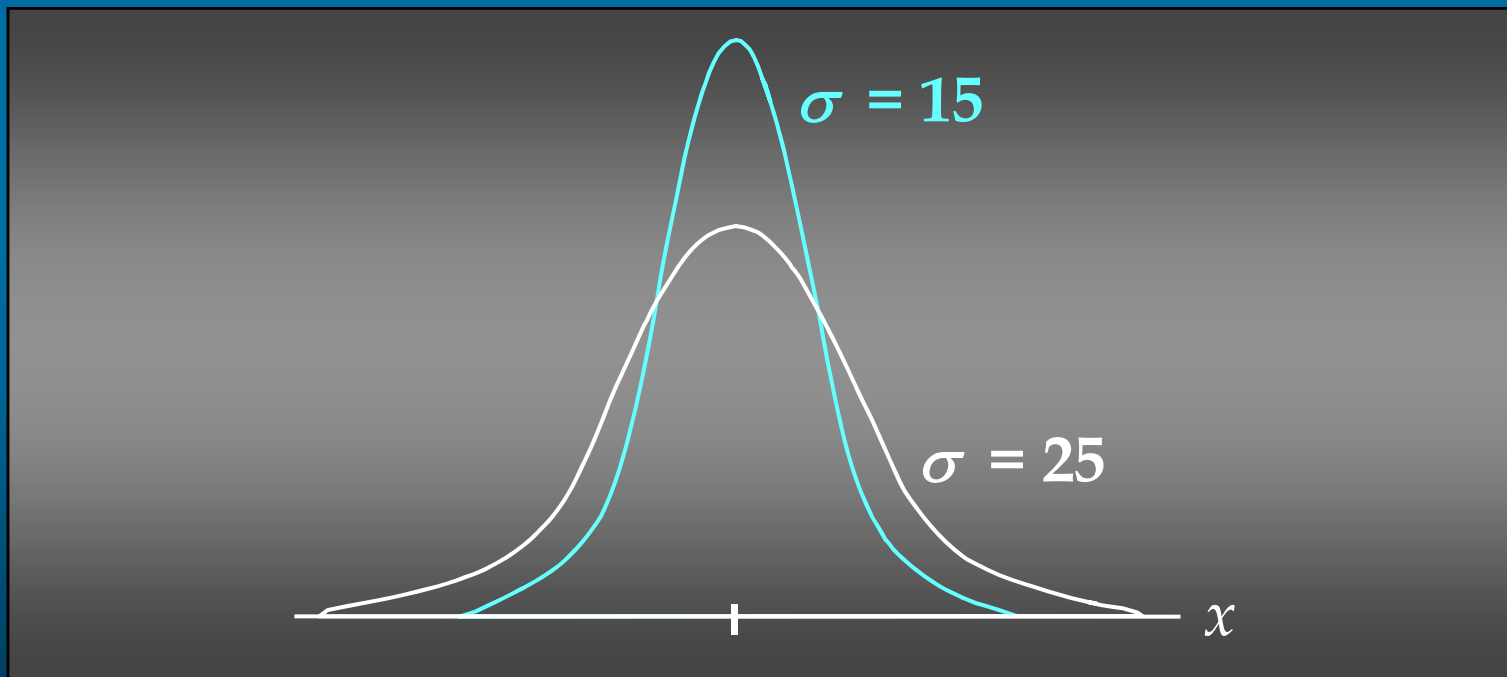
- ▶ The mean can be any numerical value: negative, zero, or positive.



# Normal Probability Distribution

## q Characteristics

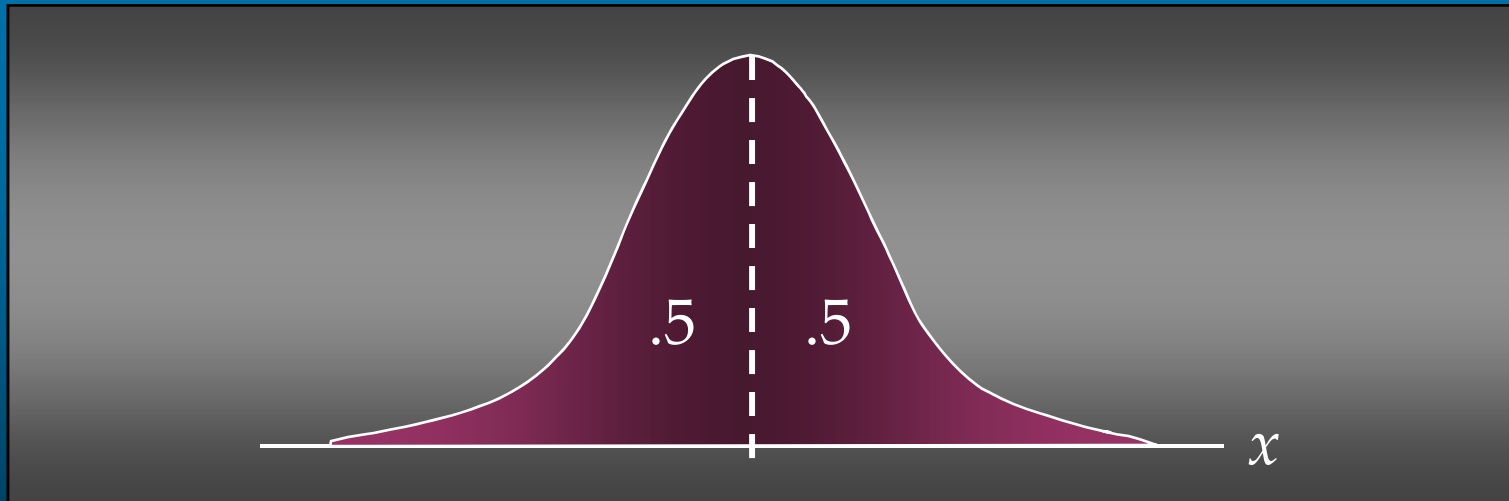
- ▶ The standard deviation determines the width of the curve: larger values result in wider, flatter curves.



# Normal Probability Distribution

## q Characteristics

Probabilities for the normal random variable are given by areas under the curve. The total area under the curve is 1 (.5 to the left of the mean and .5 to the right).



# Normal Probability Distribution

## q Characteristics

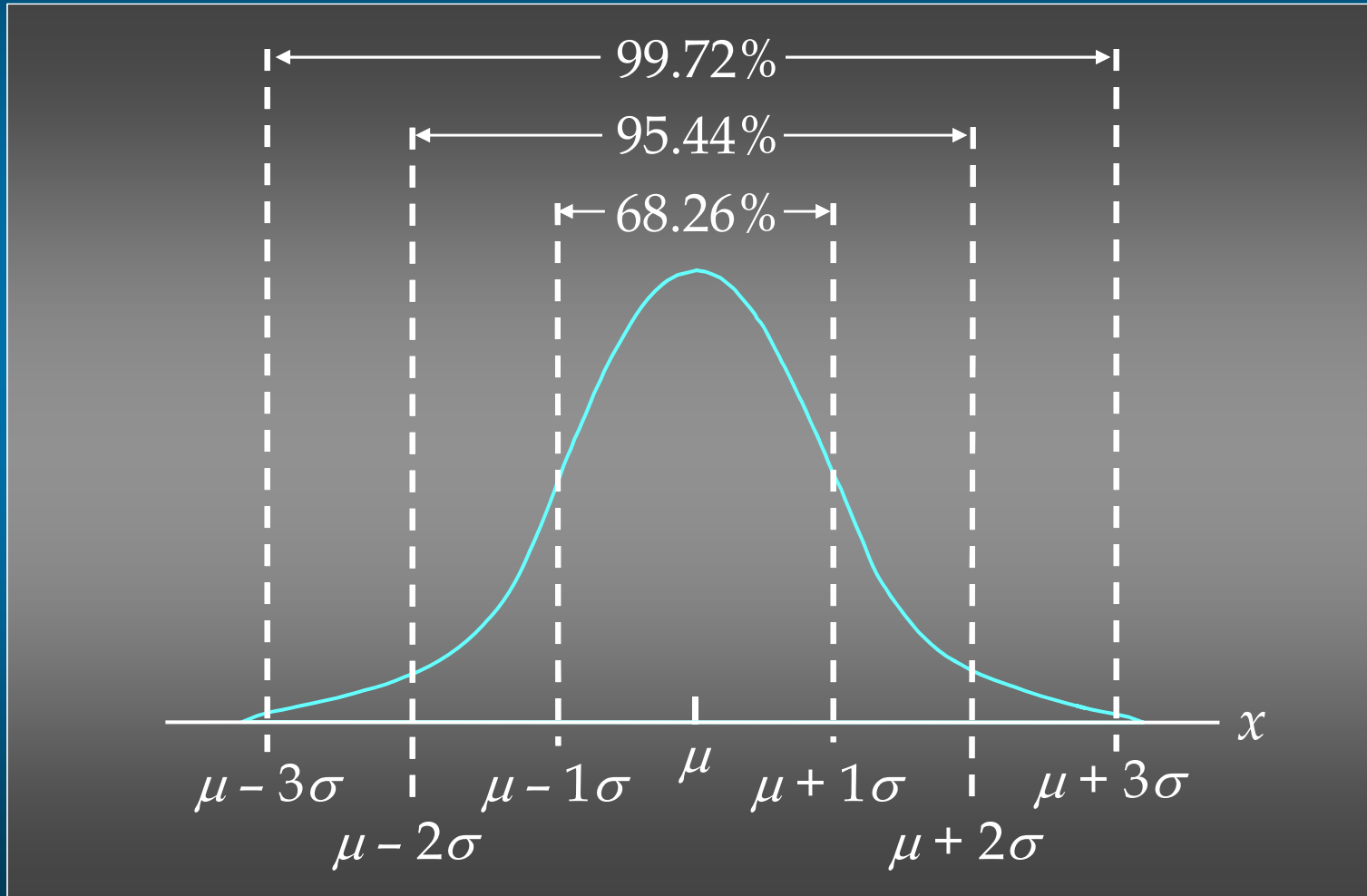
▶ 68.26% of values of a normal random variable are within  $\pm 1$  standard deviation of its mean.

▶ 95.44% of values of a normal random variable are within  $\pm 2$  standard deviations of its mean.

▶ 99.72% of values of a normal random variable are within  $\pm 3$  standard deviations of its mean.

# Normal Probability Distribution

## Characteristics

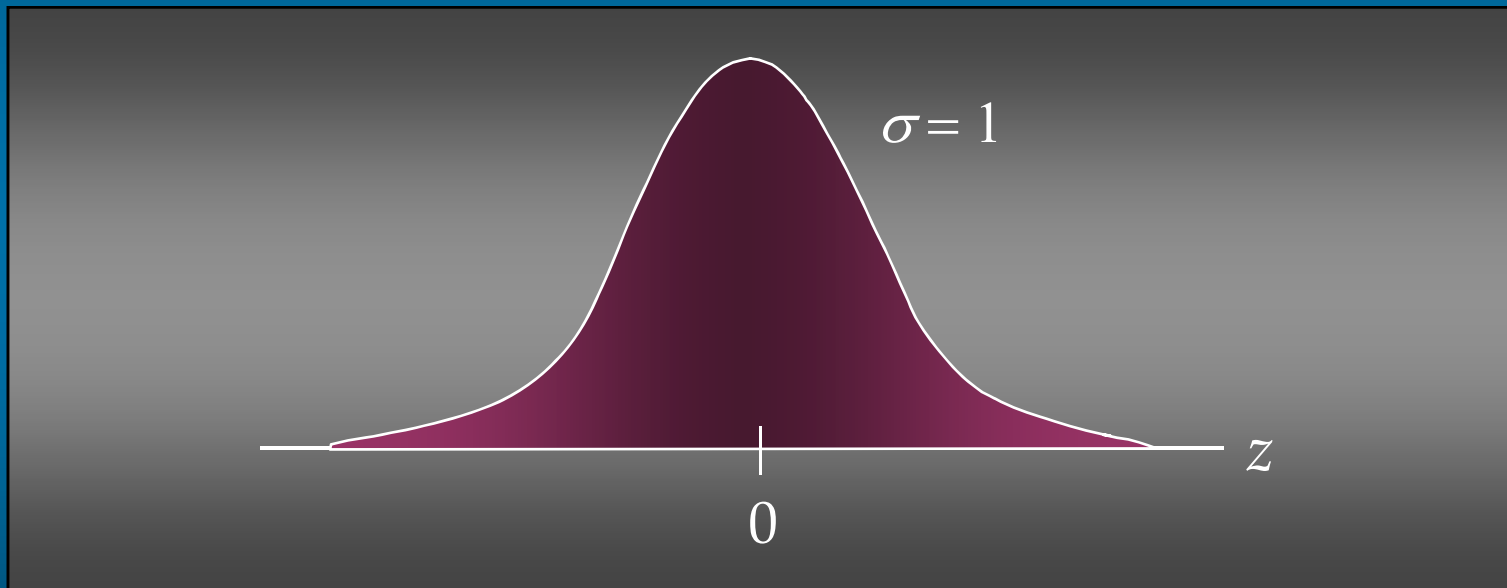


# Standard Normal Probability Distribution

- ▶ A random variable having a normal distribution with a mean of 0 and a standard deviation of 1 is said to have a standard normal probability distribution.

# Standard Normal Probability Distribution


▶ The letter  $z$  is used to designate the standard normal random variable.





# Standard Normal Probability Distribution

## q Converting to the Standard Normal Distribution


$$z = \frac{x - \mu}{\sigma}$$

We can think of  $z$  as a measure of the number of standard deviations  $x$  is from  $\mu$ .

# Standard Normal Probability Distribution

## q Example: Pep Zone

- ▶ Pep Zone sells auto parts and supplies including a popular multi-grade motor oil. When the stock of this oil drops to 20 gallons, a replenishment order is placed.

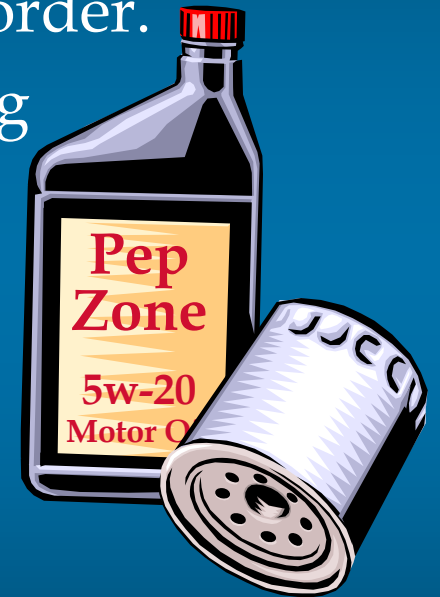


# Standard Normal Probability Distribution

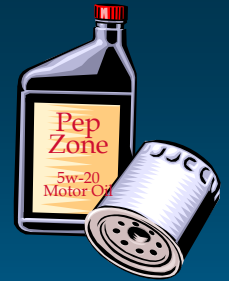
## □ Example: Pep Zone

- ▶ The store manager is concerned that sales are being lost due to stockouts while waiting for an order. It has been determined that demand during replenishment lead-time is normally distributed with a mean of 15 gallons and a standard deviation of 6 gallons.

The manager would like to know the probability of a stockout,  $P(x > 20)$ .



# Standard Normal Probability Distribution



## q Solving for the Stockout Probability

- ▶ Step 1: Convert  $x$  to the standard normal distribution.

$$\begin{aligned}z &= (x - \mu) / \sigma \\ &= (20 - 15) / 6 \\ &= .83\end{aligned}$$

- ▶ Step 2: Find the area under the standard normal curve to the left of  $z = .83$ .

see next slide

# Standard Normal Probability Distribution

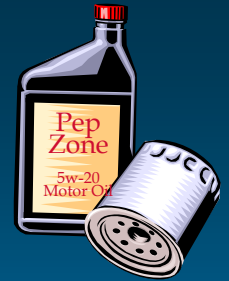


q Probability Table for  
the Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.	.	.	.	.	.	.	.	.	.	.
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
.	.	.	.	.	.	.	.	.	.	.

$$P(0 \leq z \leq .83)$$

# Standard Normal Probability Distribution



## q Solving for the Stockout Probability

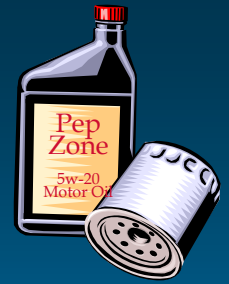
- ▶ Step 3: Compute the area under the standard normal curve to the right of  $z = .83$ .

$$\begin{aligned} P(z > .83) &= .5 - P(z \leq .83) \\ &= .5 - .2967 \\ &= .2033 \end{aligned}$$

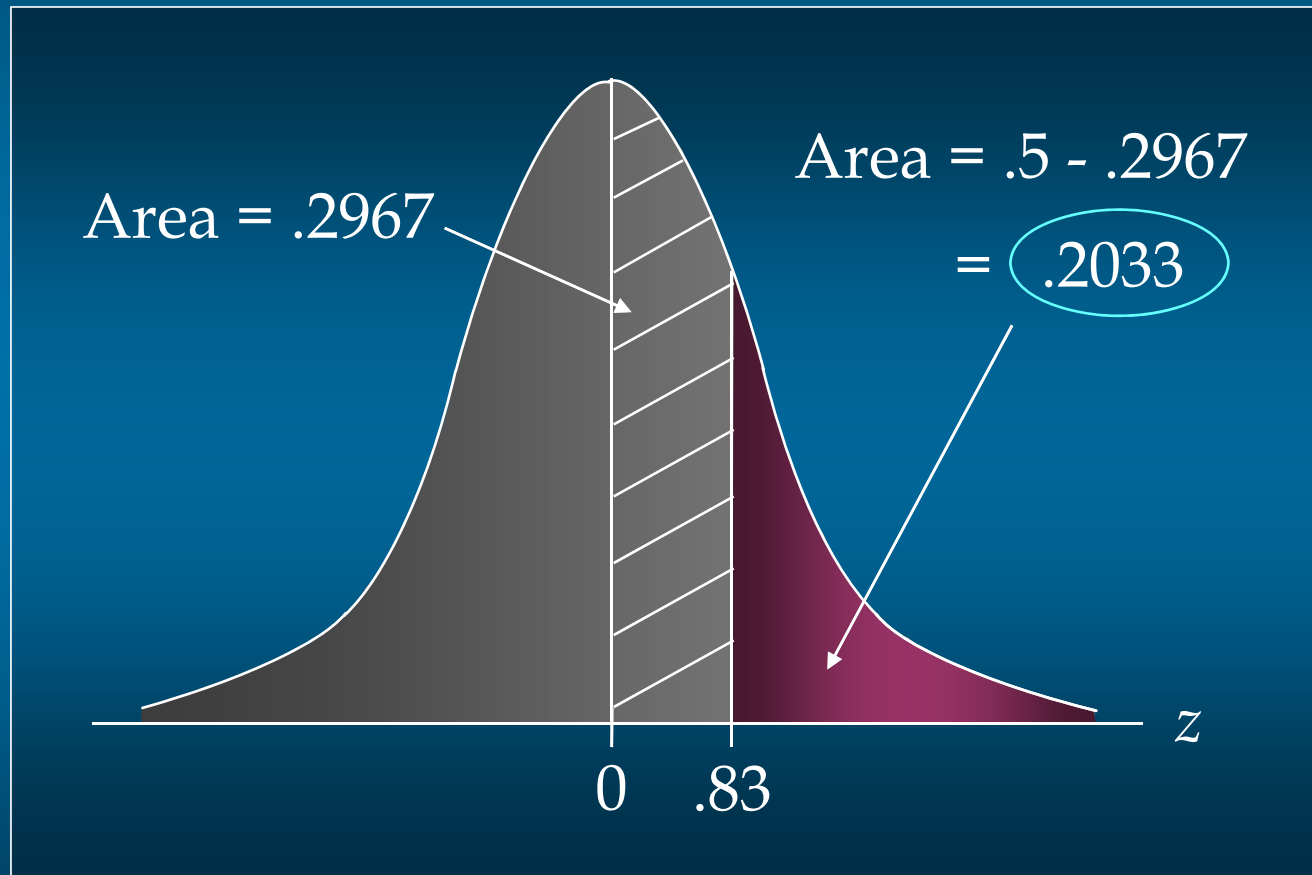
Probability  
of a stockout

$P(x > 20)$

# Standard Normal Probability Distribution



q Solving for the Stockout Probability



# Standard Normal Probability Distribution



## q Standard Normal Probability Distribution

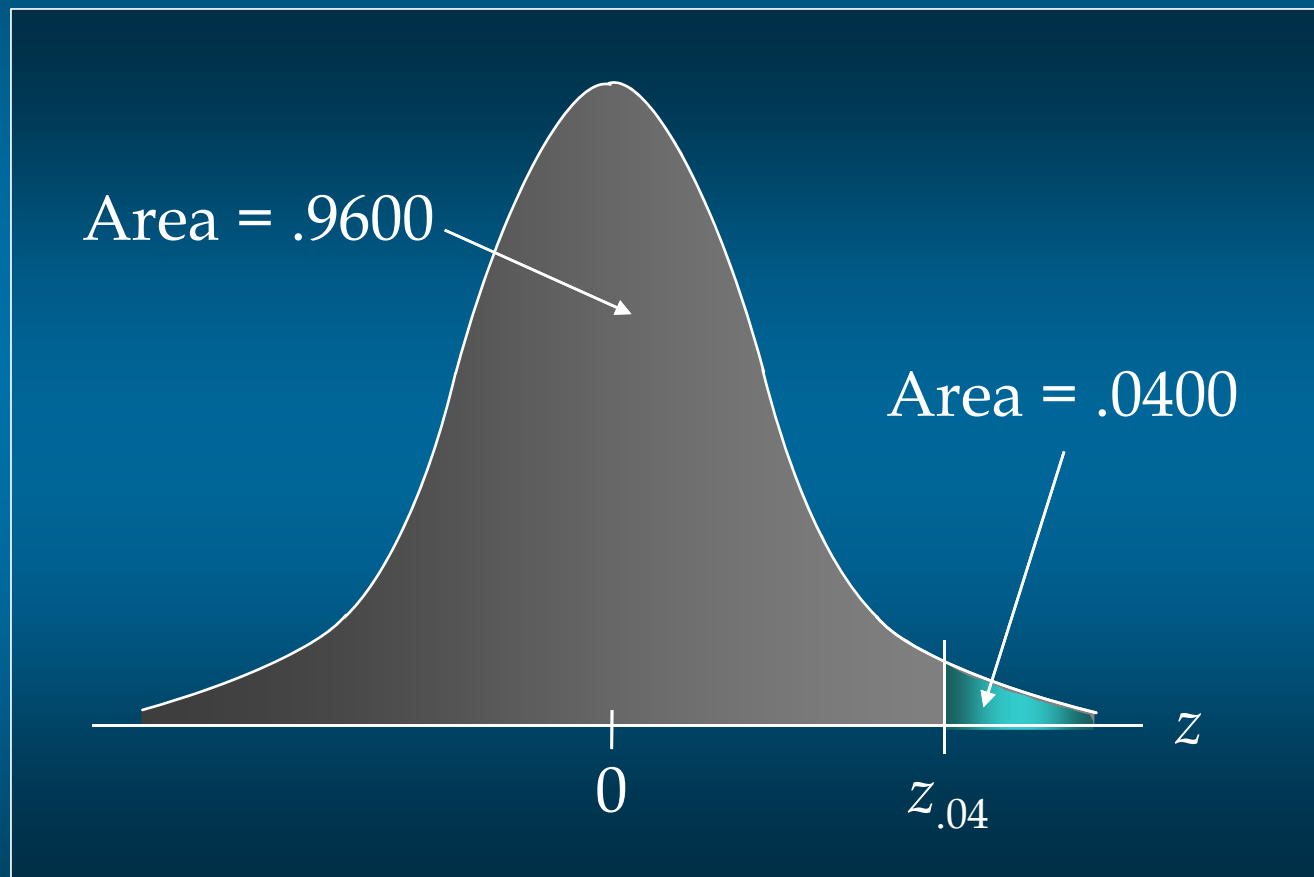
If the manager of Pep Zone wants the probability of a stockout to be no more than .04, what should the reorder point be? Assuming that it has been determined that demand during replenishment lead-time is normally distributed with a mean of 15 gallons and a standard deviation of 6 gallons.



# Standard Normal Probability Distribution



## q Solving for the Reorder Point



# Standard Normal Probability Distribution



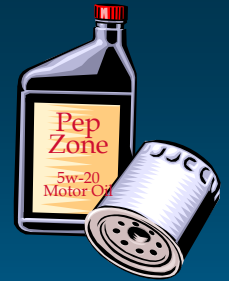
## q Solving for the Reorder Point

▶ Step 1: Find the z-value that cuts off an area of .04 in the right tail of the standard normal distribution.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.	.	.	.	.	.	.	.	.	.	.
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
.	.	.	.	.	.	.	.	.	.	.

We look up the complement of the tail area ( $1 - .04 = .96$ )

# Standard Normal Probability Distribution



## □ Solving for the Reorder Point

▶ Step 2: Convert  $z_{.04}$  to the corresponding value of  $x$ .

$$\begin{aligned}x &= \mu + z_{.05}\sigma \\ &= 15 + 1.75(6) \\ &= 25.5 \text{ or } 26\end{aligned}$$

A reorder point of 26 gallons will place the probability of a stockout during leadtime at (slightly less than) .04.

# Standard Normal Probability Distribution



## □ Solving for the Reorder Point

By raising the reorder point from 20 gallons to 26 gallons on hand, the probability of a stockout decreases from about .20 to .04.

This is a significant decrease in the chance that Pep Zone will be out of stock and unable to meet a customer's desire to make a purchase.