

# Chapter 9

## Hypothesis Testing

- ▶ ■ Developing Null and Alternative Hypotheses
- ▶ ■ Type I and Type II Errors
- ▶ ■ Population Mean:  $\sigma$  Known

# Developing Null and Alternative Hypotheses

- ▶ ■ Hypothesis testing can be used to determine whether a statement about the value of a population parameter should or should not be rejected.
- ▶ ■ The null hypothesis, denoted by  $H_0$ , is a tentative assumption about a population parameter.
- ▶ ■ The alternative hypothesis, denoted by  $H_a$ , is the opposite of what is stated in the null hypothesis.
- ▶ ■ The alternative hypothesis is what the test is attempting to establish.

# Developing Null and Alternative Hypotheses

## q Testing Research Hypotheses

- ▶ • The research hypothesis should be expressed as the alternative hypothesis.
- ▶ • The conclusion that the research hypothesis is true comes from sample data that contradict the null hypothesis.

# Developing Null and Alternative Hypotheses

## □ Testing the Validity of a Claim

- ▶ • Manufacturers' claims are usually given the benefit of the doubt and stated as the null hypothesis.
- ▶ • The conclusion that the claim is false comes from sample data that contradict the null hypothesis.

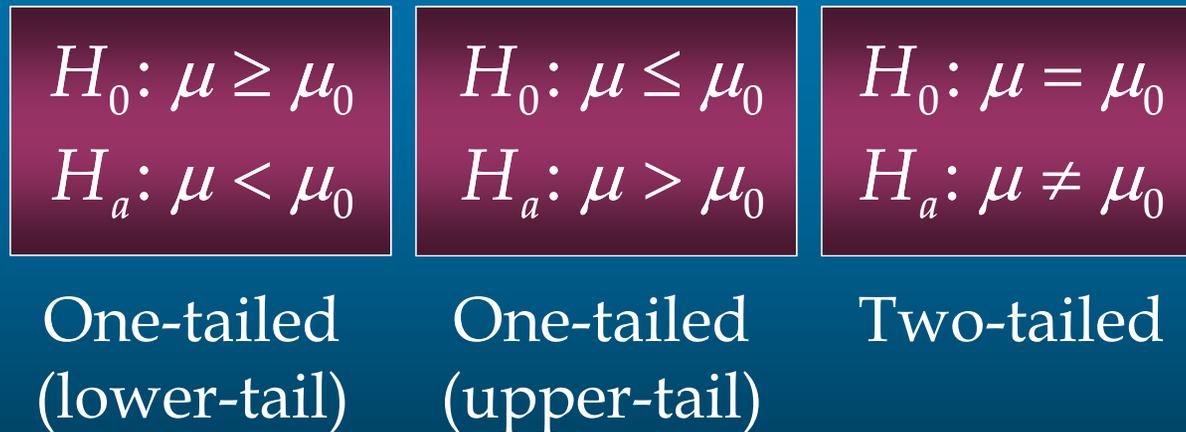
# Developing Null and Alternative Hypotheses

## q Testing in Decision-Making Situations

- ▶ • A decision maker might have to choose between two courses of action, one associated with the null hypothesis and another associated with the alternative hypothesis.
- ▶ • Example: Accepting a shipment of goods from a supplier or returning the shipment of goods to the supplier

# Summary of Forms for Null and Alternative Hypotheses about a Population Mean

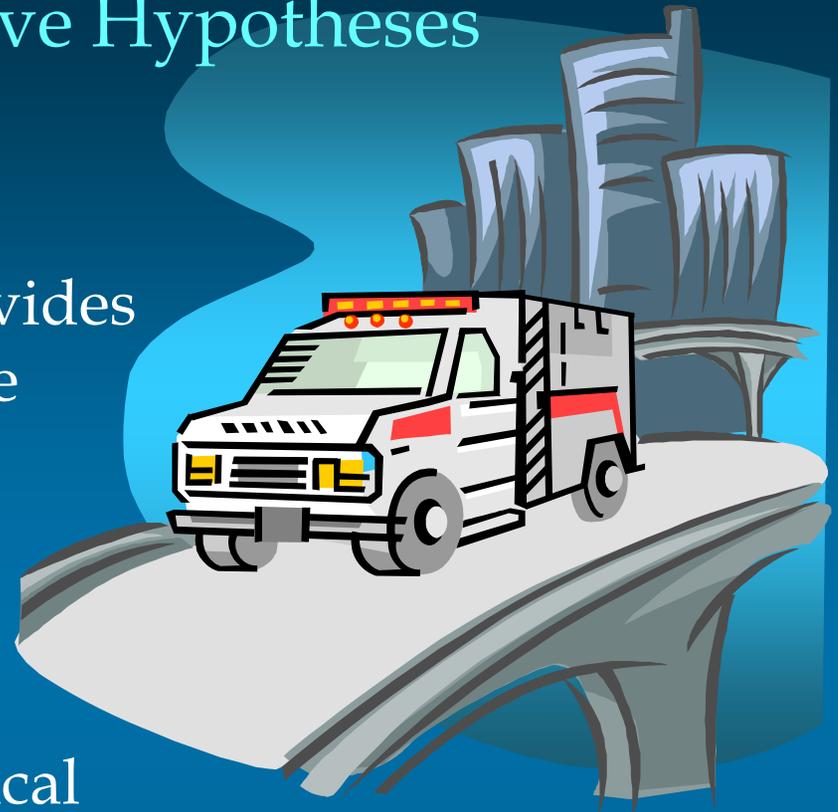
- ▶  $\alpha$  The equality part of the hypotheses always appears in the null hypothesis.
- ▶ ■ In general, a hypothesis test about the value of a population mean  $\mu$  must take one of the following three forms (where  $\mu_0$  is the hypothesized value of the population mean).



# Null and Alternative Hypotheses

## Example: Metro EMS

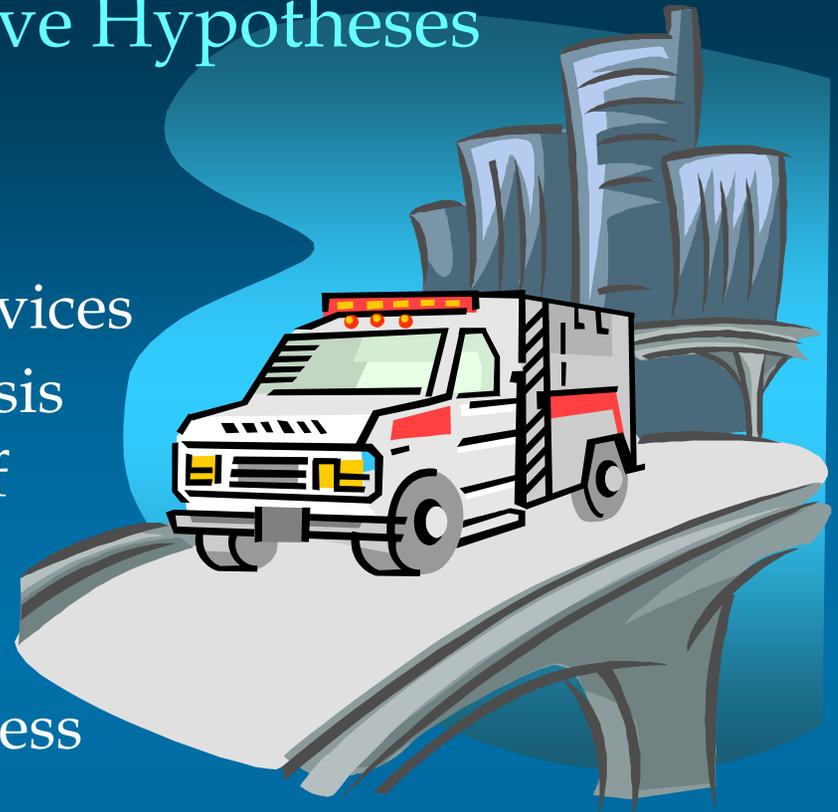
- ▶ A major west coast city provides one of the most comprehensive emergency medical services in the world.
- ▶ Operating in a multiple hospital system with approximately 20 mobile medical units, the service goal is to respond to medical emergencies with a mean time of 12 minutes or less.



# Null and Alternative Hypotheses

## □ Example: Metro EMS

- ▶ The director of medical services wants to formulate a hypothesis test that could use a sample of emergency response times to determine whether or not the service goal of 12 minutes or less is being achieved.



# Null and Alternative Hypotheses



- ▶  $H_0: \mu \leq 12$  The emergency service is meeting the response goal; no follow-up action is necessary.
- ▶  $H_a: \mu > 12$  The emergency service is not meeting the response goal; appropriate follow-up action is necessary.

where:  $\mu$  = mean response time for the population of medical emergency requests

# Type I Error

- ▶ ■ Because hypothesis tests are based on sample data, we must allow for the possibility of errors.
- ▶ ◻ A Type I error is rejecting  $H_0$  when it is true.
- ▶ ◻ The probability of making a Type I error when the null hypothesis is true as an equality is called the level of significance.
- ▶ ◻ Applications of hypothesis testing that only control the Type I error are often called significance tests.

## Type II Error

- ▶  $\alpha$  A Type II error is accepting  $H_0$  when it is false.
- ▶  $\alpha$  It is difficult to control for the probability of making a Type II error.
- ▶  $\alpha$  Statisticians avoid the risk of making a Type II error by using “do not reject  $H_0$ ” and not “accept  $H_0$ ”.

# Type I and Type II Errors



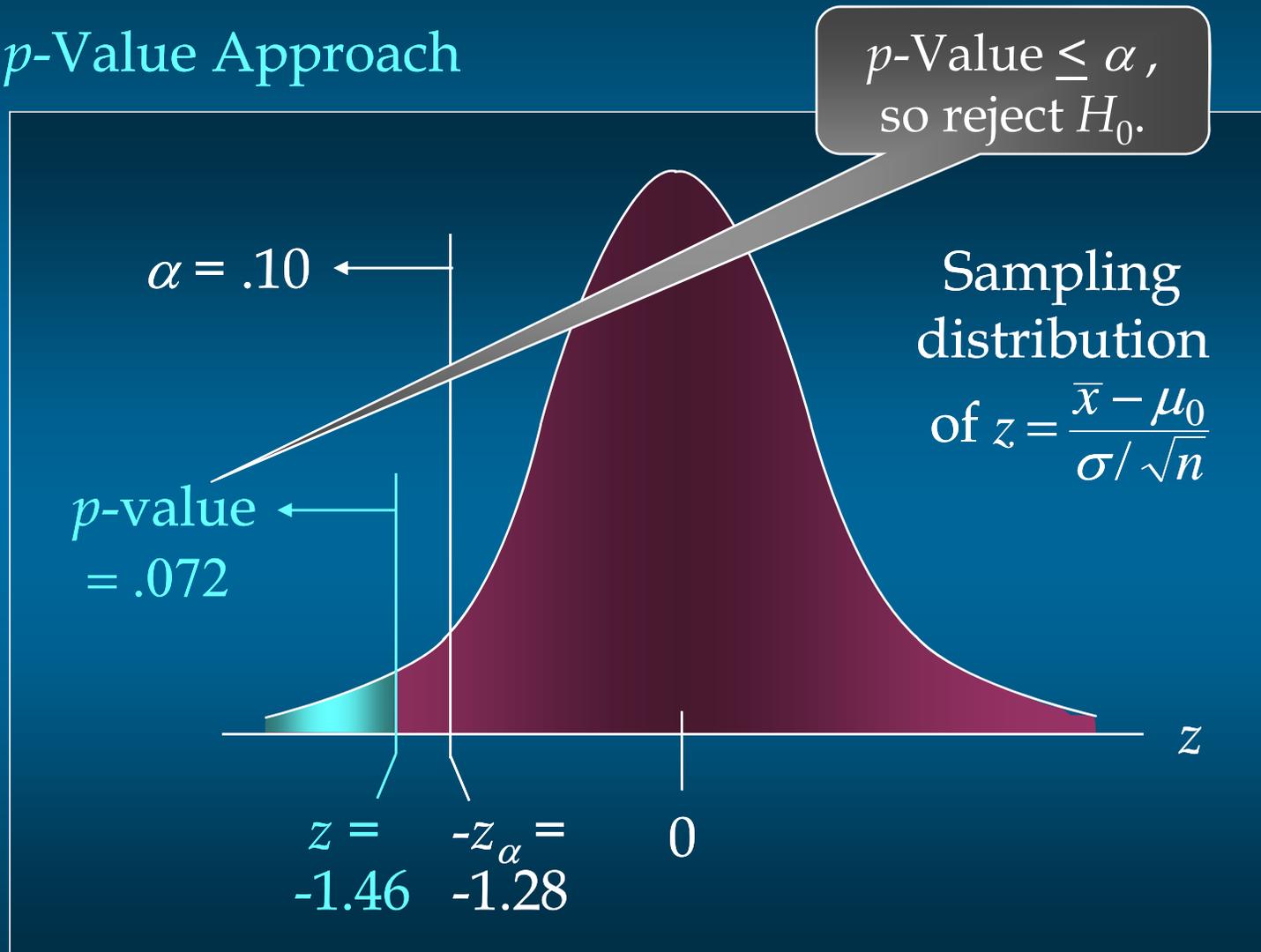
		Population Condition	
		$H_0$ True ( $\mu \leq 12$ )	$H_0$ False ( $\mu > 12$ )
Conclusion	Accept $H_0$ (Conclude $\mu \leq 12$ )	Correct Decision	Type II Error
	Reject $H_0$ (Conclude $\mu > 12$ )	Type I Error	Correct Decision

## $p$ -Value Approach to One-Tailed Hypothesis Testing

- ▶ ■ The  $p$ -value is the probability, computed using the test statistic, that measures the support (or lack of support) provided by the sample for the null hypothesis.
- ▶ ■ If the  $p$ -value is less than or equal to the level of significance  $\alpha$ , the value of the test statistic is in the rejection region.
- ▶ ■ Reject  $H_0$  if the  $p$ -value  $\leq \alpha$ .

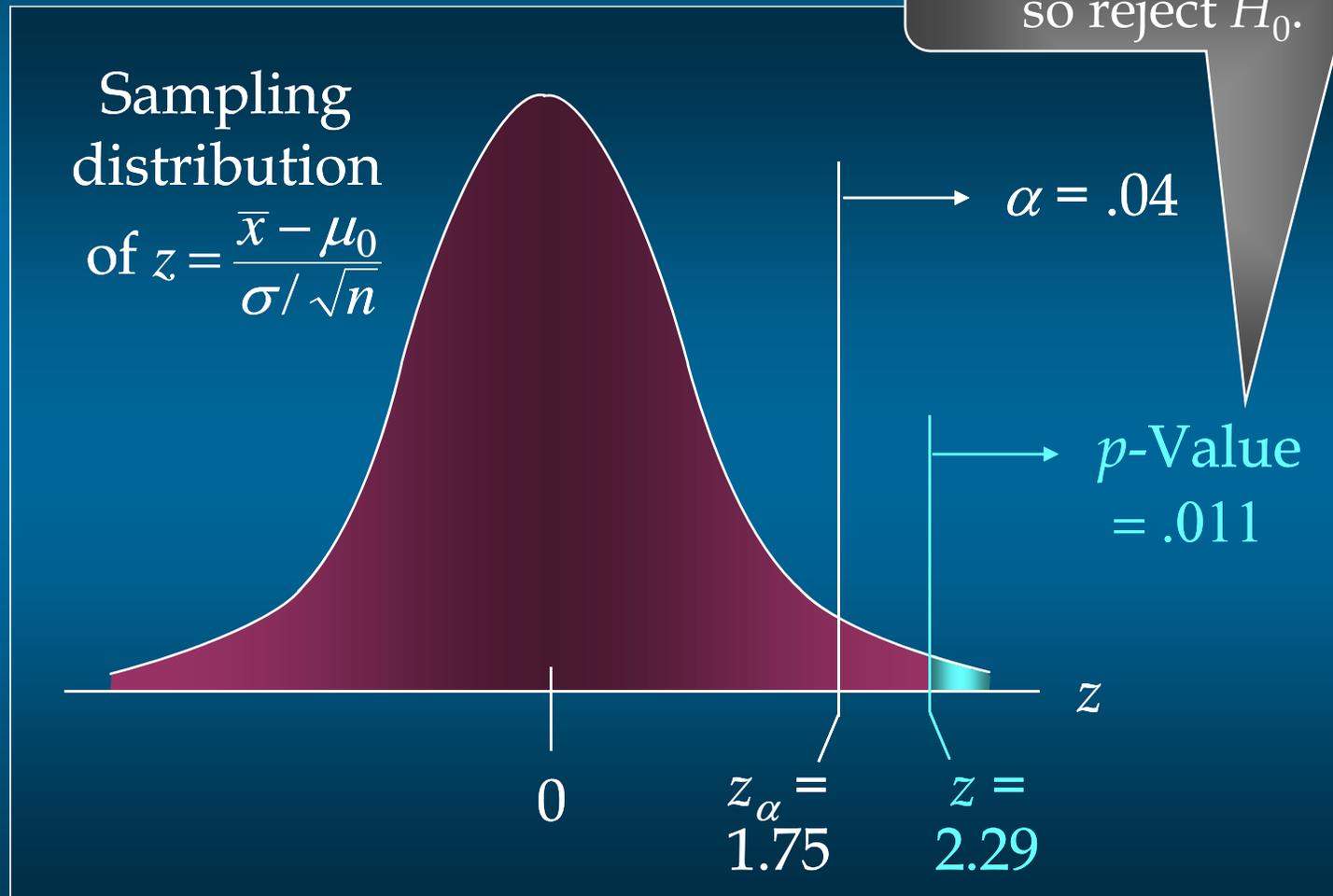
# Lower-Tailed Test About a Population Mean: $\sigma$ Known

## q $p$ -Value Approach



# Upper-Tailed Test About a Population Mean: $\sigma$ Known

## q $p$ -Value Approach

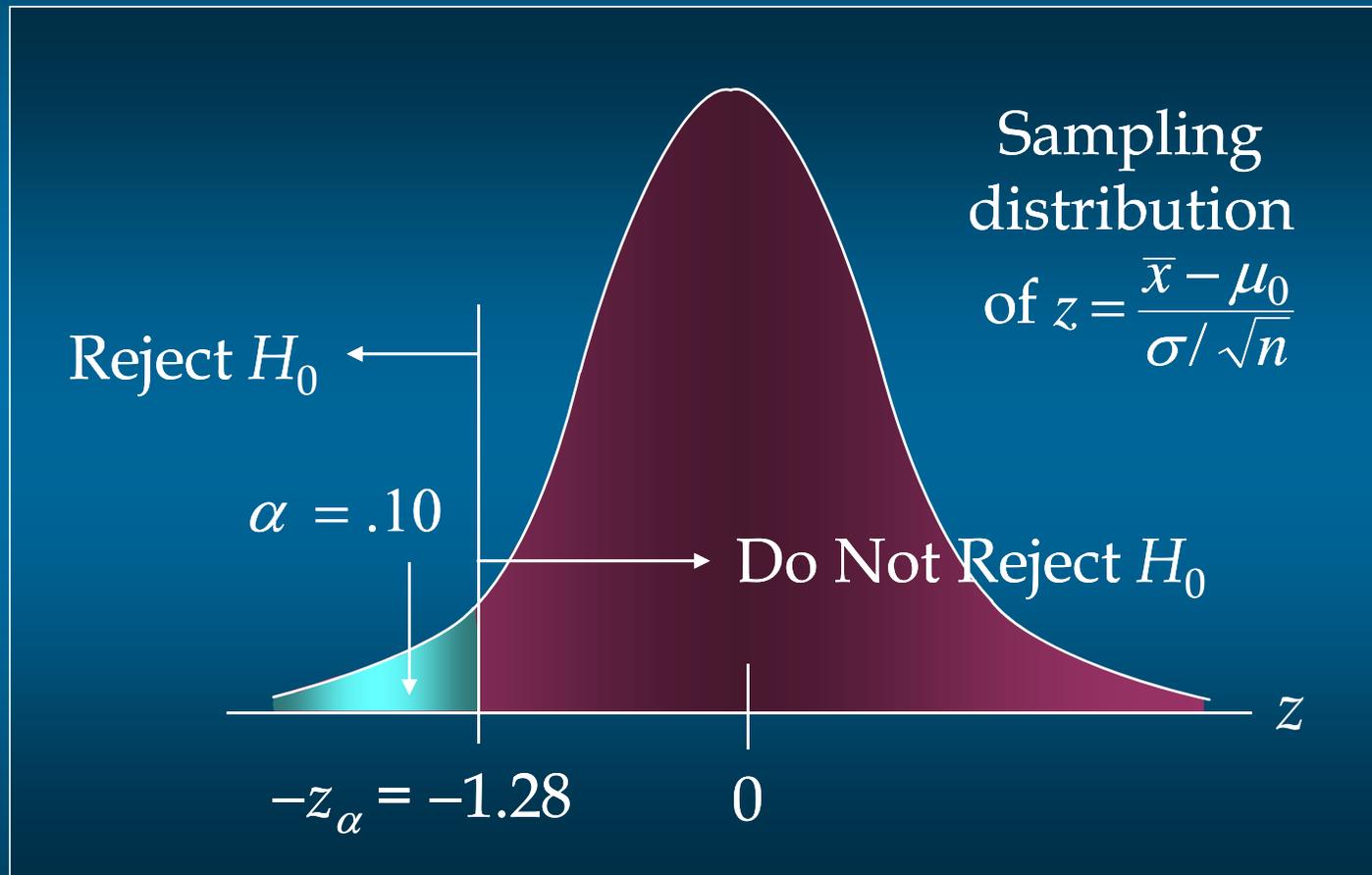


# Critical Value Approach to One-Tailed Hypothesis Testing

- ▶ ■ The test statistic  $z$  has a standard normal probability distribution.
- ▶ ■ We can use the standard normal probability distribution table to find the  $z$ -value with an area of  $\alpha$  in the lower (or upper) tail of the distribution.
- ▶ ■ The value of the test statistic that established the boundary of the rejection region is called the critical value for the test.
- ▶ ◻ The rejection rule is:
  - Lower tail: Reject  $H_0$  if  $z \leq -z_\alpha$
  - Upper tail: Reject  $H_0$  if  $z \geq z_\alpha$

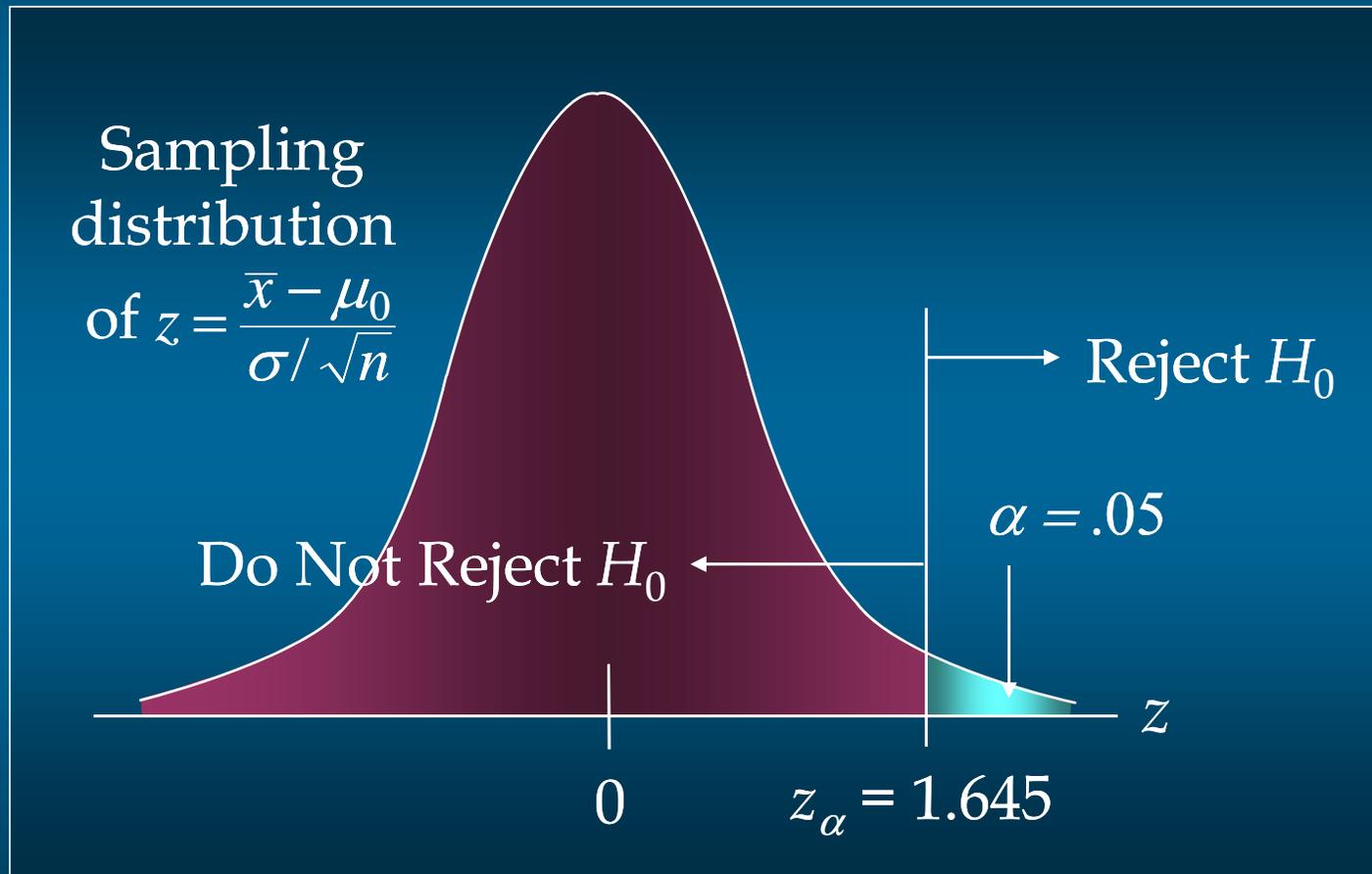
# Lower-Tailed Test About a Population Mean: $\sigma$ Known

## Critical Value Approach



# Upper-Tailed Test About a Population Mean: $\sigma$ Known

## Critical Value Approach



# Steps of Hypothesis Testing

- ▶ Step 1. Develop the null and alternative hypotheses.
- ▶ Step 2. Specify the level of significance  $\alpha$ .
- ▶ Step 3. Collect the sample data and compute the test statistic.

## *p*-Value Approach

- ▶ Step 4. Use the value of the test statistic to compute the *p*-value.
- ▶ Step 5. Reject  $H_0$  if *p*-value  $\leq \alpha$ .

# Steps of Hypothesis Testing

## Critical Value Approach

- ▶ **Step 4.** Use the level of significance to determine the critical value and the rejection rule.
- ▶ **Step 5.** Use the value of the test statistic and the rejection rule to determine whether to reject  $H_0$ .

# One-Tailed Tests About a Population Mean: $\sigma$ Known

## Example: Metro EMS

- ▶ The response times for a random sample of 40 medical emergencies were tabulated. The sample mean is 13.25 minutes. The population standard deviation is believed to be 3.2 minutes.

The EMS director wants to

- ▶ perform a hypothesis test, with a .05 level of significance, to determine whether the service goal of 12 minutes or less is being achieved.



# One-Tailed Tests About a Population Mean: $\sigma$ Known



## ■ $p$ -Value and Critical Value Approaches

- ▶ 1. Develop the hypotheses.  $H_0: \mu \leq 12$   
 $H_a: \mu > 12$

- ▶ 2. Specify the level of significance.  $\alpha = .05$

- ▶ 3. Compute the value of the test statistic.

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{13.25 - 12}{3.2 / \sqrt{40}} = 2.47$$

# One-Tailed Tests About a Population Mean: $\sigma$ Known



## ■ $p$ -Value Approach

- ▶ 4. Compute the  $p$ -value.

For  $z = 2.47$ , probability = .4932.

$$p\text{-value} = .5 - .4932 = .0068$$

- ▶ 5. Determine whether to reject  $H_0$ .

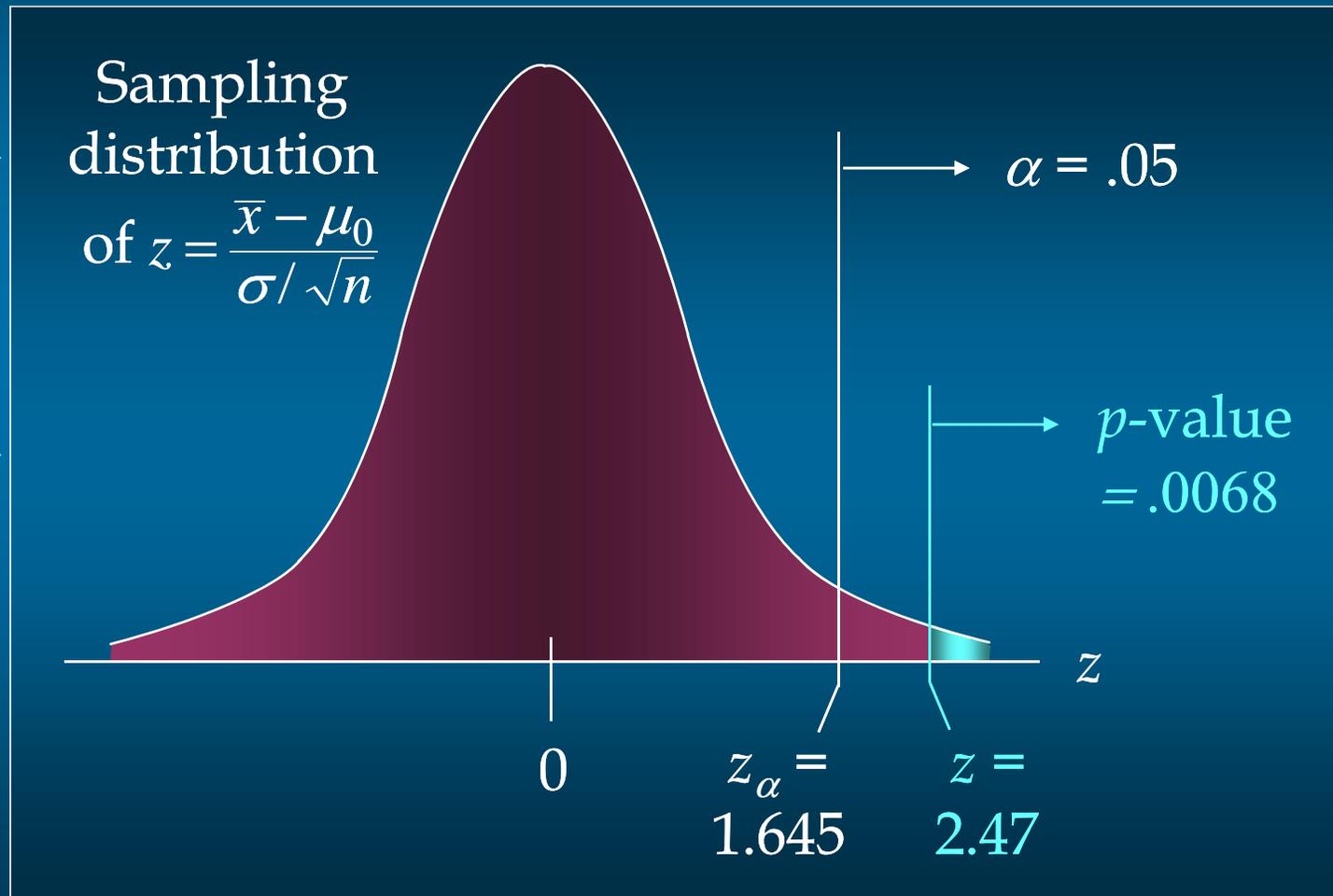
Because  $p\text{-value} = .0068 \leq \alpha = .05$ , we reject  $H_0$ .

We are at least 95% confident that Metro EMS is not meeting the response goal of 12 minutes.

# One-Tailed Tests About a Population Mean: $\sigma$ Known



q  $p$ -Value Approach



# One-Tailed Tests About a Population Mean: $\sigma$ Known



## ■ Critical Value Approach

- ▶ 4. Determine the critical value and rejection rule.

For  $\alpha = .05$ ,  $z_{.05} = 1.645$

Reject  $H_0$  if  $z \geq 1.645$

- ▶ 5. Determine whether to reject  $H_0$ .

Because  $2.47 \geq 1.645$ , we reject  $H_0$ .

We are at least 95% confident that Metro EMS is not meeting the response goal of 12 minutes.

## $p$ -Value Approach to Two-Tailed Hypothesis Testing

- ▶ ■ Compute the  $p$ -value using the following three steps:
  - ▶ 1. Compute the value of the test statistic  $z$ .
  - ▶ 2. If  $z$  is in the upper tail ( $z > 0$ ), find the area under the standard normal curve to the right of  $z$ .  
If  $z$  is in the lower tail ( $z < 0$ ), find the area under the standard normal curve to the left of  $z$ .
  - ▶ 3. Double the tail area obtained in step 2 to obtain the  $p$ -value.
- ▶ ■ The rejection rule:  
Reject  $H_0$  if the  $p$ -value  $\leq \alpha$ .

## Critical Value Approach to Two-Tailed Hypothesis Testing

- ▶ ■ The critical values will occur in both the lower and upper tails of the standard normal curve.
- ▶ ■ Use the standard normal probability distribution table to find  $z_{\alpha/2}$  (the  $z$ -value with an area of  $\alpha/2$  in the upper tail of the distribution).
- ▶ ◻ The rejection rule is:  
Reject  $H_0$  if  $z \leq -z_{\alpha/2}$  or  $z \geq z_{\alpha/2}$ .

## Example: Glow Toothpaste

### q Two-Tailed Test About a Population Mean: $\sigma$ Known

- ▶ The production line for Glow toothpaste is designed to fill tubes with a mean weight of 6 oz. Periodically, a sample of 30 tubes will be selected in order to check the filling process.
- ▶ Quality assurance procedures call for the continuation of the filling process if the sample results are consistent with the assumption that the mean filling weight for the population of toothpaste tubes is 6 oz.; otherwise the process will be adjusted.



## Example: Glow Toothpaste

### q Two-Tailed Test About a Population Mean: $\sigma$ Known

- ▶ Assume that a sample of 30 toothpaste tubes provides a sample mean of 6.1 oz. The population standard deviation is believed to be 0.2 oz.
- ▶ Perform a hypothesis test, at the .03 level of significance, to help determine whether the filling process should continue operating or be stopped and corrected.



# Two-Tailed Tests About a Population Mean: $\sigma$ Known



## ■ $p$ -Value and Critical Value Approaches

▶ 1. Determine the hypotheses.  $H_0: \mu = 6$   
 $H_a: \mu \neq 6$

▶ 2. Specify the level of significance.  $\alpha = .03$

▶ 3. Compute the value of the test statistic.

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{6.1 - 6}{.2 / \sqrt{30}} = 2.74$$

# Two-Tailed Tests About a Population Mean: $\sigma$ Known



## ■ $p$ -Value Approach

- ▶ 4. Compute the  $p$ -value.

For  $z = 2.74$ , cumulative probability = .4969

$$p\text{-value} = 2(.5 - .4969) = .0062$$

- ▶ 5. Determine whether to reject  $H_0$ .

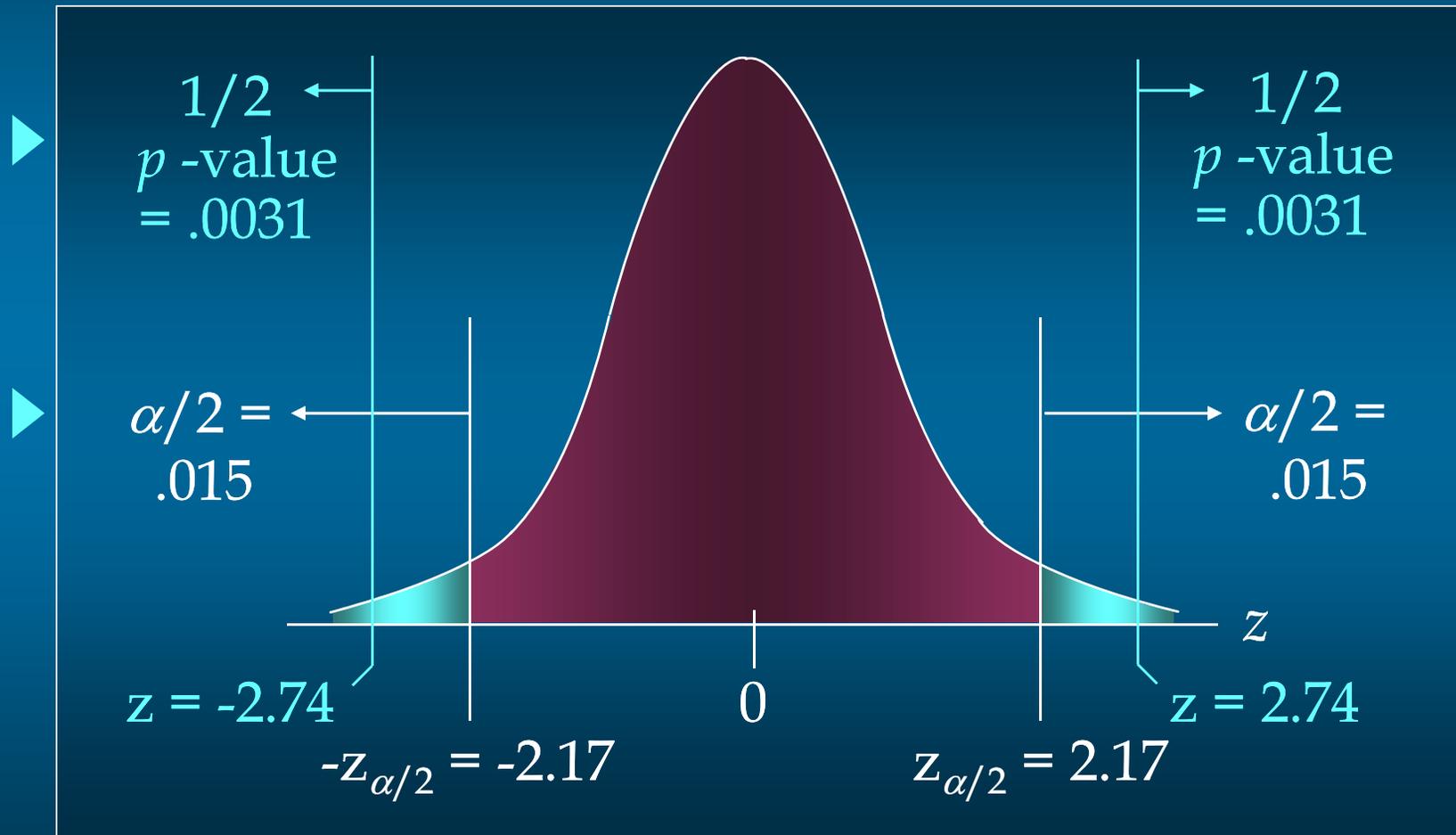
Because  $p\text{-value} = .0062 \leq \alpha = .03$ , we reject  $H_0$ .

We are at least 97% confident that the mean filling weight of the toothpaste tubes is not 6 oz.

# Two-Tailed Tests About a Population Mean: $\sigma$ Known



## ■ $p$ -Value Approach



# Two-Tailed Tests About a Population Mean: $\sigma$ Known



## ■ Critical Value Approach

- ▶ 4. Determine the critical value and rejection rule.

For  $\alpha/2 = .03/2 = .015$ ,  $z_{.015} = 2.17$

Reject  $H_0$  if  $z \leq -2.17$  or  $z \geq 2.17$

- ▶ 5. Determine whether to reject  $H_0$ .

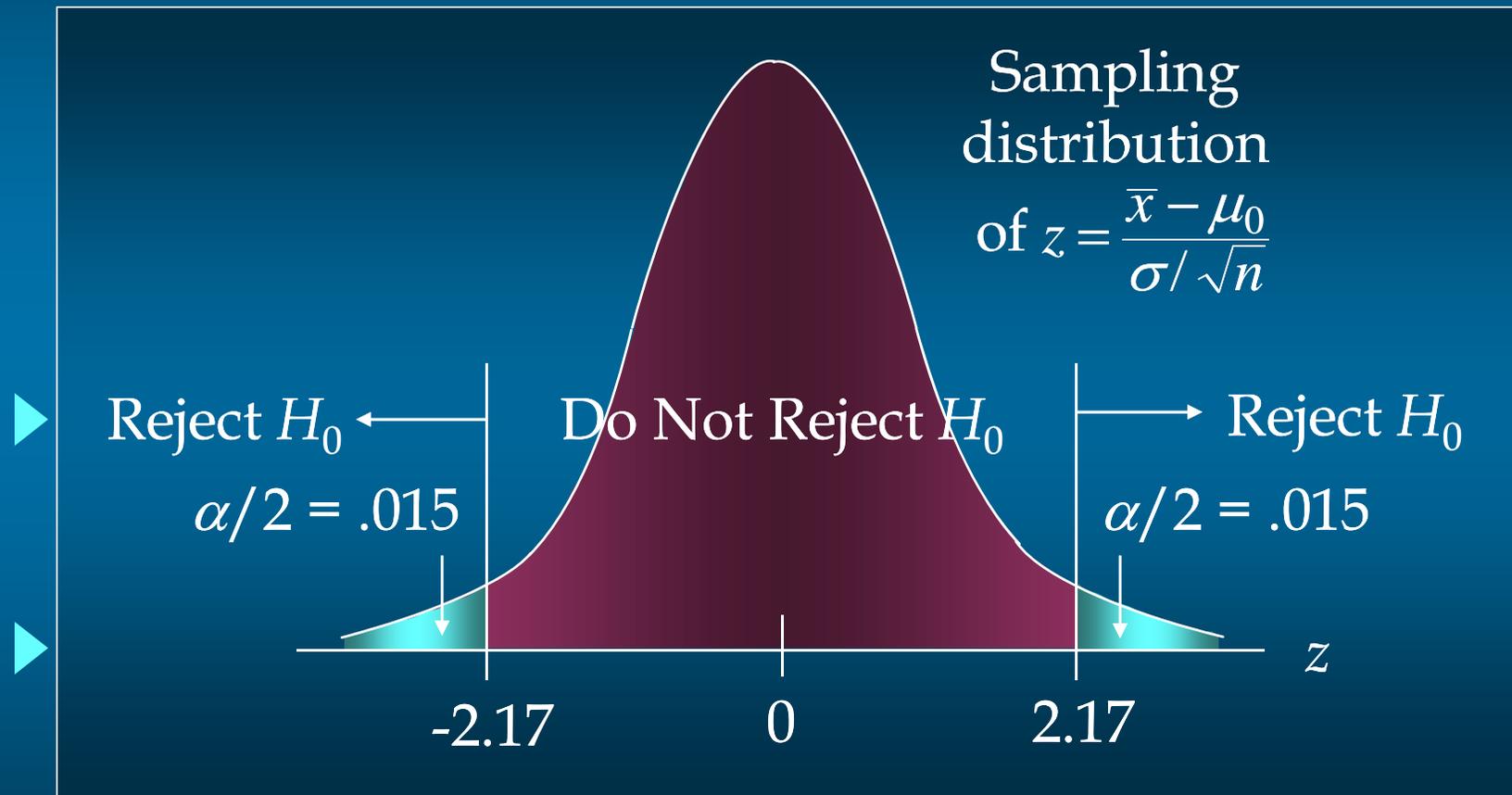
Because  $2.47 \geq 2.17$ , we reject  $H_0$ .

We are at least 97% confident that the mean filling weight of the toothpaste tubes is not 6 oz.

# Two-Tailed Tests About a Population Mean: $\sigma$ Known



## ■ Critical Value Approach



## Confidence Interval Approach to Two-Tailed Tests About a Population Mean

- ▶ ■ Select a simple random sample from the population and use the value of the sample mean  $\bar{x}$  to develop the confidence interval for the population mean  $\mu$ . (Confidence intervals are covered in Chapter 8.)
- ▶ ■ If the confidence interval contains the hypothesized value  $\mu_0$ , do not reject  $H_0$ . Otherwise, reject  $H_0$ .

# Confidence Interval Approach to Two-Tailed Tests About a Population Mean

The 97% confidence interval for  $\mu$  is



▶ 
$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 6.1 \pm 2.17(.2/\sqrt{30}) = 6.1 \pm .07924$$

or 6.02076 to 6.17924

▶ Because the hypothesized value for the population mean,  $\mu_0 = 6$ , is not in this interval, the hypothesis-testing conclusion is that the null hypothesis,  $H_0: \mu = 6$ , can be rejected.