

## Further Notes on Probability

### I. Qualitative Measures of Probability and Comparison

The typical examples of qualitative expressions of probability are **certain**, **impossible**, **likely** and **unlikely**.

An event that is **certain** will certainly happen. Examples: When you drop a coin it will fall. It is certain that it will rain in Corpus Christi in the next year.

An event that is **impossible** is one that will never happen. Examples: When you drop a coin it will land on its edge. It is impossible that it will snow in Corpus Christi before the end of the month.

An event that is **likely** will happen more times than not. Examples: Our class has more female than male students. A student chosen at random is likely to be female. A die with six sides (numbered 1,2,3,4,5,6) is rolled. It is likely that the number is less than 6. These events are not certain since a male can be chosen and a 6 can be rolled.

An event that is **unlikely** won't happen more times than not. Examples: A student chosen at random is unlikely to be male. A die rolled is unlikely to show a 6.

Of course, all qualitative expressions of probability can be qualified. A weatherman might say, it is very likely that we will have showers today.

Comparisons can be made between events without any reference to their specific probabilities. We can say that a student chosen at random is more likely to be female than male. It is less likely to roll a six than a number that is not a six.

### II. Trees and arrays

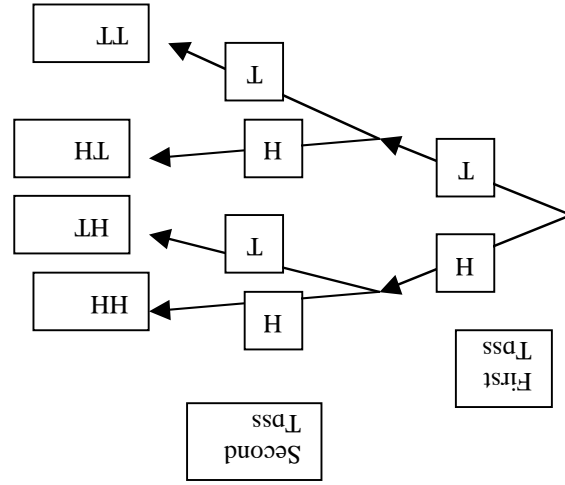
Compound events represent a sequence of simple events. An example is rolling two dice. A simple event can be rolling a 3 with the first die or rolling an even number with the second die. A compound event made from these simple events is rolling a 3 on the first die AND rolling an even number on the second. We saw that it is possible to list the outcomes of a compound event.

The least complicated example of listing outcomes of a compound event is two successive coin tosses. The sample space is  $S = \{HH, HT, TH, TT\}$ , where XY represents tossing an X on the first toss and Y represents tossing a Y on the second. We might not be sure that we have listed all possible outcomes. The structure of trees and arrays help us to make sure that we have considered all possible outcomes.

ARRAY (or TABLE): The possible results of the first toss are listed across the top of the array. The possible results are listed down the left side of the array. The remaining cells represent the possible outcomes in the Sample Space. The H or T directly above them form the first letter, the H or T directly to the left form the second letter.

First Roll -- >	H	T
Second Roll	HH	HT
	TH	TT

TREE: In the tree below, each toss is represented by a branching of the tree. There are two branches at each toss since there are two possible results from that toss, H or T.



Note that any one of the ways of representing the sample space make it easy to identify the probabilities of given events, such as First toss a head ( $2/4 = 1/2$ ), or First toss a head and second toss a tail ( $1/4$ ).

Arrays versus Trees: The selection of one over the other is not important if there are two turns in a sequence. Trees are easier to use more than two choices are to be made.

Exercises. A restaurant serves burgers, steaks and fried chicken as entrees and peas, string beans and carrots as side dishes.

1. Make a tree to represent the sample space for a patron choosing one entrée and one side dish. Use it to determine the probability of a patron randomly choosing a beef entrée and a green side dish. (answer:  $4/9$ ).
2. Extend this example to where the patron must choose one entrée and two different side dishes. Determine the probability of choosing a chicken as the entrée and carrots as one of the dishes (answer:  $4/18 = 2/9$ ).