Exercises on Complex Numbers
MATH 5333.001
Summer 2000

Let **C** = { a + bi : a and b are real numbers}, where $i^2 = -1$, represent the complex numbers. It is claimed in the text that the complex numbers form a field. The operations on complex numbers are addition ((a + bi)+ (c + di) = (a+b) + (c+d)i) and multiplication ((a + bi)*(c + di) = (ac - bd) + (ad+bc)i).

1. Let $c_1 = 2 + 1I$ and $c_2 = 3 - 2i$.
   a) Find $c_1 + c_2$, $c_1 - c_2$, and $c_1 * c_2$.
   b) Show that 0 = 0 + 0 i is the additive identity for the field of complex numbers and that 1=1 + 0 i is the multiplicative identity for the field of complex numbers.
   c) Find a complex number c such that $c * c_1 = 1$. The complex number c would be the multiplicative inverse of c_1.
   d) Find a general formula for the multiplicative inverse of a + bi.

2. For a complex number a + bi, write $\overline{a + bi} = a - bi$, or the conjugate of the complex number a + bi.
   a) Find the complex conjugates of the numbers c_1 and c_2 from problem 1.
   b) Find the complex conjugate of the multiplicative inverse of c_1, and find the multiplicative inverse of the complex conjugate of c_1 you found in (a). How are they related?
   c) Prove any conjecture you have based on (b).
   d) Find $\overline{\overline{a + bi}}$, that is the conjugate of the conjugate, for c_1 and c_2. What happens in general? Prove it.

3. Let $\| a + bi \| = \sqrt{a^2 + b^2}$ be the absolute value of a complex number.
   a) Find the absolute values of the complex numbers c_1 and c_2.
   b) Find the absolute value of $\overline{c_1}$. How does it compare to the absolute value of c_1?
   c) Prove any conjecture you have based on (b).