ISSN 1223-9631

BULLETIN

OF THE

TRANSILVANIA UNIVERSITY
OF
BRASOV

Vol. 1 (36) - New Series
Series A

- MECHANICS
- ELECTROTECHNICS AND ELECTRONICS
- MATERIALS PROCESSING
- WOOD INDUSTRY
- SILVICULTURE

OFF - PRINT

Published by
TRANSILVANIA UNIVERSITY PRESS
Brasov, România
1994

NUMERICAL SYNTHESIS OF THE STEPHENSON II FUNCTION GENERATING MECHANISMS

P. A. Simionescu*

Abstract: *The paper presents a synthesis method of the STEPHENSON II function generating mechanism. The novelty is that, by considering a variable length of one of its elements, the structure of the mechanism is modified, so that the input and output members can be driven exactly in accordance with the imposed function. The synthesis problem becomes one of optimization, i.e. of choosing the parameters of the mechanism, on which the root mean square (RMS.), or the TCHEBYSHEFF norm of this variable length is minimum.*

Key words: *function generator, variable length, optimization algorithm.*

1. Introduction

The study of the six-bar STEPHENSON mechanism attracted a number of investigators, being, as known, a problem of high complexity.

By a proper choice of dimensions, a mechanism can be synthesized so that a given input motion (in this case, rotation) can produce a specified output motion. In comparison with the classical four-bar function generating mechanism, the six-bar STEPHENSON mechanism can produce a function closer to the imposed one, due to the increased number of parameters that define its geometry. However, the linkage can not be made to produce exactly the desired function, and the designer should aim at minimizing the departure from the required function, which is called the structure error of the mechanism.

2. The STEPHENSON II function generating mechanism

The synthesis of a STEPHENSON II function generator, for a minimum structure error, is usually carried out either by the precision point approach [2] or by applying optimization techniques to the synthesis equation of the mechanism. For the six-bar STEPHENSON II mechanism, the synthesis equation can be obtained from the closed loop equation (and solved using numerical techniques), or can be applied a suitable kinematic inversion of the mechanism [1]. Both methods are difficult to use due to the increased number of calculations involved. Furthermore, in case of an iterative optimization algorithm, it is important that the CPU time required for each valuation of the objective function be as short as possible.

* Machine Elements, Mechanisms and Robotics Dept. "Transilvania" University of Brasov.

As in papers [3] and [7] on a four-bar mechanism, we impose the six-bar STEPHENSON II mechanism to generate the function:

$$y = -\frac{x}{8}(x+2), \tag{1}$$

in the range $0<x<6$. The four-bar mechanisms proposed in these papers, generate the above function (1) on a maximum structure error of 1.88°. and 1.477° respectively.

An important requirement is to design a mechanism having uniform scales for both x and y. Thus the relationship between x and y and the rotation angle φ_{in} and φ_{out} of the respective input and output members are:

$$\varphi_{in} = \frac{x - x_s}{x_f - x_s}\Delta\varphi_{in} + \varphi_{ins},$$
$$\varphi_{out} = \frac{y - y_s}{y_f - y_s}\Delta\varphi_{out} + \varphi_{outs}, \tag{2}$$

where the input limits are x_s and x_f and the corresponding output limits are $y_s=y(x_s)$ and $y_f=y(x_f)$, while φ_{ins} and φ_{outs} are the initial angles of the input and output members.

3. The synthesis method applied to the STEPHENSON II mechanism

For the six-bar STEPHENSON II mechanism, the synthesis method proposed in this paper considers the modified mechanism with two degrees of freedom (fig. 1 and 2). An advantageous way to increase the degree of freedom of the mechanism (because of the simpler analytical calculations involved) is to consider a variable angle (CAE for the mechanism in fig. 1, or CBE in case of fig. 2) of the triple jointed, non adjacent to the frame element. In this case, the input and output members can be driven (with some limitations) exactly in accordance with the imposed transmission law, as that given by relation (1).

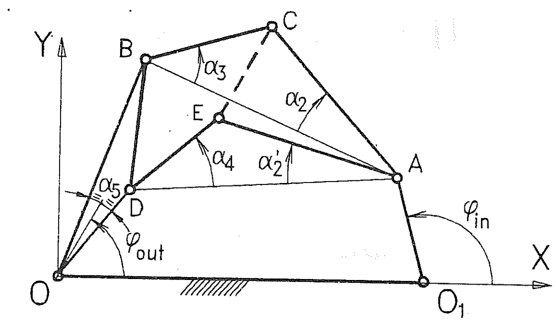

Fig. 1

The problem becomes now one of optimization, i.e. to determine the parameters that define the mechanism in which, by a separate driving of the input and output members (in accordance with the imposed function) the variance of the distance between joints C and E is minimum. In such a case, the structure error of the single degree of freedom mechanism, with a stiffened CE rod, will also be of a small value.

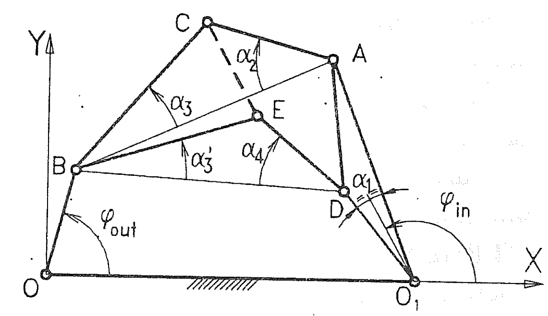

Fig. 2

For an intermediate position φ_{inj}, φ_{outj} of the two extreme elements, driven in accordance with the imposed function, the distance between the released joints is calculated with the known formula

$$CE_j = \sqrt{\left(X_{Cj} - X_{Ej}\right)^2 + \left(Y_{Cj} - Y_{Ej}\right)^2} \,. \quad (3)$$

The coordinates (X_{Cj}, Y_{Cj}) and (X_{Ej}, Y_{Ej}) of joints C and E of the mechanism in fig. 1, can be obtained by solving the following system of equations:

$$\begin{aligned}
&\left(X_{Cj} - X_{Aj}\right)^2 + \left(Y_{Cj} - Y_{Aj}\right)^2 = AC^2, \\
&\left(X_{Cj} - X_{Bj}\right)^2 + \left(Y_{Cj} - Y_{Bj}\right)^2 = BC^2, \\
&\left(X_{Ej} - X_{Aj}\right)^2 + \left(Y_{Ej} - Y_{Aj}\right)^2 = BE^2, \\
&\left(X_{Ej} - X_{Dj}\right)^2 + \left(Y_{Ej} - Y_{Dj}\right)^2 = DE^2.
\end{aligned} \quad (4)$$

We considered point C as an intersection of the circle of radius AC and center A with the circle of radius BC and center B, and in the same way, point E as an intersection of the circle of radius AE and center A with the circle of radius DE and center D.

For the mechanism in fig. 2, the coordinates of joints C and E can be valued by solving the similar system of equations (5)

$$\begin{aligned}
&\left(X_{Cj} - X_{Aj}\right)^2 + \left(Y_{Cj} - Y_{Aj}\right)^2 = AC^2, \\
&\left(X_{Cj} - X_{Bj}\right)^2 + \left(Y_{Cj} - Y_{Bj}\right)^2 = BC^2, \\
&\left(X_{Ej} - X_{Bj}\right)^2 + \left(Y_{Ej} - Y_{Bj}\right)^2 = BE^2, \\
&\left(X_{Ej} - X_{Dj}\right)^2 + \left(Y_{Ej} - Y_{Dj}\right)^2 = DE^2.
\end{aligned} \quad (5)$$

Because it has been proven that the more angles we use to define the geometry of the mechanism, the better it is[1] (think of the fact that any two angles and a length determine a triangle, while three lengths are not always determining one), we considered the mechanism in fig. 1 defined by the following parameters: OO_1, O_1A, φ_{ins}, α_2, $\alpha_{2'}$, α_3 , α_4, OB, OD, α_5 and φ_{outs}, and the same for the mechanism in fig. 2: OO_1, O_1A, O_1D, α_1, φ_{ins}, α_2, α_3, $\alpha_{3'}$, α_4, OB, and φ_{outs}.

The lengths AC, BC, AE or BE and DE in the given system of equations (4) and (5), can be calculated (in the reference position φ_{ins}, φ_{outs}) from the triangles ABC and ADE (fig. 1), or ABC and BDE in case of the mechanism in fig. 2.

We have considered a positive orientation of angles $\alpha_1 \dots \alpha_5$ as shown, and the double sign in a solution of equations (4) and (5) of the form:

$$y_{Cj} = \frac{-P_C \pm \sqrt{Q_C}}{R_C}, \qquad y_{Ej} = \frac{-P_E \pm \sqrt{Q_E}}{R_E}, \quad (6)$$

is chosen in accordance with the positive or negative sign of angles α_2 and α_4, respectively.

The coordinates of joints A, B and D in an intermediate j position, can be calculated for the mechanism in fig. 1 as follows:

$$X_{Aj} = OO_1 + O_1A\cos\varphi_{inj}, \qquad Y_{Aj} = O_1O\sin\varphi_{inj}, \quad (7)$$

$$X_{Bj} = OB\cos\left(\varphi_{outj} + \frac{\alpha_5}{2}\right), \qquad Y_{Bj} = OB\sin\left(\varphi_{outj} + \frac{\alpha_5}{2}\right) \quad (8)$$

and

$$X_{Dj} = OD\cos\left(\varphi_{outj} - \frac{\alpha_5}{2}\right), \qquad Y_{Dj} = OD\sin\left(\varphi_{outj} - \frac{\alpha_5}{2}\right). \quad (9)$$

The same goes for the mechanism in fig. 2

$$X_{Aj} = OO_1 + O_1A\cos\left(\varphi_{inj} - \frac{\alpha_1}{2}\right), \qquad Y_{Aj} = O_1A\sin\left(\varphi_{inj} - \frac{\alpha_1}{2}\right), \quad (7')$$

[1] The range of the function will be wider.

$$X_{Dj} = OO_1 + OD\cos\left(\varphi_{inj} + \frac{\alpha_1}{2}\right),$$
$$Y_{Dj} = OO_1 + OD\sin\left(\varphi_{inj} + \frac{\alpha_1}{2}\right), \quad (8')$$

and

$$X_{Bj} = OB\cos\varphi_{outj},$$
$$Y_{Bj} = OB\sin\varphi_{outj}. \quad (9')$$

For j=1..n positions that correspond to angles φ_{inj} and φ_{outj} in the ranges

$$\varphi_{ins} \le \varphi_{inj} \le \varphi_{ins} + \Delta\varphi_{in},$$
$$\varphi_{outs} \le \varphi_{outj} \le \varphi_{outs} + \Delta\varphi_{out},$$

we have defined an objective function F1 of value the RMS. of the variable distance between joints E and C, i.e.

$$F1(\ldots) = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(CE_j - CE_0\right)^2}. \quad (10)$$

The design parameters are the lengths and angles that define the geometry of the mechanism. We have nominated with CEo the average of the n lengths CEj calculated for each j intermediate position

$$CE_0 = \frac{1}{n}\sum_{j=1}^{n} CE_j, \quad (11)$$

which is the length of the rod CE in case of the single degree of freedom mechanism, we also search for.

In order to make some comparisons, another objective function F2, of the same arguments, but of value: the Tchebysheff norm ‖..‖ of the same variable length CE, have been considered i.e.

$$F2(\ldots) = \|\delta CE_0\| = \max\left|CE - CE_0\right|. \quad (12)$$

In a computer algorithm it is easier to calculate the value of this objective function as the maximum of j=1..n discrete values $\delta CE_j = |CE_j - CE_0|$. The reference length CE_0 is the same average given by relation (11).

4. Results and conclusions

In case of function (1), some numerical applications have been made for the mechanism in fig. 1, in case of the unit length frame OO_1. We have also imposed the initial angles φ_{ins} and φ_{outs} of fixed values 80° and -20° respectively, and both input and output members to rotate 90° for the chosen range of x, namely $\Delta\varphi_{in}$=90° and $\Delta\varphi_{out}$=90°.

The study of these F1 and F2 objective functions proved that there are several local minimums, separated mainly by the geometry of the ABCD loop. This loop can be concave one, with angles α_2 and α_4 of the same sign, or convex, in case of α_2 and α_4 of different signs.

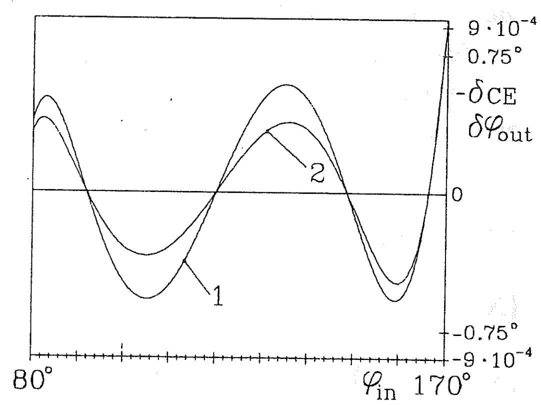

Fig. 3

An important remark is that the minimum of the F2 objective function, usually ensures a less maximum structure error then the F1 objective function. This can be seen from the following two solutions, obtained as minimums of the F1 and F2 objective functions respectively, selected from the same 50,000 sets of randomly generated parameters. The number of intermediate positions, equidistant in the input member range, was considered n=90.

The first mechanism obtained as the minimum of F1, is of parameters: OO_1=1, O_1A=0.4652, AC=0.7051, AE=0.5221, CE_0=0.2139, BC=0.3335, DE=0.5106, OB=0.3021, α_5=-4.052°, OD=0.3586 (and positive angles α_2 and α_2'), and ensures a maximum structure error of 0.9273° (for the input member in its final position). The corresponding RMS and Tchebysheff norm of the variable length CE are 421 10^{-6} and 947 10^{-6}, respectively. A whole range graphical representation of the δCE (curve 1) and of the structure error $\delta\varphi_{out}$ (curve 2) is given in fig. 3.

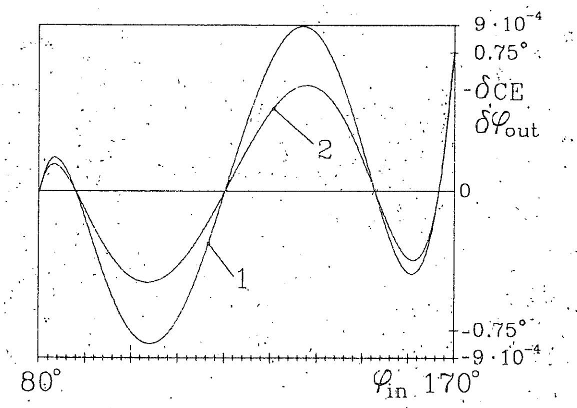

Fig. 4

The second mechanism of parameters: OO_1=1, O_1A=0.4774, AC=0.7029, AE=0.5259, CE_0=0.2116, BC=0.3397, DE=0.5138, OB=0.3134, α_5=-4.052°, OD=0.3628 (the same positive angles α_2 and α_2'), ensures a less output error of 0.7175° (for the input member in its final position too). The value of the RMS was of 537 10^{-6} in this case, greater than before, while the value of the function (the Tchebysheff norm) was 894 10^{-6}.

A similar graphical representation of the δCE and of the structure error $\delta\varphi_{out}$ are given in fig. 4. In fig. 5 is also given a drawing of this former mechanism, in its initial and final positions.

In both these examples, a resemblance of the shape of $\delta CE(\varphi_{in})$ and of $\delta\varphi_{out}(\varphi_{in})$ can be observed. The fact that both δCE and $\delta\varphi_{out}$ become zero for the same φ_{in} is obvious, but it can be observed that by a proper scaling of one of this curves, it can not be exactly superposed over the other. This proves that the mechanism that ensures the absolute minimum in $\| \delta CE \|$, will not ensure the absolute minimum in $\| \delta\varphi_{out} \|$.

The angular structure error $\delta\varphi_{out}$ for the graphical representations in fig. 3 and 4 has been numerically determined, by searching the value of the angle $\varphi_{outREAL}$ in the neighborhood of the corresponding theoretical angle φ_{out}, (given by relation (2)), on which the deviation $CE-CE_0$ equals zero [4], [6].

Both solutions obtained after the 50.000 iterations Monte Carlo search, ensure more accurate approximation of function (1), than the four-bar mechanism proposed in papers [3] and [3].

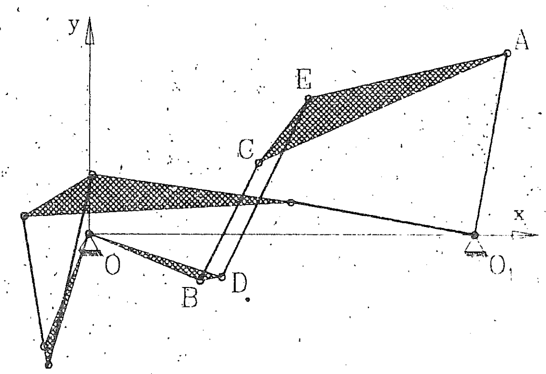

Fig. 5

Better solutions of STEPHENSON II functions generators can be obtained in case of the same function, both in output precision and in transmission angles, by, for instance, varying the φ_{ins} and φ_{outs} initial angles, and also by using more advanced optimization subroutines.

References

1. Biswas A.., Kinzel, G.L. An Automated Inversion Approach to Closed-Form Kinematic Analysis of Planar Mechanisms *Journal of Mechanical Design,* **DE-Vol. 146**, 1992, pp. 293-299.
2. Dhingra A.K., Cheng J.C., Kohli D. Complete Solutions to Synthesis of Six-Link, Slider-Crank and Four-Bar Mechanisms for Function, Path and Motion Generation, Using Homotopy With M-Homogenization *Journal of Mechanical design*, **DE-Vol. 46**, 1992, pp. 281-291.
3. Fang, Y., F., Davies, T.,H., Synthesis of Four-Bar Function Generator, Using Generalized Matrix Inverse Approach *The 5-th International Symposium Theory and practice of Mechanisms* Bucharest, 1989, **Vol. I 1**, pp. 227-242.
4. Maros, D., Mecanisme (Mechanisms) *Cluj-Napoca University Publishing House*, 1980, pp. 46-49.
5. Perju, D. On the Linkages Synthesis for Function Approximation, *The Bulletin of The 3-rd International Symposium Theory and Practice of Mechanisms*, Bucharest, 1981, **Vol. I-1**, pp. 203-213.
6. Pommersheim, A., Crudu, M. The Use of some Computer Programs in the Kinematic Analysis of the Superior Kinematic Groups, *The 5-th National Symposium PRASIC'94*, Brasov 1994, **Vol. 1** pp. 237-242.
7. Simionescu, P.A. Synthesis of the Four-Bar Function Generator Using the Method of Increasing the Degree of Freedom of the Mechanism, *The 5-th National Symposium PRASIC'94*, Brasov, 1994, **Vol. 1** pp. 261-268.

Sinteza numericã a mecanismelor STEPHENSON II generatoare de functii

Rezumat: *Lucrarea prezintã o nouã metodã de sinteza a mecanismelor generatoare de functii, cu triada si tetrada, derivate din lantul cinematic STEPHENSON. Noutatea metodei consta in aceea ca mecanismul este modificat prin considerarea unuia dintre elemente de lungime variabila. In acest fel elementele de intrare si de iesire pot fi antrenate in concordanta cu legea impusa, problema sintezei mecanismului revenind la a determina acei parametri geometrici pentru care abaterea medie patratica sau norma Cebasev, a lungimii elementului considerat variabil este minima.*

Cuvinte cheie: *mecanism generator de functii, lungime variabila, algoritm de optimizaretin.*

Recenzent: prof.dr.ing. Petre Alexandru.