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Synthesis of Function Generators Using the Method of Increasing the Degree of Freedom of the Mechanism

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Abstract : *According to the method proposed in the paper, the mechanism under synthesis is modified by dismounting one of its middle rods, so that the degree of freedom of the mechanism becomes two. For the fictitious mechanism, the input and output members can be exactly driven in accordance with the imposed function, the synthesis problem being one of optimization i.e. of choosing the parameters of the mechanism on which the variable distance or angle determined by the released joints is minimum. In case of a dismounted rod joined to the frame, it is shown that is better to search for those parameters on which the other joint draws a curve closest to an arc of circle. Finally, the paper presents three examples: of four-bar, STEPHENSON II and STEPHENSON III function generators synthesis.*

Keywords : Function generator, Variable length/angle, Optimization algorithm

Introduction

Function generators is a class of mechanisms in which a given input motion (usually rotation) will produce a specified output motion. The problem of determining the optimal parameters of a linkage that fulfill such requirement is classical to the mechanism synthesis, and has been addressed by several researchers in the past.

Without considering the graphical methods [5] that are of low precision and ineffective for multiple loop mechanisms, the synthesis methods can be broadly classified into two categories: precision point approach [2], [5] and optimization techniques [1], [3] and [8]. The precision point synthesis assures a function exactly matches the desired one at a limited number of points. In between these points the output error varies in an unpredictable way, the maximum output error depending upon the number and disposition of these points. It usually leads to a nonlinear system of equations, which can only be solved using numerical methods. The optimization techniques allow one to specify more points in which the departure between the actual and desired function is controlled. Moreover, extra conditions upon angles or maximum gauge can be imposed [7], as compared with the precision point approach, on which these conditions can only be verified at the end of synthesis. The convergence of the method depends upon the searching algorithm employed and of the choice of the starting points, because of the pronounced no-linear, non-monotonic objective function, and of the existing of multiple local minima [4].

Description of the method of increasing the degree of freedom of the mechanism (IDFM)

The optimization synthesis methods attempt in one way or another to minimize the departure between the actual and desired function, i.e. the output error of the mechanism:

$$\delta\varphi_{out} = \varphi_{outREAL} - \varphi_{out} . \qquad (1)$$

Hence it becomes necessary to solve the displacement equation of the mechanism for each of the points considered, which is some times a difficult problem.

A different approach is proposed in [1] where the objective function is defined as a weighted sum of the local errors (as defined by the authors) evaluated at the considered points.

The method proposed in this paper resembles in a way the lather, because the same operates with a fictitious mechanism. This mechanism has 2 DOF and is obtained from the initial one by properly dismounting one of the rods from its joints. Thus the input and output members can be exactly driven in accordance with the imposed function, while the distance between the released joints will correspondingly vary. Sometimes, a variable angle can be identified, corresponding to this variable length. The synthesis problem is now one of determining (using an optimization algorithm) those parameters on which, by the simultaneous driving of the input and former output members, the variation of the distance between the released joints, or of the corresponding angle, is a minimum.

The rod to be dismounted (generally a middle one) must be chosen such as for the 2 DOF mechanism, the distance or variable angle, in case of known positions of the input and output members, can be calculated with ease. If the dismounted rod has one of its ends joined to the frame, it is easier to search for those parameters of the 2 DOF mechanism, on which the trajectory drawn by the mobile released joint is the closest to an arc of circle. Thus the coordinates of the other released joint (located to the frame), will result from the optimization procedure, and will not be among the design parameters.

Further are given three examples of application of the method in case of the four-bar, STEPHENSON II and STEPHENSON III function generators synthesis. Also given are modified relations of the objective functions, which yelled good approximations of the output error - this is, in fact, the characteristic of the desired mechanism which is required to be a minimum.

The four-bar function generator synthesis

The problem of synthesis of the four-bar function generator is not a difficult one. In this case, the synthesis equation can be obtained analytically [5], and based on it an objective function can be defined. The only difficulty is that the definition domain of the objective function has some discontinuities that correspond to the cases in which the loop of the mechanism can not be closed.

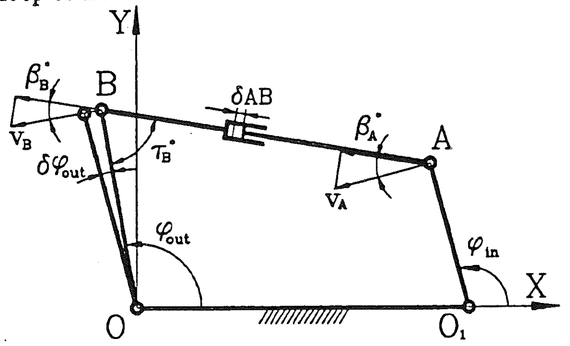

Fig. 1

In order to apply the IDFM method, the mechanism must be transformed into a 2 DOF one, as shown in Fig. 1. To maintain the structure of a mechanism, the coupler has been replaced with a RTR dyad. This fictitious mechanism is defined by the lengths OO_1, O_1A and OB. For a given function $y=f(x)$ to be generated in the range $x_s..x_f$, and for uniform scale factors both in x and y, the theoretical angles of the input and output members should be[1] [5]:

$$\varphi_{in} = \varphi_{ins} + \frac{\varphi_{inf} - \varphi_{inf}}{x_f - x_s}(x - x_s)$$
$$\varphi_{out} = \varphi_{outs} + \frac{\varphi_{outf} - \varphi_{outs}}{y_f - y_s}(y - y_s). \quad (2)$$

For an intermediate theoretical position (φ_{in} ; φ_{out}) with $\varphi_{in} \in [\varphi_{ins}..\varphi_{inf}]$ and $\varphi_{out} \in [\varphi_{outs}..\varphi_{outf}]$, and for a reference distance AB_0, a variation of the distance between the joints A and B can be calculated

$$\delta AB = AB - AB_0. \quad (3)$$

The distance AB is given by the known formula

$$AB = \sqrt{(x_A - x_B)^2 + (y_{A} - y_B)^2} \quad (4)$$

while

$$x_A = OO_1 + O_1A\cos\varphi_{in} \quad ; \quad y_A = O_1A\sin\varphi_{in}$$
$$x_B = OB\cos\varphi_{out} \quad \text{and} \quad y_B = OB\sin\varphi_{out}. \quad (5)$$

Considering the design parameters OO_1, O_1A, OB, φ_{ins}, $\Delta\varphi_{in}=\varphi_{inf}-\varphi_{ins}$, $\Delta\varphi_{out}=\varphi_{outf}-\varphi_{outs}$ and φ_{in0} a reference angle of the input member (in the domain $\varphi_{in0} \in [\varphi_{ins}..\varphi_{inf}]$) that will give an exact point to the four-bar synthesized mechanism and allow the calculation of AB_0, we can define a first objective function:

$$F_{10}(..) = \max|AB - AB_0| = \|\delta AB\|. \quad (6)$$

A Tchebyscheff norm $\|.\|$ have been chosen and not the classical RMS for the objective function, because it has been proven that this assures a less maximum deviation from the imposed function [8]. In a computer algorithm, the value of this objective function is calculated as the minimum from j=1..n discrete positions. Obviously, the greater the value of n the more precise (these observations are valid for all the next defined objective functions).

In paper [8] we have shown that the absolute minimum of an objective function such as F_{10} differs slightly from the absolute minimum of an objective function based upon the classical output error. For this reason we have searched for an adjusted expression of F_{10}, that gives a good approximation of the output error, but keeps the advantages of simple calculations and wider monotony (100% in case of the four-bar mechanism) i.e.[2]

$$F_{11}(..) = \left\| \frac{\delta AB}{OB \cdot \cos\beta_B^*} \right\| = \|\delta\varphi^*{}_{out}\|. \quad (7)$$

The approximate pressure angles β_B^* (Fig. 1) are calculated for the joints A and B in the corresponding theoretical position (the closer to the optimum in a searching iteration, the better the approximation). In case we impose restrictions upon the transmission angles, by for instance applying penalty functions method, the same approximation may be done, where $\tau^* = 90° - \beta^*$ and:

$$\tau_B{}^* = \sin^{-1}\left|\overline{OB} \times \overline{AB}\right| / (|OB| \cdot |AB|). \quad (8)$$

Regarding the expression (7), it can be obtained from the following relation, valid due to the PROJECTION THEOREM, applied to the mechanism in Fig. 1 with a stiffened O_1A link:

$$\frac{\delta\varphi_{out}}{\delta t} OB\cos\beta_B{}^* = \frac{\delta AB}{\delta t}. \quad (9)$$

The STEPHENSON II function generator synthesis

The synthesis of STEPHENSON II mechanism is considered a problem of high complexity, because the displacement equation of the mechanism can be solved only using numerical techniques, or by applying a suitable kinematic inversion.

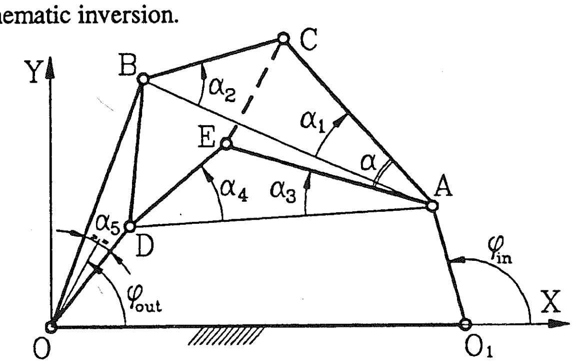

Fig. 2

According to the IDFM method, the associated fictitious 2 DOF mechanism shown in Fig. 2, is obtained by disassembling the CE rod. In this case, by simultaneously driving O_1A and (OBD) members according to relations (1) and (2), the angle CAE will become a variable one. If we name this angle with α, the similar F_{10} objective function will be:

[1]In whole paper subscript s signifies start and f signifies finish.

[2]Exponent * signifies aproximate values of the respective size.

$$F_{20}(..) = \|\alpha - \alpha_0\| = \|\delta\alpha\|. \tag{10}$$

where α_0 is a reference value of CAE angle, that will correspond to the stiffened CAE element of the single DOF final mechanism.

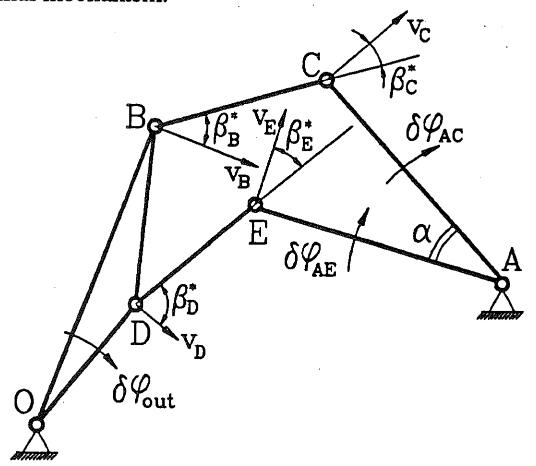

Fig. 3

The design parameters of this objective function can be chosen as: lengths OO_1, O_1A, OB, OD, angles α_1, α_2, α_3, α_4, α_5, φ_{ins} φ_{outs}, $\Delta\varphi_{in}$, $\Delta\varphi_{out}$, and similarly a reference angle φ_{in0}, used in calculation of lengths AC, BC, AE, DE and of reference angle α_0. For any intermediate position, the angle α can be evaluated as

$$\alpha = \sin^{-1}\left|\overline{AC}\times\overline{AE}\right| / \left(\left|\overline{AC}\right|\cdot\left|\overline{AE}\right|\right). \tag{11}$$

The coordinates of joints C and E result as intersection of circles, for example in case of X_C and Y_C:

$$\begin{aligned}(X_C - X_A)^2 + (Y_C - Y_A)^2 &= BC^2\\ (X_C - X_B)^2 + (Y_C - Y_B)^2 &= AC^2\end{aligned} \tag{12}$$

Lengths AC, BC and AE, DE are determined from the triangles ABC and ADE for the 2 DOF mechanism in the reference position and employing $\alpha_1 \ldots \alpha_4$ design parameters. A good synthesis can be performed using this objective function [8], but we further give a corrected form of it, based on the same observation that we actually search for a minimum in the output error:

$$F_{21}(..) = \left\| \frac{\delta\alpha}{\dfrac{OB\cos\beta_B^*}{AC\cos\beta_C^*} - \dfrac{OD\cos\beta_D^*}{AE\cos\beta_E^*}} \right\| = \|\delta\varphi^*_{out}\|. \tag{13}$$

The relation for approximate output error $\delta\varphi^*_{out}$ is obtained from the (14) given system of equations. In Fig. 3 is drawn the equivalent mechanism obtained from the one in Fig. 2 by stiffening the element O_1A. Considering a $\delta\varphi_{out}$ input at the ternary (OBD) link, based on the same PROJECTION THEOREM, the corresponding relative motion between links AC and AE is obtained.

$$\begin{aligned}\frac{\delta\varphi^*_{out}}{\delta t} OB\cos\beta_B^* &= \frac{\delta\varphi_{AC}}{\delta t} AC\cos\beta_C^*\\ \frac{\delta\varphi^*_{out}}{\delta t} OD\cos\beta_D^* &= \frac{\delta\varphi_{AE}}{\delta t} AE\cos\beta_E^*\\ \delta\varphi_{AC} - \delta\varphi_{AE} &= \delta\alpha\end{aligned} \tag{14}$$

The angles β^* in equations (14) can be calculated using a relation similar to (8), the corresponding increase in CPU time being very small.

In case of the STEPHENSON II mechanism with a ternary (OBD) input link, the calculation of an objective function similar to F_{20} is the same. The relation of an homologous to F_{21} objective function will be different from that in (13), and can be obtained using PROJECTION THEOREM.

The STEPHENSON III function generator synthesis

In case of STEPHENSON III mechanism with an adjacent to the input member ternary link, the displacement equation can be obtained analytically, otherwise a proper inversion can be applied.

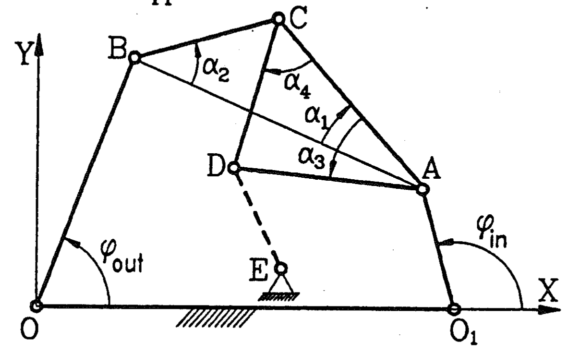

Fig. 4

Use of the IDFM method is not dependent on which of the links is the input member. The 2 DOF associated mechanism is shown in Fig. 4. It can be seen that the chosen dismounted rod is DE, which facilitate searching for those parameters on which the joint D draws a curve approximating to an arc of circle. The center of this approximating circle will give the coordinates of the joint E, while the length of the rod DE will be the radius of this circle. There is also the possibility to dismount rod CD or BC instead of DE, with the disadvantage that both X_E and Y_E coordinates should be considered as design parameters (the synthesis problem being one of minimizing the angle CAD or lengths CD or BC respectively).

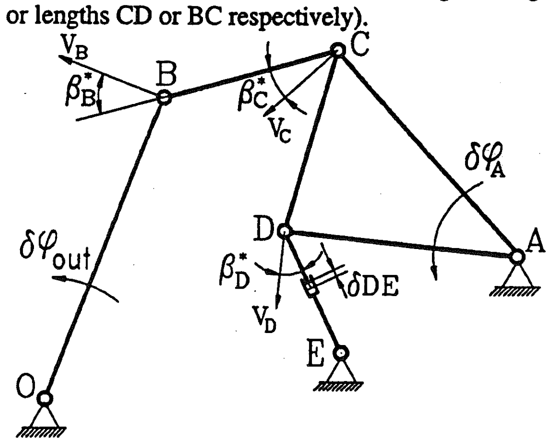

Fig. 5

Regarding the 2 DOF mechanism in Fig. 4, and the trajectory of point D, can be considered the following design parameters: lengths OO_1, O_1A, OB, angles α_1, α_2, α_3, α_4, φ_{ins} φ_{outs}, $\Delta\varphi_{in}$, $\Delta\varphi_{out}$, and a reference position φ_{in0}. In this reference position are calculated the lengths

AC, BC, AD and DC. The coordinates of joint C and D are obtained each time by solving equations similar to (12). The coordinates of joint E have been considered as the center of curvature of the trajectory of joint D in the neighborhood of this reference position, while the radius of curvature is the length DE_0. Further a deviation of distance can be calculated, and a first objective function is defined as:

$$F_{30} = \left\| DE - DE_0 \right\| = \|\delta DE\| \quad (15)$$

Similar to the four-bar and STEPHENSON II function generators, we introduce a corrected objective function

$$F_{31}(..) = \left\| \frac{\delta DE \cdot AC \cdot \cos\tau_C^*}{AD\cos\tau_D^* \cdot OB\cos\tau_B^*} \right\| = \left\|\delta\varphi^*_{out}\right\| \quad (16)$$

In Fig. 5 is given the mechanism resulting from the 2 DOF one, by stiffening the O_1A link, used in deducing the expression of $\delta\varphi^*_{out}$. The corresponding approximate relations based on the PROJECTION THEOREM are :

$$\frac{\delta\varphi_A}{\delta t} AD\cos\beta_D^* = \frac{\delta DE}{\delta t}$$
$$\frac{\delta\varphi_A}{\delta t} AC\cos\beta_C^* = \frac{\delta\varphi^*_{out}}{\delta t} OB\cos\beta_B^* \quad (17)$$

In this case a correction of the F_{30} function proved to be necessary, at least by dividing δDE with AD, to avoid the convergence to the case in which joint D coincides with joint A for the 2 DOF mechanism (which is the absolute minima).

For the case of the mechanism with a OB input member, the coordinates of joint D are calculated in the same way, some differences appearing in the expression of the corrected objective function.

Results and conclusions

Based on the IDFM method described, the authors have made some numerical applications and have synthesized the four-bar, STEPHENSON II and STEPHENSON III mechanisms to generate the same function as in referred paper [3]:

$$y = -x/8(x+2) \quad (18)$$

and considering the same: $-6 \le x \le 6$ range of x, fixed values of angles φ_{ins}=80°, φ_{outs}=-20°, and fixed ranges $\Delta\varphi_{in}$=90°, $\Delta\varphi_{out}$=90° of the input and output members respectively.

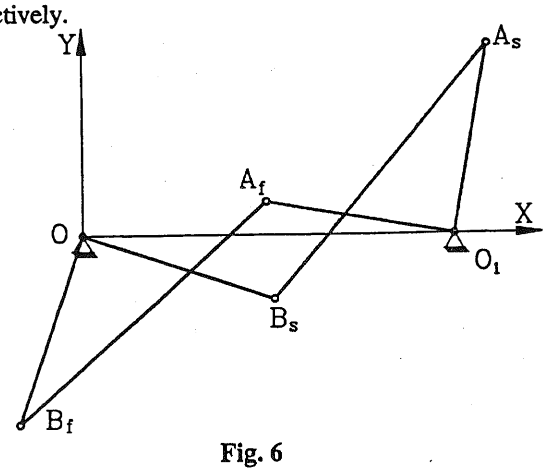

Fig. 6

The four bar mechanism obtained is shown Fig. 6. In this case only, the authors have imposed restriction upon transmission angles. During the search, the approximate τ_A^* and τ_B^* are calculated, and it proved to be necessary that a smaller value (for instance 29° instead of 30°) had to be taken for comparison in a penalty function.

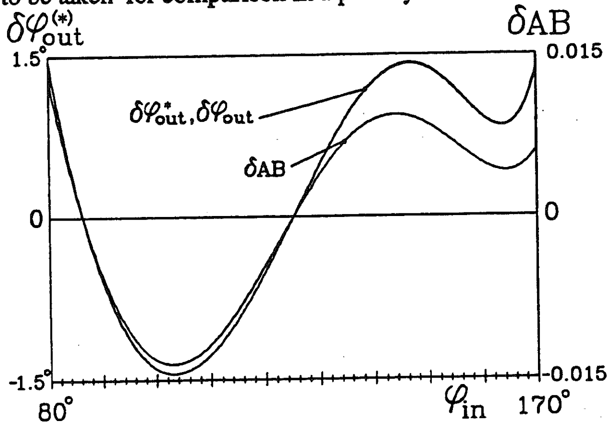

Fig. 7

The parameters of the mechanism obtained as optimum are OO_1=1, O_1A=0.51779, OB=0.53899, AB=0.89417, while the maximum output error is of 1.4486° (obtained for φ_{in}=103.26°) and the minimum transmission angles are τ_A=30.350° and τ_B=29.153° (exactly calculated values). A whole range of graphical representation of δAB, $\delta\varphi^*_{out}$ and $\delta\varphi_{out}$ are given in Fig. 7. It is to be observed that the mechanism obtained is of only 2 exact points, while that in paper [3] is of 4 exact points (and close as proportions), but of less precision (1.9 ° maximum output error).

For the same function, initial angles and ranges of x, $\Delta\varphi_{in}$ and $\Delta\varphi_{out}$ a STEPHENSON II function generator has been synthesized. A scale representation of the optimum mechanism and graphics of $\delta\alpha$, $\delta\varphi^*_{out}$ and $\delta\varphi_{out}$ are given in Fig. 8 and 9 respectively.

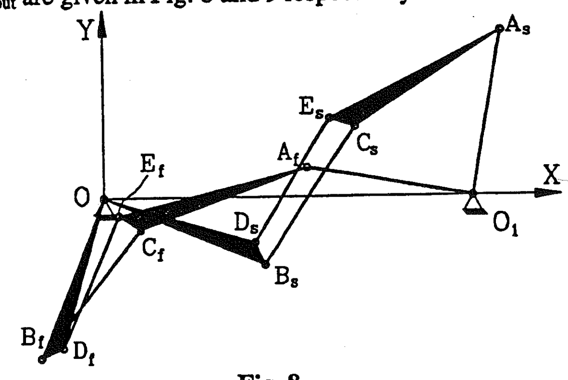

Fig. 8

The corresponding parameters obtained as optimum are: OO_1=1, O_1A=0.45687, OB=0.46852, OD=0.42294, AC=0.47878, BC=0.44565, AE=0.52472, α_5=-6.2541° and α=6.1743°. The rigorous value of $\delta\varphi_{out}$ was determined by solving (in the neighborhood of the corresponding $\delta\varphi^*_{out}$) the equation:

$$\alpha(\varphi_{inREAL}) - \alpha_0 = 0. \quad (19)$$

The maximum output error is of 0.8316° obtained for φ_{in}=159.174°.

Finally, the authors have synthesized the STEPHENSON III mechanism, to generate the same function and imposing the same conditions.

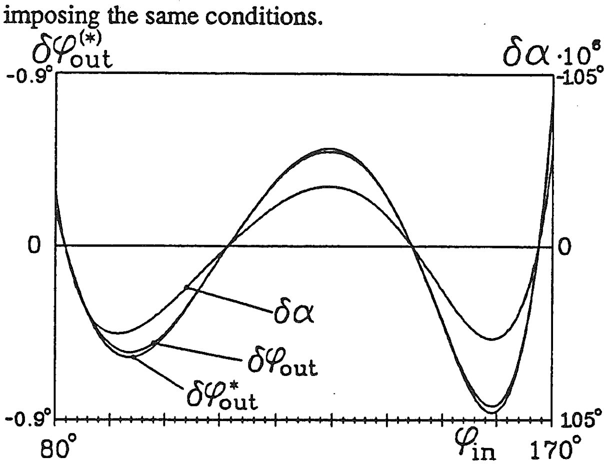

Fig. 9

The mechanism obtained and graphical representation of δDE, $\delta\varphi^{*}_{out}$ and $\delta\varphi_{out}$ are given in Fig. 10 and 11. The corresponding optimum parameters are: $OO_1=1$, $O_1A=0.51989$, $OB=0.53327$, $AC=0.48216$, $BC=0.44379$, $AD=0.13650$, $DC=0.35986$, $DE=0.75187$ $X_E=1.04814$ and $Y_E=0.39954$. This mechanism assures a maximum output error of 0.6091° for $\varphi_{in}=159.212°$.

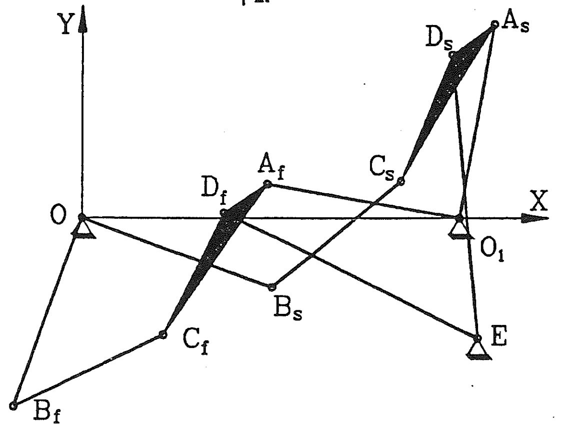

Fig. 10

For this mechanism, the rigorous values of φ_{out} were calculated using the displacement equation of the mechanism, available in this case in an analytical form.

In all three cases, the exact values of maximum error were calculated employing some post-optimal subroutines that minimize the function $F(\varphi_{in}) = -|\delta\varphi_{out}|$.

From the graphical representations in Fig. 7, 9 and 11 it can be seen that $\delta\varphi^{*}_{out}$ is a good approximation of the real output error. The same it can be seen that the shape of δAB, $\delta\alpha$ and δDE resembles respective $\delta\varphi_{out}$ but there are greater differences between them (without considering the units, by proper scaling of say δAB, it can less exactly be superpose over corresponding $\delta\varphi_{out}$) so the introduction of the corrected objective functions F_{11}, F_{21} and F_{31} have good sense.

All the results have been obtained for n=90 intermediate positions, and employing a simple MONTE-CARLO searching algorithm. From this point of view, advanced optimization subroutines can lead to better solutions.

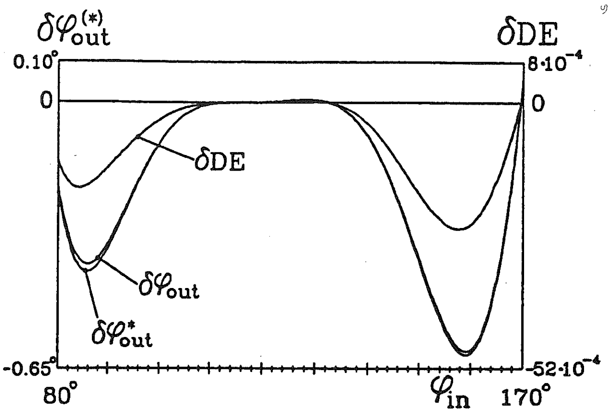

Fig. 11

Acknowledgments

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