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KINEMATIC SYNTHESIS AND ANALYSIS OF FIVE-LINK REAR WHEEL SUSPENSION SYSTEM OF AUTOMOBILES

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Introduction

The rear independent wheel or axle guiding mechanisms importance upon car stability and ride comfort is well accepted. They are, in the sense of mechanisms theory, spatial motion generators (also called rigid body guidance mechanisms). The research work on motion generators analysis and synthesis has been carried out on both abstract and applied mechanisms by several researchers in the past.

An exact point approach is proposed by Suh and Kang for independent wheel guiding mechanisms synthesis [3], [9]. In the earlier published paper [9], two positions are considered of combined finite and instantaneous displacements for an RSSR-SS double wishbone suspension mechanism, by using displacement, differential displacement matrices and constraint equations. Further on, in paper [3], the synthesis of an RSCS-SS Mc Pherson strut suspension mechanism has been performed for three finite positions of the spindle-tire assembly by using the same displacement matrix method.

A general formulation of the mechanism synthesis problem is proposed by Aviles et al for path, function and rigid body guidance, based on optimization techniques [1]. However, numerical examples given refer to the synthesis of plane four bar path generators only. In this cited paper, a global error function to be minimized is defined as a weight sum of some local error functions. These local error functions are previously minimized with respect to the basic points Cartesian coordinates of the mechanism. The so called "basic points" are the centers of the joints and the points on the links required to generate certain paths through the working range of the mechanism. However, the big number of variables in an objective function can be disadvantageous, as well as the exclusion from the design parameters of the coordinates positioning the ground joints.

An interesting approach to the RSSR-SS spatial motion generators design is that of Shandor et al [7], where a combination of exact and approximate synthesis is performed. According to the method, a part of the prescribed points are considered exact points and the corresponding synthesis equations are solved. The free choices in these equations are further considered design variables in an objective function, penalized with the conditions of avoiding branching, achieving correct sequence of prescribed positions and observing the shortest and longest links ratios to be within the prescribed limits. The objective function is defined as a weighted sum of the motion errors calculated for each link. For the weight factors, the authors of the paper haven't make any recommendations of how to be chosen, their values being essential in gaining appropriate results for the synthesized mechanism.

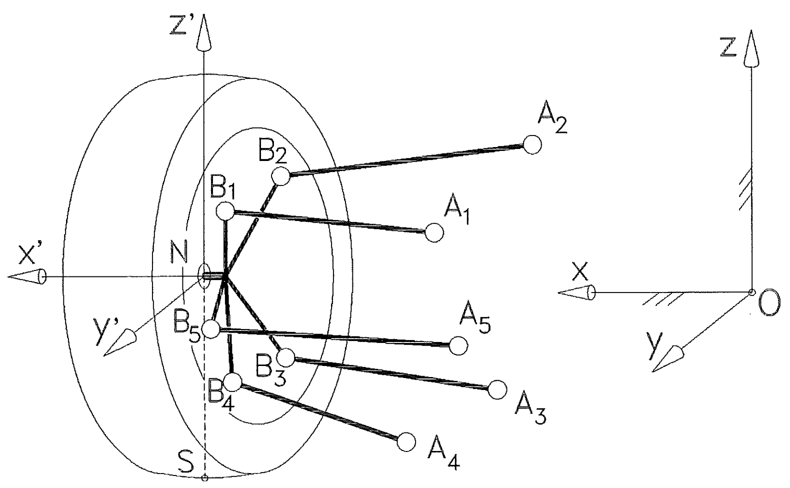


Fig. 1

Rear wheel guiding mechanism working requirements and problem formulation

The aim of this paper is to present a rapid synthesis method for the five-link rear wheel guiding mechanism shown in **Fig. 1**, and some numerical results obtained by applying it.

According to the referred paper [2], the requirements upon rear wheels movement, which can be transposed into kinematic synthesis conditions of the suspension mechanism, are:

-minimum steering like rotation during compression and rebound;

-avoid excessive outward camber thrusts on corners;

-avoid excessive sideways thrust and consequent rear end steering impulses on single wheel bump or rebounds.

Supplementary, the suspension elements must assure a minimum intrusion into the passengers and luggage accommodation. This last condition can be translated into some constrains upon the joints placement on the chassis and on the wheel carrier.

The contribution of the elasticity of the rear wheel suspension upon the car ride behavior is important and in a dynamic analysis they must be necessary considered. However, in order to simplify the kinematic formulation of both the synthesis and analysis procedures, we make the assumptions that the joints have no clearances nor elasticity and also that the vehicle chassis and the suspension elements are rigid.

Taking the first three above mentioned conditions, it seems that the ideal wheel movement during its operation travel must be a pure vertical translation relative to the car body; thus the 5(S-S) suspension mechanism synthesis can be formulated as an optimization problem i.e. of finding the minimum of the following objective function (**Fig. 1**):

$$F\left(X_{Ai}, Y_{Ai}, Z_{Ai}, X'_{Bi}, Y'_{Bi}, Z'_{Bi}\right) = \sum_{i=1}^{5}\sum_{j=1}^{n}\left(l_{i0} - \left(A_iB_i\right)_j\right)^2 \qquad (1)$$

for j =1..n intermediate positions of the wheel carrier (released from its five links) on the mentioned ideal trajectory. We have noted with l_{i0} a reference length of the i-th link, determined as the distance between the joints A_i and B_i for the wheel in an initial position (corresponding to the car unloaded and in rest) for which (*xyz*) and (*x'y'z'*) reference frames axes are parallel and the origin N of the one attached to the wheel carrier has the coordinates X_{N0}, Y_{N0} and Z_{N0}. In case of the coordinates of the ball joints B_i given relative to the reference frame $(x'y'z')$, the following transformations must be applied in order to evaluate $(A_iB_i)_j$ distances:

$$\begin{bmatrix} X_{Bi} \\ Y_{Bi} \\ Z_{Bi} \end{bmatrix}_{xyz} = \begin{bmatrix} X'_{Bi} \\ Y'_{Bi} \\ Z'_{Bi} \end{bmatrix}_{x'y'z'} + \begin{bmatrix} X_{N0} \\ Y_{N0} \\ Z_{N} \end{bmatrix}_{xyz} \tag{2}$$

The variable coordinate $Z_N = Z_{N0} \pm j\ \Delta Z_N$ is calculated considering a step $\Delta Z_N = (Z_{Nmax} - Z_{Nmin})/n$ and the same j =1..n discrete positions. For j = 0 relation (2) permits the calculation of the coordinates of joints B_i needed to evaluate the reference lengths l_{i0}.

The inequality constrains considered are of the type:

$$\begin{array}{llllll} X_{Ai\min} \le X_{Ai} \le X_{Ai\max} & \text{and} & X_{Bi\min} \le X_{Bi} \le X_{Bi\max} \\ Y_{Ai\min} \le Y_{Ai} \le X_{Ai\max} & \text{and} & Y_{Bi\min} \le Y_{Bi} \le Y_{Bi\max} \\ Z_{Ai\min} \le Z_{Ai} \le X_{Ai\max} & \text{and} & Z_{Bi\min} \le Z_{Bi} \le X_{Bi\max} \end{array} \tag{3}$$

and are taken from the chassis and wheel carrier blueprints and express some constructive limitations.

The objective function F(..) together with the restrictions (3), can be minimized using a proper optimization subroutine. The design parameters are 30 of maximum but their number can be reduced by keeping some of the coordinates fixed, or by imposing zero searching spans to some of the restrictions above.

Analysis of the optimum obtained mechanism

The guiding mechanism under consideration has 6 degree of freedom, of which 5 are trivial rotations of the links around the line determined by their ball-joint centers. Correspondingly the motion of the rigid body is described by one independent parameter. Let this be the coordinate Z_N of the origin N of the $(x'y'z')$ reference frame, relative to (xyz). This means that the other five parameters that describe the position and orientation of the guided rigid body can be determined by solving the (4) given system of the equations of constrains, which express the condition of the distance between joints A_i - B_i to remain constant during the working travel of the mechanism:

$$(X_{Ai} - X_{Bi})^2 + (Y_{Ai} - Y_{Bi})^2 + (Z_{Ai} - Z_{Bi})^2 = l_{i0}^2 \qquad (i = 1..5) \tag{4}$$

where the coordinates X_{Bi}, Y_{Bi} and Z_{Bi} are determined by applying a general transformation to the $(x'y'z')$ reference frame attached to the wheel spindle assembly.

$$\begin{bmatrix} X_{Bi} \\ Y_{Bi} \\ Z_{Bi} \end{bmatrix} = \left[R_{\beta\alpha\gamma}\right] \begin{bmatrix} X'_{Bi} \\ Y'_{Bi} \\ Z'_{Bi} \end{bmatrix} + \begin{bmatrix} X_{N} \\ Y_{N} \\ Z_{N} \end{bmatrix} \tag{5}$$

$[R_{\beta\alpha\gamma}]$ is the spatial rotation matrix [8] obtained by successively rotating the wheel carrier relative to the fixed frame by the pitch angle* β, yaw angle α and roll angle γ, and which have the form:

$$\left[R_{\beta\alpha\gamma}\right] = \left[R_{\gamma,x}\right]\left[R_{\alpha,z}\right]\left[R_{\beta,y}\right] = \begin{bmatrix} C\alpha C\beta & -S\alpha & C\alpha S\beta \\ S\alpha C\beta C\gamma + S\beta S\gamma & C\alpha C\gamma & S\alpha S\beta C\gamma - C\beta S\gamma \\ S\alpha C\beta S\gamma - S\beta C\gamma & C\alpha S\gamma & S\alpha S\beta S\gamma + C\beta C\gamma \end{bmatrix} \tag{6}$$

* Yaw α, pitch β, and roll γ angles are different from those describeing the car body movement !

where $[R_{\alpha,z}]$, $[R_{\beta,y}]$ and $[R_{\gamma,x}]$ are the basic rotation matrices [8], and $C\alpha=\cos\alpha$, $S\alpha=\sin\alpha$, and so forth. The (4) system of equations with the unknowns α, β, γ, X_N and Y_N can be solved by minimizing the following objective function:

$$F_0(\alpha,\beta,\gamma,X_N,Y_N)=\sum_{i=1}^{5}\left[(X_{Ai}-X_{Bi})^2+(Y_{Ai}-Y_{Bi})^2+(Z_{Ai}-Z_{Bi})^2-l_{i0}^2\right]^2 . \qquad (7)$$

The solution of the system of equations (4) corresponds to the global minimum of $F_0(..)$ which will be zero only if the independent parameter Z_N is in the working range of the analyzed mechanism. This function is differentiable and an efficient gradient based optimization procedure can be employed. The starting point in the searching algorithm may be the previously determined minimum point (considering a successive position analysis of the mechanism with $Z_N = Z_{N0} \pm j\,\Delta Z_N$ and starting twice from j=0) or, the ideal position from the synthesis corresponding to the same discrete Z_N bump-rebound intermediate positions

Results and conclusions

Based on the described procedure, the synthesis of a five-link rear wheel independent suspension system was performed; some numerical data taken from paper [4] and corresponding to Mercedes 190 rear suspension components and have been used in describing the searching domains of the joint centers coordinates i.e.:

Tab. 1.

$190 \le X_{A1} \le 225$	$92 \le Y_{A1} \le 112$	$221 \le Z_{A1} \le 241$
$486 \le X_{A2} \le 506$	$-331 \le Y_{A2} \le -311$	$241 \le Z_{A2} \le 261$
$394 \le X_{A3} \le 414$	$-219 \le Y_{A3} \le -199$	$286 \le Z_{A3} \le 306$
$427 \le X_{A4} \le 457$	$-219 \le Y_{A4} \le -199$	$392 \le Z_{A4} \le 412$
$346 \le X_{A5} \le 366$	$-5 \le Y_{A5} \le 15$	$406 \le Z_{A5} \le 426$
$-53 \le X_{B1} \le 672$	$33 \le Y_{B1} \le 53$	$-110 \le Z_{B1} \le -84$
$-83 \le X_{B2} \le 642$	$-54 \le Y_{B2} \le -34$	$-149 \le Z_{B2} \le -129$
$656 \le X_{B3} \le 676$	$-151 \le Y_{B3} \le -131$	$-43 \le Z_{B3} \le 4$
$652 \le X_{B4} \le 672$	$-88 \le Y_{B4} \le -68$	$85 \le Z_{B4} \le 105$
$656 \le X_{B5} \le 676$	$-5 \le Y_{B5} \le 15$	$115 \le Z_{B5} \le 135$

The origin of the (*x'y'z'*) coordinate system in the reference position is $N_0(705,0,302)$ and the tyred wheel radius, that determines the coordinates of the center of the road-tyre contact patch S is R=302 mm. By imposing to the wheel carrier a vertical travel of ±125 mm, the following optimum parameters have been obtained, of which the ball-joint coordinates have been rounded to the closest practical values:

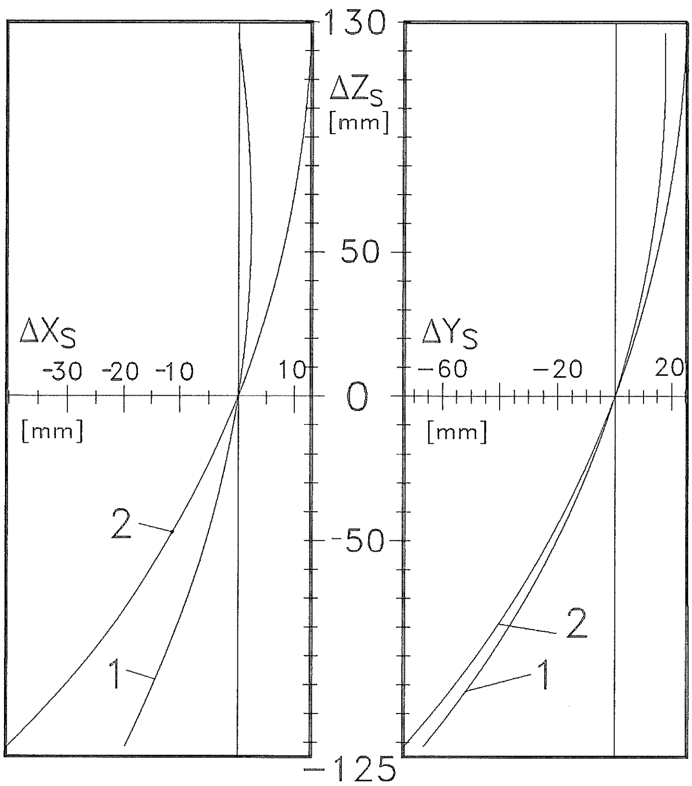
Tab. 2.

	A_1	A_2	A_3	A_4	A_5		B_1	B_2	B_3	B_4	B_5
x:	192.0	486.0	395.0	427.0	353.0		-37.0	-63.0	-29.0	-34.0	-63.0
y:	95.0	-330.0	-216.0	-210.0	-3.5		39.0	-34.0	-132.0	-77.0	-4.5
z:	228.0	241.0	287.0	398.0	419.0		-89.0	-129.0	-23.0	95.0	115.5

l_1 = 479.576 mm	l_2 = 341.638 mm	l_3 = 292.582 mm	l_4 = 277.217 mm	l_5 =289.118 mm

In **Fig. 2** are given the road-tyre path center S lateral and longitudinal variations ΔX_S and ΔY_S versus ΔZ_N and can be seen that are of smaller values as compared to the original solution. The corresponding α and β angles variations, which describe the undesired wheel plane vertical rotations, are still kept within small limits (less than 0.1 of a degree) i.e.:

proposed mechanism:	$0.078° \le \alpha \le 0.035°$	$-0.026° \le \beta \le -0.059°$	for	$-121.5 \le \Delta Z_S \le 126.1$ mm
initial mechanism:	$0.033° \le \alpha \le 0.003°$	$0.037° \le \beta \le 0.101°$	for	$-121.8 \le \Delta Z_S \le 126.3$ mm



1–proposed mechanism
2–initial mechanism (MERCEDES 190)

Fig. 2

The precision of the determination of the wheel-carrier position was estimated by calculating the difference:

$$\delta_{ij} = \left(A_i B_i\right)_j - l_{i0} \qquad (8)$$

In case of the optimization algorithm chosen by the author (based on the simplex method of Nelder and Mead and for n=40 intermediate positions), the values of δ_{ij} vary from 10^{-13} to 10^{-15}, which is quite satisfying.

As a general conclusion, it can be observed the simplicity of the approach, for both synthesis and analysis, as well as the generality of the problem formulation. This permits for example, (if the axle roll movement relative to the car body is not wished to be controlled) the synthesis of a similar 5(S-S) guiding mechanism of a rigid axle suspension system [10]. However, in this case, the joints elasticity must be some how considered, because their contribution is essential to a combined rotational translational real motion of the axle (the ideal mechanism with no elastic elements having only one degree of freedom).

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DIE KINEMATISCHE SYNTHESE UND ANALYSE DER SELBSTÄNDIGEN FÜNF-TEILIGEN HÄNGEWERKSYSTEME DER FAHRZEUGHINTERRÄDER

ZUSAMMENFASSUNG

In der Arbeit bietet man eine einheitliche Methode für kinematische Synthese und Analyse der 5(K-K) Raumgetrieben der selbständigen Fahrzeughinterräder dar. Die Synthese des Mechanismus ist als eine Optimierungsfrage formuliert, wo die Objectivfunktion die Abweichung gegenüber einer idealen, senkrechten, echten Translationbewegung des Radträgers beschreibt, und wo die Einschränkungen baumäßig auferlegte Grenzen, der Kugelgelenke vom Fahrgestell und vom Radträger sind. Die Analyse der realen Bewegung des optimalen Mechanismus verwirklicht man auch durch die Anwendung eines Optimierungalgorithmus. Am Ende sind einige Ergebnisse, die man für ein konkretes Fahrzeug erhalten hat, dargeboten.