Calculus II Final, MATH 2414.001, Fall 2002 NAME:

Exam Time: 11 AM to 1:30 PM, Tuesday, December 10. Each of the 6 numbered problems is worth a possible 20 points. There are 120 points possible on the test; your score will be recorded out of 100. You may use calculators, text, homework, and notes: any material except another person or another person's work. Good luck!

1. For each of the definite integrals below, find a number larger and a number smaller than the actual value so that the larger and smaller numbers differ by less than 0.1. For each of your answers, explain carefully which method you used to obtain it, and show how you know the answer is as accurate as required.

a) $\int_{\pi/2}^{5\pi/2} e^{\sin x}\, dx$ b) $\int_{0}^{10} \ln(x^2+1)\, dx$

2. The density of a compressible liquid it 40(5 – h) kg/m^3 at a height of h meters above the bottom. The liquid is put into the two containers pictured below. The shapes are triangular prisms with cross sections being isosceles triangles. What is the mass (in kilograms) of the liquid that fills each container?

(In both cases the shapes are triangular prisms with length 10 meters. The base of the isosceles triangle is 1 meter and the height of the isosceles triangle is 1 meter.)

3. A repeating decimal can always be expressed as a fraction. This problem shows how writing a repeating decimal as a geometric series enables you to find the fraction.

a) Use the fact that 0.47474747… = 0.47 + 0.0047 + 0.000047 + … to write the decimal number 0.47474747… as a geometric series. Use the formula for the sum of a geometric series to express the decimal number 0.47474747… as a fraction.
b) Use the fact that 0.6474747… = (1/10)*(6.474747…) = (1/10)*(6 + 0.474747…) to express the decimal number 0.6474747… as a fraction.
c) Write .43825825825… as a fraction

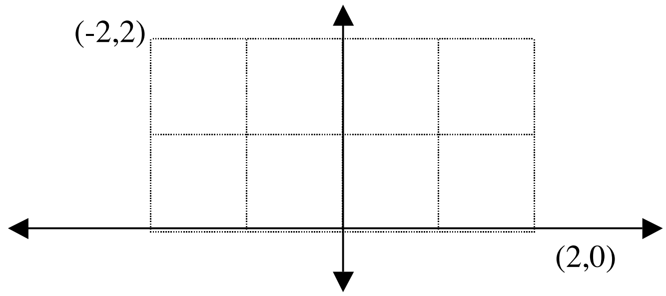
4. Consider the function f(x) = 1 + cos x.

a) Find the Taylor series for f(x) for x near π.
b) Based on the Taylor series from part (a), what do you conclude about the value of the following limit: $\lim_{x \to \pi} \frac{1+\cos x}{(x-\pi)^2}$

5. This problem concerns the differential equation $\frac{dy}{dx} = \frac{2x}{y}$

Note: You can do part (d) without parts (a)-(c). I can provide a solution to (a) at the cost of credit for that part of the problem.

a) Sketch the slope field for the differential equation at the 14 points in the first and second quadrants: $-2 \le x \le 2$; $0 \le y \le 2$ where x and y are integers and (0, 0) is excluded.



b) Sketch in a solution curve passing through (1,2). Do you expect it to show any symmetry? Explain

c) Without plotting any more slopes in the third and fourth quadrants, describe what the slope field must look like there, in terms of the slope field in the first and second quadrants.

d). Find an equation for the solution to the differential equation $\frac{dy}{dx} = \frac{2x}{y}$ passing through the point (1,2). Is it consistent with your equation to part (b)?

6. TRUE/FALSE: For each statement, circle true or false. Each correct answer will earn +2 points. Each incorrect answer will earn –2 points. Questions left unanswered earn 0 points. The total points, somewhere between –20 and 20, will be rounded up to zero if it is negative.

A. **TRUE FALSE** If the left-hand sum, LEFT(n) for $\int_a^b f(x)dx$ is too large for one value of n, it will be too large for all values of n.

B. **TRUE FALSE** $y = x^2 + x$ is a solution to $\frac{dy}{dx} = 2(y - x^2) + 1$

C. **TRUE FALSE** If $\sum a_k = a_0 + a_1 + \cdots$ is the sum of a series of numbers and $\lim_{x \to \infty} a_k = 0$, then the series converges.

D. **TRUE FALSE** If a power series $\sum a_k x^k = a_0 + a_1 x + \cdots$ converges at x = 1 and at x = 2, then it converges at x = -1.

E. **TRUE FALSE** The solution of $\frac{dy}{dx} = x + 1$ passing through (0,1) is the same as the solution passing through (0,0), except that it has been shifted one unit upward.

F. **TRUE FALSE** $\int_1^2 \sin x^2 \, dx > 3$

G. **TRUE FALSE** If the average value of f(x) on the interval [2,5] is between 0 and 1, then f is between 0 and 1 on the interval [2,5].

H. **TRUE FALSE** The solutions of $\frac{1}{P}\frac{dP}{dt} = k(L - P)$, where k and L are constants, are always concave down.

I. **TRUE FALSE** If $f'(x) > g'(x)$ for all $a < x < b$, then the left-hand Riemann sum approximation of $\int_a^b f(x)dx$ will have larger error than the left-hand Riemann sum for $\int_a^b g(x)dx$