NAME: _____

MATH 1470   Fall 2003 Tintera

TEST 3: Two-Species, Epidemic and Linear Programming Models. Covers Chapters 7-9

You may use calculators and one 8.5 by 11 inch page of notes. Please show all work on this test booklet. Partial credit is awarded only for work shown. Each problem is worth as indicated. Good luck!

1. An incomplete S-I-R model for a measles epidemic is as follows: $\dfrac{\Delta I}{\Delta t} = 0.0024\,SI - 0.33I$

a) (10 points) Write the change equations for S and R to complete the model

b) (5 points) According to this model, how long does it take to recover from the measles?

c) (5 points) On one day there are 1000 Susceptible people and 20 Infected and none are Recovered. How many will there be the next day in each category.

d) (5 points) For which values of S will I increase (ie, have $\Delta I > 0$) ?

2. Consider the following two-species model:

$$\frac{\Delta x}{\Delta t} = 0.7x - 0.003x^2 - 0.001xy, \qquad \frac{\Delta y}{\Delta t} = 0.6y - .004y^2 - 0.006xy$$

a) (5 points) Is either species logistic? How can you tell?

b) (5 points) Is this a predator prey model or competing species model? How can you tell?

c) (5 points) If the starting populations are x = 100 and y = 100, will the species x and y increase or decrease in the near future?

d) (10 points) Either
      (i)   Compare and contrast the two species in terms of their equations. How do they compare with regard to interactions? How do they compare with regard to other terms?
      (ii)   Or Sketch a phase plane for the two species.

3. You and your roommate got a contract to advertise local sports franchises this summer. You will walk around with a sandwich board showing off arena football on the front and minor league baseball on the back. Each minute you spend downtown, you will be seen by 5 people interested in football and 12 people interested in baseball. Each minute spent at the beach will expose you to 10 people interested in football and 7 people interested in baseball. You will be seen by 1 person interested in football and 5 interested in baseball for each minute spent at the university. Your contract requires you to be exposed to 1500 people interested in football and 2000 people interested in baseball. Also, you need to spend more time at the university than at the beach. Of course, you want to spend part of your time this summer doing other things. Decide how to minimize the total time you spend wearing the sandwich board.

    DO NOT SOLVE THIS PROBLEM. Instead name:

a) (6 points) Appropriate variables to solve the problem (eg, x1 = inches of rain, x2 = miles of flooded streets, x3 = dollars in flood damage).

b) (7 points) An objective function (with formula) and indicate whether it should be minimized or maximized.

c) (12 points) Constraints on the solutions to the objective function. You don't need to include non-negativity constraints.

4. (25 points) For the linear program

$$\text{Maximize } Z = 7\ x + 6\ y$$

subject to

$$14\ x + \quad 5\ y \le 70$$
$$12\ x + 10\ y \ge 60$$
$$x,\ y > 0$$

solve the problem by graphing. Partial credit is awarded for work shown. Solution: $Z =$ _____   is a maximum for $x =$ _____ and $y =$ _____