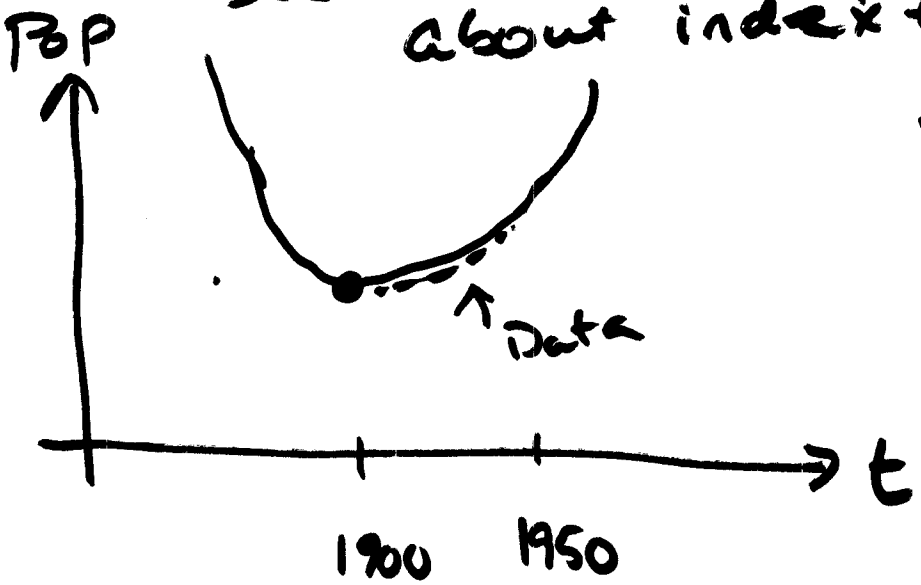
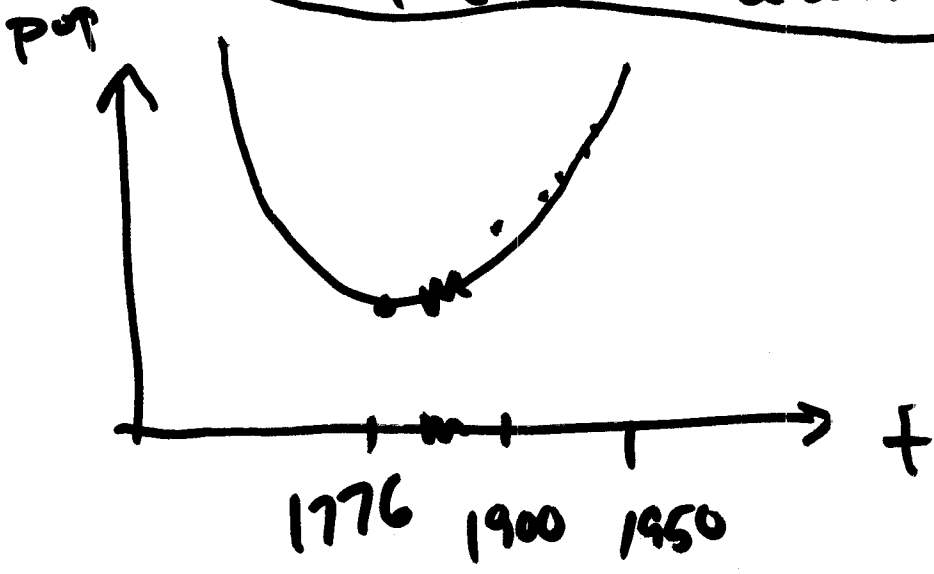
10-18-05

→ See note on last page about index formula ←



In this example data is close to vertex.

In this case the model clearly shows the pop. decreasing & then increasing.



Here vertex is far from data points

So the vertex has no real meaning.

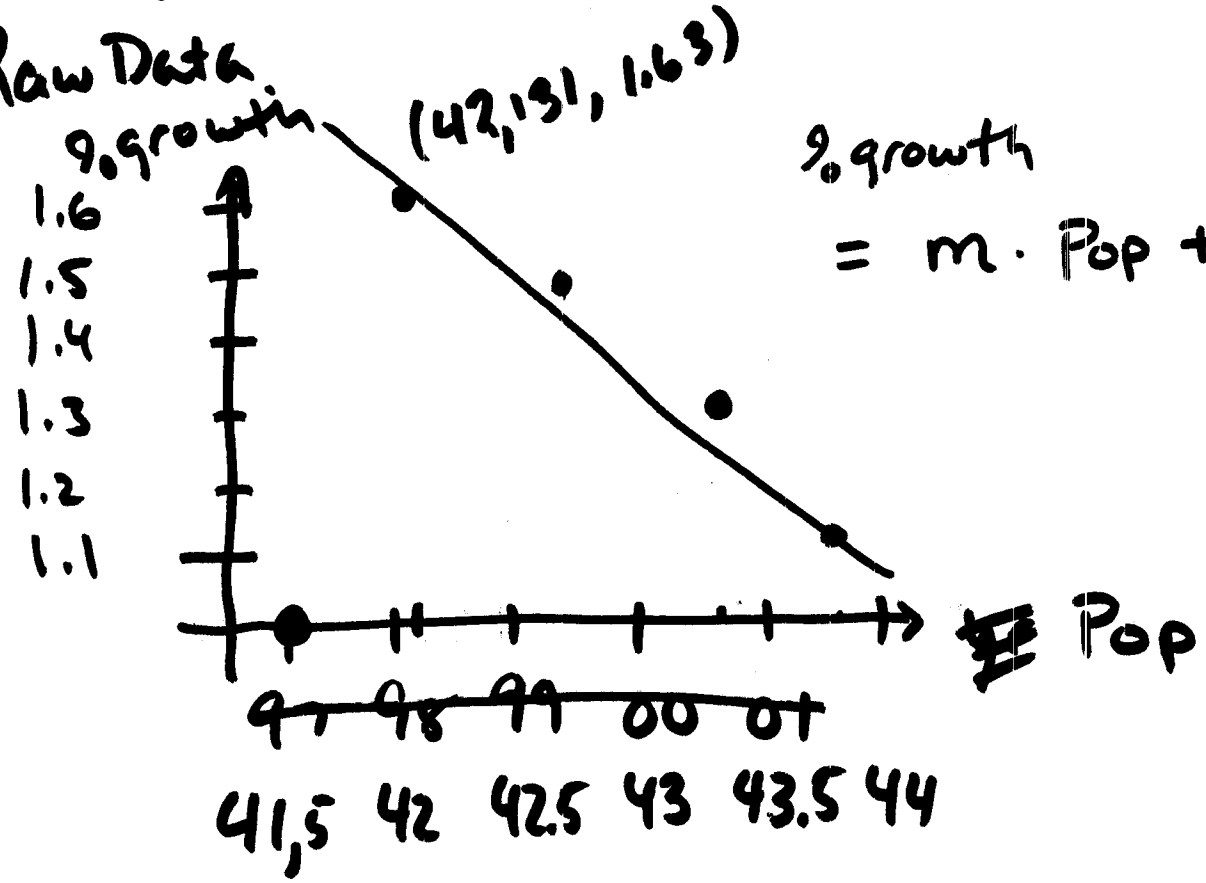
Lab F Logistic Models

Given Data, how to find

p314 r, L - parameters in model.

t	Pop	%growth rate = ΔPop/Pop
97	41,456	
98	42,131	(42131 − 41,456) ÷ 41456 = 1.63%
99	42,754	1.48
00	43,309	1.3
01	43,792	1.12

Raw Data



%growth
= m · Pop + b

$$\frac{\Delta Pop}{\Delta t} = r \cdot Pop\left(1 - Pop/L\right)$$

Let $\Delta t = 1$ Divide by Pop, distribute r

$$\%growth = \frac{\Delta Pop}{Pop} = \left(-\frac{r}{L}\right) Pop + r$$

So $r = b$ $\quad -r/L = m$

$$L = -\frac{r}{m} = -\frac{b}{m}$$

$r = b =$ ~~3.088~~ $1.466\,E-01$ ← 1 d. place to left

↖ scientific Notation

$= .1466$

$r = 14.669\%$

$$L = \left(\frac{-1.466\,E-1}{-3.088\,E-6}\right) = .475\,E5$$

5 d. places to right.

$-1-(-6)$

$L = 47{,}500$

$$\frac{\Delta y}{\Delta t} = r y (1 - y/L)$$

$\Delta t = 1$

Δy = change in y

= new y - old y

new y - old y = $r y (1 - y/L)$

(old y in $y(1 - y/L)$)

new y = old y + $r \cdot y \cdot (1 - y/L)$

Review Sessions

Wed 1-3 BH 103

Thurs 10-Noon CCH 110

5

NAME: Key

MATH 1470 Fall 2004 Tintera

TEST 2: Malthus, Demographic Transition and Logistic Models. Covers Chapters 5-6

You may use calculators and one 8.5 by 11 inch page of handwritten notes. Please show all work on this test booklet. Partial credit is awarded only for work shown. Each problem is worth as indicated. Good luck!

For the first three questions, choose the best answer by circling the letter for that answer.

1. Which of the following correctly relate Malthus' terminology to modern terminology:

   A. Arithmetic growth is the same as linear growth.
   B. Geometric growth is the same as logistic growth.
   C. Arithmetic growth is the same a exponential growth.
   D. Geometric growth is the same as linear growth.

2. Which of the following is NOT true about indexing of a population

   A. Indexing a population helps make it harder to compare between data sets.
   B. Indexing a population helps make it easier to see growth rates.
   C. Indexing linear data keeps the data linear.
   D. If a year's population is less than the population during the base year, the index will be less than 100.

3. Which of the following is true about demographic transitions:

   A. Birth rates fall in a country before the death rates.
   B. The growth rates rise and then fall.
   C. The birth rate rises as a result of prosperity.
   D. The death rate rises as a result of industrialization.

4. Below are the current birth and death rates for two countries. For each of them find the growth rate expressed as a percentage and your best guess as to the stage of the demographic transition of the given country. Your answer should indicate that you understand what a demographic transition is.

Country 1: Angola

Current Birth Rate: 46.2 births per thousand Current Death Rate: 24.4 deaths per thousand.

a) Current Growth Rate $B - D = 46.2 - 24.4 = 21.8$ people thous.

$= 2.18$ PPO/hundred $= 2.18\%$

b) Stage: Stage II - The birth rate remains high while the death rate has fallen. In moving from stage I, We'd expect the birth rate to fall in Stage III

Country 2: Denmark

Current Birth Rate: 11.7 births per thousand Current Death Rate: 10.8 deaths per thousand.

c) Current Growth Rate 11.7 - 10.8 = .9/thou = .09%

d) Stage: III - Both B & D rates are at low levels, a sign of industrialization.

5. The following data is about the population and food production in Panama:

Year	Population	Food	Food per Capita
1962	1193	46.8	0.039229
1966	1341	54.6	0.040716
1970	1506	69.7	0.046282
1974	1679	76	0.045265
1978	1860	86.5	0.046505
1982	2037	92	0.045164
1986	2212	92.6	0.041863
1990	2398	102.5	0.042744

ΔFood/Δt

(54.6 - 46.8)/4 = 1.95

⋮

(102.5 - 92.6) ÷ 4 = 2.475

a) Does the production of food in Panama match what Malthus said about food production in general? Be clear about what he said, what you see and your conclusion.

M. said Food prod. grows linearly.

It seems that the rate increase of food production increases rather than stays constant. This does not match what M said.

b) Does the country of Panama appear to be suffering from the Post WWII definition of Malthusianism. Be clear about what it is, what you see and your conclusion.

M. M'ism is where Pop outgr in creases faster than Food. From '62 to '90

Since linear growth has a constant rate of change

% growth Pop = (2398 - 1193) ÷ 1193 = 101%

% growth Food = (102.5 - 46.8) ÷ 46.8 = 119%

In Panama, Food has grown faster than Pop. so there's no modern M'ism there.

(7)

6. The number, x, of convenience stores in Corpus Christi seems to be governed by a the logistic model:

$$\frac{\Delta x}{\Delta t} = 0.0625x - 0.0000125x^2$$

a b

a) Find the maximum number of convenience stores in Corpus Christi predicted by the model.

P 120

$L = \frac{a}{b} = \frac{.0625}{.0000125} = 5000$ is the carrying capacity for conv. stores in CC.

b) If there were 80 convenience stores in Corpus Christ one year, how many would there be the next year?

New x = old x + Δx

$= 80 + .0625(80) - .0000125(80)^2$

$= 84.92$

c) When would there be the greatest increase in convenience stores in Corpus Christi?

When $x = L/2 = \frac{5000}{2} = 2500$

7. Sales of sodas at the ALCS baseball game increased about 1.5% per minute and sales per minute peaked after 1300 gallons had been sold.

a) What type of model is appropriate for this situation? Explain.

Logistic - There is an early exponential growth & a peak with later leveling off.

b) Write the equation for the model you chose. Be sure to explicitly define the variables used.

$\frac{\Delta y}{\Delta t} = .015\, y \cdot \left(1 - \frac{y}{2600}\right)$

Peak at $y = 1300$, $L = 2600$

y = # sodas sold

t = time (minutes)

8

8. Below is a spreadsheet of the population and food supply for Libya for the years given.

	A	B	C	D	E	F
1	Year	Libya Pop.	Food	3 Yr Centered Mvg Avg—Pop	Per Capita Food	Indexed Pop (Base = 1970)
2	1962	1452	22.7			
3	1966	1688	35.6			
4	1970	1986	35.8			
5	1974	2344	55.2	?		
6	1978	2783	70.4		?	
7	1982	3536	87.8			?
8	1986	3924	91.3			

1970 →

a) For each of the cells below, show the formulas as they would be entered into an Excel spreadsheet. Where appropriate, put $ signs to indicate values that don't change.

D5: =average(B4:B6)

E6: =C6÷B6

F7: =B7÷B$4*100

not done in class!

Base always same needs $

b) Into which cells in the table above could the formula in cell D5 be copied? You should assume that row 8 is the last row in the table.

D3 to D7