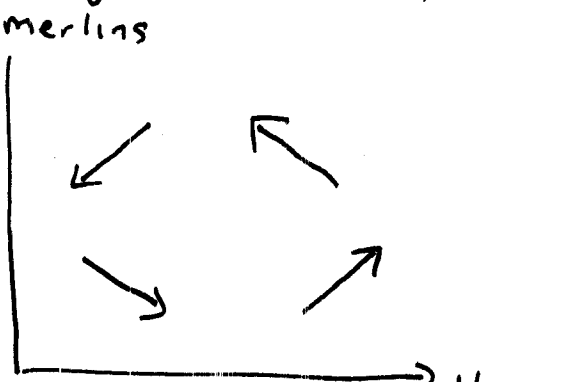
Two Species Models

Predator-Prey Interactions → "Phase Plane" Qualitatively



Lotka Volterra Equations

$$\frac{\Delta r}{\Delta t} = a \cdot r - b \cdot rm$$

$$\frac{\Delta m}{\Delta t} = c \cdot rm - d\,m$$

Excel

Algebra ↓

Better Understand Phase Plane

- Identify Equilibrium Points
- Regions ~~w~~ of consistent change

$$\frac{\Delta r}{\Delta t} = .9r - .03rm$$

$$\frac{\Delta M}{\Delta t} = .00001667rM - .01667M$$

Equilibrium Points: $\Delta r = 0$ & $\Delta M = 0$

$$\begin{cases} .9r - .03rm = 0 & (1) \\ \& \quad .00001667rm - .01667m = 0 & (2) \end{cases}$$

Equation (1) There is an r common to both terms: $r \cdot (.9 - .03M) = 0$

Either $r = 0$ or $.9 - .03M = 0$

$-.03M = -.9$, $M = \frac{-.9}{-.03} = 30$

To keep rats constant either have $r = 0$ or $M = 30$

Equation (2) There is an m common to both terms: $m(.00001667r - .01667) = 0$

So $m = 0$ or $.00001667r - .01667 = 0$, $r = \frac{.01667}{.00001667} = 1000$

$\Delta r = 0$ 2 factors

$\Delta M = 0$ 2 factors

	$r = 0$	$M = 30$
$M = 0$	$(r, M) = (0, 0)$	X
$r = 1000$	X impossible	$(r, M) = (1000, 30)$

$$\frac{\Delta x}{\Delta t} = x - .01xy \qquad \text{(prey)}$$

$$\frac{\Delta y}{\Delta t} = .005xy - .2y \qquad \text{(predator)}$$

$\Delta x = 0$

Eq points

$x = 0$, $\quad y = 100$

$1 - .01y = 0$

$-.01y = -1$

$y = \frac{-1}{-.01}$

$y = 0$ $\quad (0,0)$

$\Delta y = 0$

$y = 100$

$x = 40$

$(40,100)$

$.005x - .2 = 0$

$.005x = .2$

$x = \frac{.2}{.005} = 40$